Exercise for Chapter 1,2

April 5, 2012

1. Construct a **simple** truth table for the statement: $[(\sim q \to p) \wedge \sim p] \to q$.

2. Give the negation of this statement without using the negation notation ($\sim$): For every ε>0, there is an $N \in \mathbb{N}$ such that for each $n \in \mathbb{N}$, if $n \geq N$ then $|a_n - a|<\varepsilon$.

3. The second principle of induction is in the form: Let $Q(\cdot)$ be a property which may or may not hold for all $n \in \mathbb{N}$. Assume that

   (i) $Q(1)$ holds,

   (ii) $Q(2)$ holds and that,

   (iii) whenever $Q(j)$ and $Q(j+1)$ hold, $Q(j+2)$ holds, where j is a natural number;

   then it follows that every natural number n has the property Q.

   (a) [10%] Show that the Principle of Induction implies the second principle of induction.

   (b) [15%] The well-known Fibonacci Sequence is defined as $a_1 = a_2 = 1$, and $a_{n+2} = a_n + a_{n+1}$ for each $n \in \mathbb{N}$. Show that

$$a_n = \frac{1}{\sqrt{5}}\left[\left(\frac{1+\sqrt{5}}{2}\right)^n - \left(\frac{1-\sqrt{5}}{2}\right)^n\right]$$

4. A Fibonacci Number is any term in the Fibonacci Sequence

$$1, 1, 2, 3, 5, 8, 13, 21, 34, \cdots$$

   Show that every natural number is a either a Fibonacci number or a sum of distinct Fibonacci numbers.

5. The first box is a box containing nothing. The second box is a box containing the first box. The third box is a box containing the first and the second box. Hence we know that to build the first box we need a box, to build the second box we need two boxes, and to build the third box we need 4 boxes. For the n-th box, the $(n+1)$-th box is a box containing all objects in the n-th box and moreover the n-th box itself. Show that to build the n-th box, we need 2^{n-1} boxes.

6. Let $x \geq 0$, $n \in \mathbb{N}$. Show that

$$[x] + [x + \frac{1}{n}] + \cdots + [x + \frac{n-1}{n}] = [nx]$$

   where [x] denotes the Gaussian Notation.