

Problem 5: The convergence of  $(\sigma_n)$ .

[Solution]

We see that

$$\begin{aligned}(c_n) &= \{\alpha, \beta, \gamma, \alpha, \beta, \dots\} \\ (s_n) &= \{\alpha, -\gamma, 0, \alpha, -\gamma, 0, \dots\} \\ (\sigma_n)_{n=1}^{\infty} &= \left\{ \frac{\alpha}{1}, \frac{\alpha - \gamma}{2}, \frac{\alpha - \gamma}{3}, \frac{2\alpha - \gamma}{4}, \frac{2\alpha - 2\gamma}{5}, \dots \right\}\end{aligned}$$

This means (by induction)

$$\begin{aligned}\sigma_{3k+1} &= \frac{(k+1)\alpha - k\gamma}{3k+1} \\ \sigma_{3k+2} &= \frac{(k+1)\alpha - (k+1)\gamma}{3k+2} \\ \sigma_{3k} &= \frac{k\alpha - k\gamma}{3k}\end{aligned}$$

Since the three subsequences above all converge to  $\frac{\alpha - \gamma}{3}$ , by zipper theorem (Problem 1),  $(\sigma_n)$  converges to the same limit.