Problem 1: f_k has the same limit as a_k, b_k, and c_k.

Proof. Let $\varepsilon > 0$. By convergence of these three sequences, there is an N'_ε such that for all $k \geq N'_\varepsilon$,

$$|a_k - \alpha| < \varepsilon$$
$$|b_k - \alpha| < \varepsilon$$
$$|c_k - \alpha| < \varepsilon$$

Choose $N_\varepsilon = 6N'_\varepsilon$. Then for all $n \geq N_\varepsilon$,

Case I If $n = 3k$, then $k \geq N'_\varepsilon$. We see that $|f_{3k} - \alpha| = |a_{k+1} - \alpha| < \varepsilon$.

Case II If $n = 3k + 1$, then $k \geq N'_\varepsilon$. We see that $|f_{3k+1} - \alpha| = |b_{k+1} - \alpha| < \varepsilon$.

Case III If $n = 3k + 2$, then $k \geq N'_\varepsilon$. We see that $|f_{3k+2} - \alpha| = |c_{k+1} - \alpha| < \varepsilon$.

Hence $\lim_{m\to\infty} = \alpha$. □