

METATHEOREMS FROM REPLACEMENT

Ἰἶἶἶ ἜἜ

August 30, 2015

1 The axiom

I think the Axiom of replacement is quite unfriendly to a beginner, for requirement of a better viewpoint of the term $H(x, y)$. Let us recall that the axiom takes the form

WHENEVER $H(x, y)$ IS DECIDED WITH THE PROPERTY THAT IF $H(x, y_u)$ AND $H(x, y_v)$ THEN $y_u = y_v$, THAT IS, WHENEVER WE'VE ALREADY PREPARED THE SENTENCE, AND A IS ANY SET, THEN THIS IS A TAUTOLOGY:

There is a set B such that for any set $x, y \in B$ if and only if $H(x, y)$ for some $x \in A$.

2 Conceptual function

According to our experience in mathematics, when such a predicate $H(x, y)$ is proved to accompany the fact $y_u = y_v$ whenever $H(x, y_u)$ and $H(x, y_v)$, we may define the SOME-NOTION of x to be y , denoted by $y = \Gamma(x)$. For example, $H(x, y)$ taken as $t \in y \Leftrightarrow t \subset x$, we show the accompanied fact. Then we say such a y is the power-set of x , denoted by $\wp(x)$. Note that the SOME-NOTION is the power-set, while the notation Γ of the notation replaced by \wp .

$\Gamma(x)$, directly called the gamma of x , is something that we might call a "conceptual function", because we do not restricted its domain and codomain. For example, intersection and union, which are two variables, and the single variable $\wp(x)$. However, the risk is the undefined or meaningless or paradoxical condition. For example, denote $\Gamma(s)$ = the set of all that contain s (as a subset) such that $t \notin t$. Everybody knows the modified Russell paradox leads to a contradiction just right after its definition.

To summary, what I call a "conceptional function" will help in set theory. Especially, it "hid-
denly" occurs in a proof of algebraic closure of a field \mathbb{F} while most people keep unaware. Let's
resume, and show its two reducing METATHEOREM:

3 The Metatheorems

METATHEOREM 1: REPLACEMENT \Rightarrow SCHEMA.

So the mission is to properly decide $H(x, y)$. Let's see below.

What we want: $\forall y, y \in B \Leftrightarrow y \in A \ \& \ \varphi(y)$;

What we have: $\forall y, y \in B \Leftrightarrow (\exists x)x \in A \ \& \ y = \Gamma(x)$

Comparing the terms , we may be inspired that $H(x, y)$ contain the condition $\varphi(y)$. To connect
to $x \in A$, that is, to support the condition $y = \Gamma(x)$ for an x . A best way is to identify $x = y$, and
then make some adjustment. Now we set $H(x, y) \Leftrightarrow x = y \ \& \ \varphi(x)$, this lead to a proof.

Note: You may give yourself an one-sentence explanation of the notion $\Gamma(x)$, for example, y is
the identical φ -ed element of x or any other name.

METATHEOREM 2: REPLACEMENT+POWERSSET \Rightarrow PAIRING.

This is my idea.

What we want: $\forall x, x \in Q \Leftrightarrow x \in u \ \text{or} \ x = v$;

What we have: $\forall y, y \in B \Leftrightarrow (\exists x)x \in A \ \& \ y = \Gamma(x)$

Here $\Gamma(x)$ is an image of what I called a "conceptional function". So $\Gamma(x)$ is meant to be u ,
or to be v . Our mission is to decide conditions of each value, that is, we're going to construct its
domain, and the mapping, and express it in a formula.

This is an answer: $H(x, y) \Leftrightarrow (x = \emptyset \ \& \ y = u) \ \text{or} \ (x = \{\emptyset\} \ \& \ y = v)$. You might complete
the proof, and it will be easy.