Pumping Lemma - an Example

Wang-Shiuan Pahngerei

April 27, 2012

Show that $L := \{x \in \{a,b\} \ : \ x \neq x^R\}$ is non-regular.

Proof. Assuming regularity, but whenever $m \in \mathbb{N}$, one might choose

$$w = a^m b a^N \in L,$$

where $N = m + m!$. Then if w is decomposed as $w = xyz$, with $|xy| \leq m$ and $|y| \geq 1$, the string

$$w_j := xy^j z,$$

where $j = 1 + \frac{m!}{|y|}$, is in the form $a^K b a^N$. Note that x, y looks like this form a^r, and z in the form $a^s b a^N$. So the number K of a's at the left hand side of the middle b of w_j is

$$K = m - |y| + |y| \cdot j = m + m!.$$

Therefore it is symmectric, and then $w_j \notin L$. This contradicts the Pumping Lemma. $\square$