

1. **Motivation** From  $e^x$ , we find  $(e^x)' = e^x$ . Immediately the question arises: Is  $e^x$  unique for  $f'(x) = f(x)$ ? How about the case  $f'(x) = \alpha f(x)$ ?
2. **Solution** By Taylor Expansion, we find analytic solutions  $c_0 \cdot e^{\alpha x}$  and by assumption that  $f(x) > 0$  we get the same solutions.
3. **Generalization** We then hope to solve  $y' + p(x)y = q(x)$ . We perform multiplication of the function  $\varphi(x)$ , getting its general form. Substituting  $-\alpha$  and 0 for  $p(x)$  and  $q(x)$  resp. , we entirely solve (1).
4. **Second – ordered** The equation  $y'' + (\alpha + \beta)y' + \alpha\beta \cdot y = 0$  can be solved by means of two-variable simultaneous equations, and that of solving first-order equations twice.
5. **Multiple – Root** Solution to  $y'' + 2r \cdot y' + r^2y = 0$  is  $y = c_0 \cdot e^{-rx} + c_1 \cdot xe^{-rx}$ .
6. **Imaginary – Roots** Two particular roots  $y_1 = \sin ax$ ,  $y_2 = \cos ax$  would lead to the general solution  $y = c_1 \sin ax + c_2 \cos ax$ .
7. **Generalization** By use of "translation" like that performed on graphs on  $\mathbb{R}^2$ , the form is  $y = e^{\rho x}(c_1 \sin \omega x + c_2 \cos \omega x)$ . Also, we might apply theories of complex variables, and imitate previous methods. The solution set remains the same.