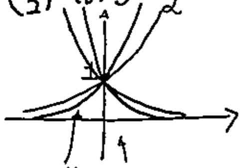
5th.	4.1	4.2	4.3	CH4
6th	4.1, 4.2	4.3	4.4	

6th 4.1 SAT Pg 7

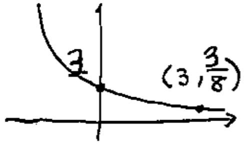
$\left(\frac{1}{2}\right)^x$ $\left(\frac{1}{3}\right)^x$ 3^x 2^x



$\left(\frac{1}{3}\right)^x$ flatter

3^x steeper grows more

$\boxed{ex3}$



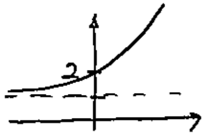
$f(x) = ?\ b a^x$ $\qquad \boxed{3\left(\frac{1}{2}\right)^x}$

$\frac{3}{8} = b a^3 \Rightarrow a = \frac{1}{2}$

$b = 3$

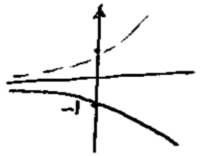
$\boxed{ex4}$ Sketch

$g(x) = 1 + 2^x$

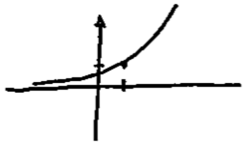


asymptote: $y = 1$

$g(x) = -2^x$



$g(x) = 2^{x-1}$



$\boxed{ex}$ Why? Java Gaddis Ch4 Pennies for Pay

\$1 million or \$0.01 & double pay for 31 days? ✓

- exponential growth!
- doubling time always the same too!

Who's bigger

$x! \gg 1.01^x \gg$ vs. x^{1000}

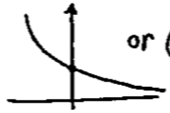
✓ eventually!

1.1^x vs. x^{10}

✓

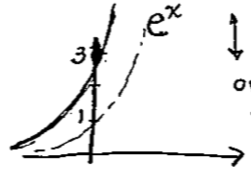
4.2 $\qquad (\text{TI } 2e^{-0.53} \approx 1.17721)$

$\boxed{ex2}$ Sketch $f(x) = e^{-x}$



or $\left(\frac{1}{e}\right)^x$

sketch $g(x) = 3e^{0.5x}$



stretch ↕ ↔

overall steeper

$\boxed{ex3}$ Logistic Growth: City population

$$v(t) = \frac{10\,000}{5 + 1245e^{-0.97t}}$$

t days

infected

a) initial infected $v(0) = \frac{10\,000}{1250} = 8$

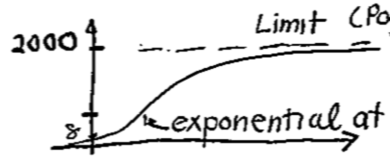
b) $v(1) = 21$

TI $\quad v(2) = 54$

ANS ^( ) $\quad v(5) = 678$

c) $\lim_{t \to \infty} v(t) = \frac{10000}{5} = 2000$

d)



Calculus $\qquad e \equiv$

OR $= \lim_{h \to 0} (1 + h)^{\frac{1}{h}}$

Who's smaller?

$\log_{10} x \ll \sqrt[100]{x}$

eventually

Interest $i = 0.08$ 8%

- P principle A_0 P

1 year A_1 $P + iP = P(1+i)$

2nd year A_2 $P(1+i) + iP(1+i) = P(1+i)^2$

$\vdots$

$A(t) = P(1+i)^t$

compounded t years Later

---

r rate per year 12%

- t years

n times per year every month

$$A(t) = P\left(1 + \underbrace{\boxed{\frac{r}{n}}}_{\text{per month}}\right)^{\boxed{nt}}$$ ← # months

- Compounded Continuously

$$A(t) = P \lim_{n\to\infty} \left(1+\frac{r}{n}\right)^{nt} \qquad h = \frac{r}{n}$$

$$= P\left[\lim_{h\to 0} (1+h)^{\frac{1}{h}}\right]^{rt} \qquad n = \frac{r}{h}$$

$$= Pe^{rt}$$

4.1 $\boxed{\text{ex7}}$ rate 6% per year, compounded daily

annual percentage yield

$$A = P\left(1+\frac{0.06}{365}\right)^{365\cdot 1} = P(1.06183)$$

$$= P(1 + \underline{\underline{i}}) \quad \Rightarrow \quad i = \boxed{6.183\%}$$

4.2 $\boxed{\text{ex4}}$ \$1000 invested, 12% per year (interest rate) compounded continuously

Amount after 3 years

$$A(3) = 1000\, e^{0.12\times 3} = \$1433.33$$

4.3 Logarithms

Logs: SAT Pg 7

Ⓠ as power of 4

- $2 = 4^{\frac{1}{2}}$
- $8 = 2 \cdot 4 = 4^{\frac{1}{2}+1} = 4^{3/2}$
- $16 = 4^2$
- $\sqrt{\frac{1}{32}} = \sqrt{\frac{1}{4\cdot 4\cdot 2}} = 4^{(-2-\frac{1}{2})\frac{1}{2}} = 4^{-\frac{5}{4}}$
- $\frac{1}{16} = 4^{-2}$

CW: $\begin{cases} 4.1 & \#20,\ 26 \sim 36 \text{ even} \\ 4.2 & 8 \sim 14 \text{ even} \end{cases}$

- Log defn.

4.3 Logarithmic Functions

SAT Pg 7

ⓔ Express as power of 4

$2 = 4^{\frac{1}{2}}$ $\quad \log_4 2 = \frac{1}{2}$ $\qquad 8 = 2\cdot 4 = 4^{\frac{1}{2}+1} = 4^{\frac{3}{2}}$ $\quad \log_4 8 = \frac{3}{2}$

$16 = 4^2$ $\quad \log_4 16 = 2$ $\qquad \sqrt{\frac{1}{32}} = 2^{-5/2} = 4^{-5/4}$ $\quad \log_4 \sqrt{\frac{1}{32}} = -\frac{5}{4}$

$\frac{1}{16} = 4^{-2}$ $\quad \log_4 \frac{1}{16} = -2$

<u>Logarithm</u> : Inverse of exponential function
The exponent y such that b raised to this power gives x

$y = \log_b x \underset{\text{inverses}}{\iff} b^y = x$ ☆ Domain $x > 0$
$b > 0 \quad b \neq 1$

ⓔ $\log_{10} 10^2 = 2$ $\qquad \log_5 1 = 0$

$\log_{10} \frac{1}{1000} = -3$ $\qquad \log_5 5 = 1$

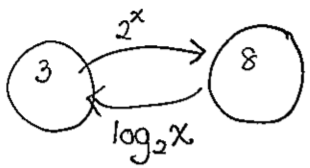
$\log_{10}(10 \times 1000) = 4$ $\qquad \log_5 5^8 = 8$
$1 + 3$

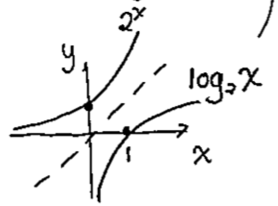
$5^{\log_5 12} = 12$

(5 to the power of 5 that gives 12
$5^r = 12$)

<u>Properties</u>

$$\boxed{\begin{aligned} &\log_b 1 = 0 \\ &\log_b b = 1 \\ &\log_b b^r = r, \ \forall r \in \mathbb{R} \\ &b^{\log_b r} = r, \ r > 0 \end{aligned}}$$

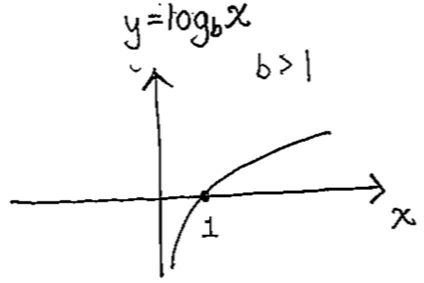




$\begin{cases} f(x) = 2^x \\ f^{-1}(x) = \log_2 x \end{cases}$ ☆ $f(f^{-1}(x)) = x$
$f^{-1}(f(x)) = x$

Graphs of logarithmic functions

Notice : Graph of inverse is reflection about $y = x$

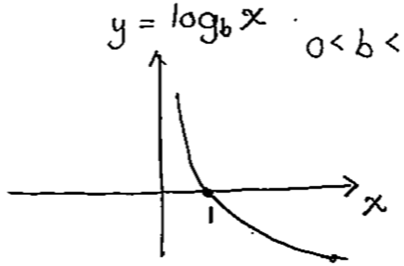


(inverse of b^x)

x	$\log_{10} x = r$, $10^r = x$	
1	0	$(10^0 = 1)$
1000	3	$(10^3 = 1000)$
flat		
0	$-\infty$	$(10^{-\infty} = 0)$

$b > 1 \Rightarrow \log_b x$ increasing
(1, 0) on graph
Asymptote : y-axis
$\lim_{x \to 0^+} \log_b x = -\infty$

Domain : $x > 0$
$(b > 1 \Rightarrow b^{\text{any power}} > 0)$

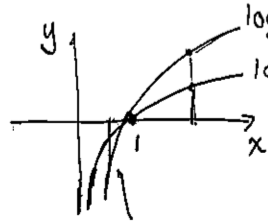


x	$r = \log_{1/10} x$	$\left(\frac{1}{10}\right)^r = x$
1	0	$\left(\frac{1}{10}\right)^0 = 1$
10	-1	$\left(\frac{1}{10}\right)^{-1} = 10$
1000	-3	
0.3	$\frac{1}{2}$	$\left(\frac{1}{10}\right)^{\frac{1}{2}} \approx 0.3$
0.977	$\frac{1}{100}$	$\sqrt[100]{\frac{1}{10}} = 0.977$
0	∞	$\left(\frac{1}{10}\right)^{\infty} = 0$

$0 < b < 1 \Rightarrow \log_b x$ decreasing
(1, 0) on graph
$\lim_{x \to 0} \log_b x = +\infty$

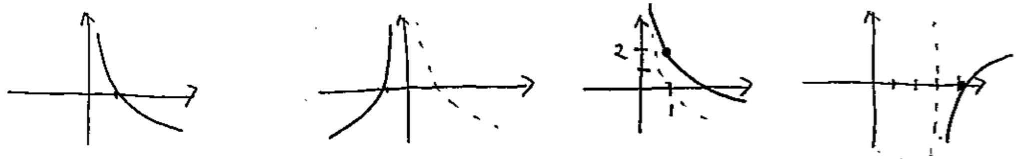
ⓔ $e^{\ln x^4} = x^4$

$\log_3 \frac{1}{27} = -3$



ex5 & 6 Sketch

$g(x) = -\log_2 x$ $\quad h(x)=\log_{0.1}(-x)$ $\quad 2+\log_{\frac{1}{5}} x$ $\quad \log_{10}(x-3)$



Why Logarithms?

- decibel
- SNR 3dB cutoff frequency
- Richter scale
- $\times \div x^p \Rightarrow \pm$

$\sqrt{\dfrac{x^2 y^5}{a^{\frac{1}{2}} b}}$

$\frac{2}{2}\ln x + \frac{5}{2}\ln y$

$-\frac{1}{4}\ln a - \frac{1}{2}\ln b$

ex8

$B = 10\log\left(\dfrac{I}{I_0}\right)$ — I: physical intensity $\frac{W}{m^2}$; I_0: reference of barely audible sound ?(0dB)

B: decibels

If $I = 100 I_0$, find the decibel level

$B = 10\log 100 = 10\times 2 = 20\text{dB}$

ex9

$\ln e^8 = 8$

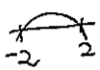
$\ln\left(\frac{1}{e^2}\right) = -2$

$\ln 5 \approx 1.609$

ex10 Domain?

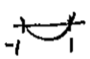
$f(x) = \ln(4-x^2)$

$4-x^2 > 0$



$\boxed{-2<x<2}$

Ⓠ Domain? $\log(\sqrt{x^2-1})$



$x^2-1 \gneq 0$ $\quad \boxed{(-\infty,-1)\cup(1,\infty)}$

$b^0 = 1$

$b^{-r} = \frac{1}{b^r}$

$b^r q^r = (bq)^r$

$b^r b^s = b^{r+s}$

$\frac{b^r}{b^s} = b^{r-s}$

$(b^r)^s = b^{rs}$

one-to-one

$\boxed{\log_b p = \log_b q \Leftrightarrow p=q}$

4.4 Laws of Logs

$\log_b 1 = 0$

$\log_b\left(\frac{1}{p}\right) = -\log_b p$

$$\boxed{\begin{aligned}\log_b(pq) &= \log_b p + \log_b q\\ \log_b\left(\frac{p}{q}\right) &= \log_b p - \log_b q\\ \log_b(p^s) &= s\log_b p\end{aligned}}$$

$\log_b p = r \quad \Rightarrow \quad b^r = p$

$\log_b q = s \quad\quad\;\; b^s = q$

$\log_b\left(\frac{p}{q}\right) = \log_b\left(\frac{b^r}{b^s}\right) = \log_b b^{r-s}$

$= r-s$

★ Change of Base

Convenient: $\log_{10}$, $\ln = \log_e$

$$\boxed{\begin{aligned}\log_b x &= \frac{\log_a x}{\log_a b}\\ \frac{\log_b x}{\log_b y} &= \frac{\log_a x}{\log_a y}\end{aligned}}$$

$a, b, x > 0$

$a \neq 1, b \neq 1$

Proof/Derivation

- $\log_b x = r$

  $b^r = x$

- $\log_a x = s$

  $a^s = x$

- $a = b^p$

$x = a^s = b^{ps}$

$\log_b x = ps$

$= \log_b a \log_a x$

evaluate

ex1 a) $\log_4 2+\log_4 32 = \log_4(64) = 3$

b) $\log_2 80-\log_2 5 = \log_2(16) = 4$

c) $-\frac{1}{3}\log 8 = \log 8^{-\frac{1}{3}} = \log\frac{1}{2} \approx -0.301$

ex2 Expand

a) $\log_2(6x) = \log_2 6+\log_2 x$

b) $\log_5(x^3y^6) = 3\log_5 x + 6\log_5 y$

c) $\ln\left(\frac{ab}{\sqrt[3]{c}}\right) = \ln a + \ln b - \frac{1}{3}\ln c$

ex3 Combine into a single log

$3\log x + \frac{1}{2}\log(x+1) = \log\left(x^3\sqrt{x+1}\right)$

ex4 Combine

$3\ln s + \frac{1}{2}\ln t - 4\ln(t^2+1) = \ln\frac{s^3\sqrt{t}}{(t^2+1)^4}$

WARNING $\log_a(x+y) \neq \log_a x+\log_a y$

$\frac{\log 6}{\log 2} \neq \log\left(\frac{6}{2}\right)$ but it is $\log_2 6$

$(\log_2 x)^3 \neq \log_2(x^3)$

ex6 Use the change of base (for calculator)

a) $\log_8 5 = \frac{\ln 5}{\ln 8} \approx 0.77398$ OR log (5, 8)

ex7 Graph $\log_6 x$ on TI

ex5 Law of forgetting

Learned with score P_0

After time t, it drops to P

$\log P = \log P_0 - c\log(t+1)$, c is a constant determined by task

months

a) Solve for P

$\log P = \log\left(\frac{P_0}{(t+1)^c}\right) \Rightarrow P = \frac{P_0}{(t+1)^c}$

b) history score 90 After 2 months? $P = \frac{90}{3^{0.2}} \approx 72$

1 year? $c = 0.2$ $P = \frac{90}{13^{0.2}} \approx 54$

- Practice HW $\begin{cases} 4.3 \\ 4.4 \end{cases}$
- $3\log_2 x = \log_2 x^3$
- Solve $9^{\log_3 x} = 4$

$3^{\log_3 x^2} = 4$

$x^2 = 4$

$\boxed{x = \pm 2}$

- solve $3^{x^2-3x} = \frac{1}{9}$

$x^2 - 3x = -2$

$x^2-3x+2 = 0$

$(x-1)(x-2) = 0$

$\boxed{x = 1, 2}$

- SAT Pg 8

- Project ideas : Logistic scatter in class

data
① Population growth
doubling time

② Sound & frequency
decibels
log scale

③ Piano hz & tones
log scale

Voice harmonics
map speech to synthesized voice
GUI & Applet

Radioactive decay
- C-14
- dinosaur

Logistic Growth
- fish in lake
- asymptote

forgetting curve
investment

ex1 & ex4
world pop. catfish

rubric
- scatter
- model
- doubling or asymp.
- predict to real

4.5

(4.4) Exponential & Logarithmic Eqns

Bring x in power down

① $2^{x} = 5$

$x \ln 2 = \ln 5$

$x = \frac{\ln 5}{\ln 2} = \log_2 5$

Put into one log

② $\log_3(x-1) = 2$

$3^2 = x-1$

$x = 3^2+1$

$= 10$

$\log_2(x+10) = \log_2 3$

$x+10=3$

$x=-7$

$x>-10$ ok

$2^{x} = 2^{x^2-2}$

$x = x^2-2$

$x^2 - x - 2 = 0$

$(x-2)(x+1) = 0$

$x = -1$ or 2

③ $\boxed{f(x_1) = f(x_2) \;\Downarrow\; x_1 = x_2 \text{ iff } f \text{ is one to one}}$

$e^x, \ln x$

$\left(a^2=b^2 \not\Rightarrow a=b\right)$

$\sin x = \sin y \not\Rightarrow x=y$

$\log_3 \overbrace{(x+1)}^{>0} - \log_3 \overbrace{x}^{>0} = 2$

$\log_3\left(\frac{x+1}{x}\right) = 2$

$3^2 = \frac{x+1}{x}$

$9x = x+1$

$8x = 1$

$x = \frac{1}{8}$ ✓

$2^{x} = 3^{x+2}$

$x \ln 2 = (x+2)\ln 3$

$x(\ln 2 - \ln 3) = 2\ln 3$

$x = \frac{2\ln 3}{\ln 2 - \ln 3} = \frac{\ln 9}{\ln\left(\frac{2}{3}\right)}$

$= \log_{\frac{2}{3}}(9)$

$\left(\frac{2}{3}\right)^x = 9$

Put into one exp first

$\log_3(x-1) = \log_9(x+1)$

$\log_3(x-1) = \frac{\log_3(x+1)}{\log_3 9}$

$2 \cdot \log_3(x-1) = \log_3(x+1)$

$2\log_3(x-1) - \log_3(x+1) = 0$

$\log_3\left[\frac{(x-1)^2}{x+1}\right] = 0$

$\frac{(x-1)^2}{x+1} = 1$

$(x-1)^2 = x+1$

$x^2 - 2x + 1 = x+1$

$x^2 - 3x = 0$

$x = 0$ or 3

---

ex1 $3^{x+2} = 7$

$(x+2)\ln 3 = \ln 7$

$x+2 = \frac{\ln 7}{\ln 3}$

$x = \frac{\ln 7}{\ln 3} - 2$

$= \log_3 7 - 2$

$3^x \, 3^2 = 7$

$3^x = \frac{7}{9}$

$x \ln 3 = \ln\left(\frac{7}{9}\right)$

$x = \frac{\ln\left(\frac{7}{9}\right)}{\ln 3}$

$= \frac{\ln 7 - \ln 9}{\ln 3}$

$= \frac{\ln 7}{\ln 3} - 2$

ex2 $8e^{2x} = 20$

$e^{2x} = 2.5$

$2x = \ln(2.5)$

$x = \frac{\ln(2.5)}{2}$

ex3 $e^{3-2x} = 4$

$3 - 2x = \ln 4$

$2x = 3 - \ln 4$

$x = 1.5 - \frac{\ln 4}{2}$

$= 1.5 + \ln\left(\frac{1}{2}\right)$

$e^{-2x} e^3 = 4$

$e^{-2x} = 4e^{-3}$

$-2x = \ln(4e^{-3})$

$x = \frac{\ln(4e^{-3})}{-2}$

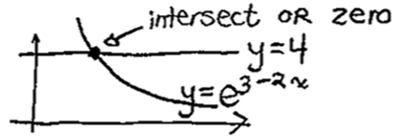
$= \frac{\ln 4 - 3}{-2}$

$= 1.5 - \frac{1}{2}\ln 4$

$= 1.5 + \ln\frac{1}{2}$

TI-89 Solve $(e^{3-2x} = 4, x)$

TI-84



ex4 $e^{2x} - e^{x} - 6 = 0$

$u^2 - u - 6 = 0$

$(u-3)(u+2) = 0$

$u = -2$ or 3

$e^x = -2$ or $e^x = 3$

$x = \cancel{\ln(-2)}$ or $\ln 3$

-2 not in domain of $\ln$

ex9 $x^2 = 2\ln(x+2)$

cannot solve by hand

Graph: $Y_1 = x^2$

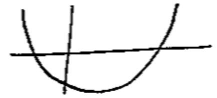
$Y_2 = 2\ln(x+2)$

"intersect"

OR

$Y_1 = x^2 - 2\ln(x+2)$

"zero"



$x = \boxed{-0.71 \text{ or } 1.60}$

ex5 $3xe^x + x^2e^x = 0$

$\underbrace{e^x}_{\neq 0} \cdot \underbrace{x(3+x)}_{=0} = 0$

$\boxed{x=0 \text{ or } x=-3}$

$(\div e^x \neq 0)$ $x(3+x) = 0$

$AB=0$	$AB \geq 0$	$AB \leq 0$
$A=0$ OR $B=0$	$A \geq 0$ & $B \geq 0$	$A \geq 0$, $B \leq 0$ OR $A \leq 0$, $B \geq 0$

ex10 $-\frac{1}{k}\ln\left(\frac{I}{I_0}\right) = x$

a) $I(x) = I_0 e^{-kx}$

b) $k = 0.025$, $I_0 = 14$, $I(20) = 14e^{-0.025(20)}$

ex6 LOGARITHMIC

a) $\ln x = 8$ $(x>0)$

$e^8 = x$

b) $\log_2(25-x) = 3$ $(x<25)$

$25 - x = 2^3$

$= 8$

$x = 17$

Check

✓ $\log_2(\underbrace{25-17}_{8}) = 3$

ex11 • Simple Interest (for one year) $A = P(1+r)$

• Interest compounded n times per year $A(\overset{\text{years}}{t}) = P\left(1+\frac{r}{n}\right)^{nt}$

• Continuously compounded $A(\overset{\text{years}}{t}) = Pe^{rt}$

ex8 $\log(x+2) + \log(x-1) = 1$ $\quad x>-2 \quad x>1$

$\log[(x+2)(x-1)] = 1$

$(x+2)(x-1) = 10$

$x^2 + x - 2 = 10$

$x^2 + x - 12 = 0$

$(x+4)(x-3) = 0$

$x = \cancel{-4}$ or 3

Check

$\underbrace{\log(-4+2)}_{\text{dne}} + \log(-4-1)$

$\log 5 + \log 2$

$= \log 10 = 1$

$P = \$5000$, $r = 5\%$ interest rate per year. Time to double compounded

a) Semi-annual

$A(t) = 2P$

$\cancel{P}\left(1+\frac{r}{n}\right)^{nt} = 2\cancel{P}$

$t = \frac{\ln 2}{n\ln(1+\frac{r}{n})}$

$= \frac{\ln 2}{2\ln(1+0.05/2)} \approx 14.04$ years

b) Continuous — e^x doubling time No matter start!

$A(t_0+T) = 2A(t_0)$

$\cancel{Pe^{rt_0}}e^{rT} = 2\cancel{Pe^{rt_0}}$

★ $T = \frac{\ln 2}{r}$

$= \frac{\ln 2}{0.05} = 13.86$ years

4.2

(ex 3) Annual Percentage Yield (for compound interest) = simple interest rate that gives same amount at end of year

$$P\left(1+\frac{r}{n}\right)^{n\overset{1\text{ year}}{t}} = P(1+\underline{R})$$

$$\left(1+\frac{r}{n}\right)^{n} = (1+R)$$

$r = 6\%$, compounded daily

$$\underbrace{\left(1+\frac{0.06}{365}\right)^{365}}_{1.06183} = (1+R)$$

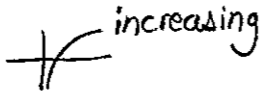
$$R = 0.06183 = 6.183\%$$

- HW 4.5
- SAT Pg 8

Inequalities

(63) 59 $\log(x-2)+\log(9-x) < 1$

$\log_{10}((x-2)(9-x)) < \log_{10} 10$



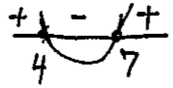
$y_1 < y_2 \Updownarrow x_1 < x_2$

$(x-2)(9-x) < 10$

$-x^2+11x-18 < 10$

$x^2-11x+28 > 0$

$(x-7)(x-4) > 0$



$x<4$ or $x>7$

$(-\infty,4) \cup (7,\infty)$

(64) 67 $3 \le \log_2 x \le 4$

$2^3 \le x \le 2^4$

$8 \le x \le 16$

2^x is increasing monotonic

$a \le b$

$\ln a \le \ln b$

$c^a \le c^b \quad (c>1)$

$a \le b$

$\pi^a \boxed{\le} \pi^b$ increase $\qquad \log_{2.5} a \le \log_{2.5} b$

$\left(\frac{1}{e}\right)^a \boxed{\ge} \left(\frac{1}{e}\right)^b$ decrease $\qquad \log_{0.2} a \ge \log_{0.2} b$

4.6

(4.5) Exponential Growth Model

Population $y(t)$, t: time $\qquad$ Start with $y_0 = y(0)$

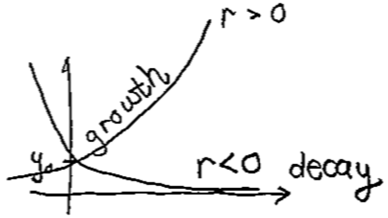
D.E. $\begin{cases} \dfrac{\Delta y}{\Delta t} = r\, y(t) \\ y(0) = y_0 \end{cases}$

$\frac{\Delta y}{\Delta t}$: Rate of growth; r: relative rate of growth (unit is time); $y(t)$: # people now

Like
- $r = 40\%$ per hour
- $\frac{\Delta y}{\Delta t}$ = grow (add) 40% of current amount per hour

$$\Rightarrow \boxed{y(t) = y_0 e^{rt}}$$

use this instead of pg 341



ex 2 In 2000
population: 6.1 billion
relative rate of growth: 1.4% per year

a) $y(t) = \boxed{6.1 e^{0.014t}}$ (y: billions, t: years)

b) Sketch

c) Population in 2050: $y(50) = 6.1e^{0.010(50)} = \boxed{10.1 \text{ billion}}$

d) $\underset{2011}{y(11)} = 6.1e^{0.014(11)} = 7.12$ billion News

e) Doubling Time

$y(t+T) = 2y(t)$

$\cancel{y_0 e^{rt}} e^{rT} = 2\cancel{y_0 e^{rt}}$

$\boxed{T = \frac{\ln 2}{r}} = \frac{\ln 2}{0.014} = 49.5$ years

* No matter when, or how many ppl, it takes 50 years for population to double!!

News: (Standing Room Only by 2801)

Bonus:
- Census: 100 years ago population ≈ by regression?
- Calculate the 2801

ex5 Bacteria Culture

Starts: 10,000 bacteria
Doubles every 40 minutes

a) $y(t) = 10{,}000\, e^{rt}$ (y: bacteria, t: minutes)

$20000 = 10000 e^{r40}$

$r = \frac{\ln 2}{40} \approx 0.01733$

$= 10\,000 e^{0.01733t}$

b) After 1 hour
$y(60) = 28\,287$

c) When to reach $50{,}000? = 10\,000 e^{0.01733t}$

$t = \frac{\ln 5}{0.01733} = 92.9$ minutes

Radioactive Decay

$y(t) = y_0 e^{rt} \quad r<0$

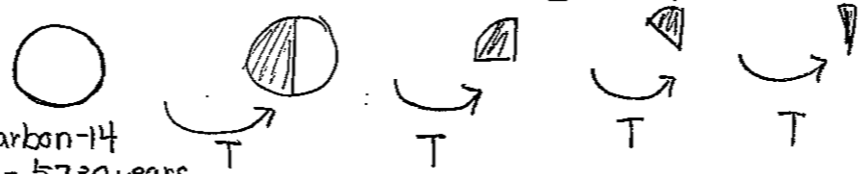
Half-Life = Time to lose half the mass (changes to another element)

$y(t+T) = \frac{1}{2} y(t)$

$\cancel{y_0 e^{rt}} e^{rT} = \frac{1}{2} \cancel{y_0 e^{rt}}$

$\boxed{T = \frac{\ln(\frac{1}{2})}{r}}$

Plutonium-239
T = 24,360
waste poses threat for thousands yrs.



Carbon-14
T = 5730 years

Radioactive Dating

(ex6) VPR 7.5

Logs in an old fort show only 70% of carbon-14 expected in living matter.
When was the fort built?

$y(t) = y_0 e^{rt}$

$\cancel{y_0} e^{rt} = 0.7 \cancel{y_0}$

$t = \frac{\ln 0.7}{r}$

$= \ln 0.7 \left(\frac{5730}{-\ln 2}\right)$

$= 2949$ years ago

T = 5730 years half-life

$\cancel{y_0 e^{rt}} e^{rT} = \frac{1}{2} \cancel{y_0 e^{rt}}$

$T = \frac{\ln(\frac{1}{2})}{r}$

$r = \frac{\ln(\frac{1}{2})}{T} = \boxed{\frac{-\ln 2}{5730}}$

Richter Scale

magnitude $M = \log \frac{I}{S}$ (I ← intensity 100km from epicenter; S ← "standard")

ex10 7.1 vs. 8.3

$S\,10^{7.1} : S\,10^{8.3}$

$10^{8.3-7.1} = 10^{1.2} \approx \boxed{16}$ times intensity

Log Scales

$pH = -\log[H^+]$ (hydrogen ion concentration) (acids have low pH < 7) (pH 0~7)

ex8 Acid Rain $pH = 2.4 \Rightarrow H^+ = 10^{-2.4} \approx 4 \times 10^{-3}$ M