2.1 (5th P.155) # 3 8 11 14 15 23 27 30 & f(a) 47 51 52 57 69

(6th P. 149) → 8 10 16 20 21, 25, 30 34 35 & f'(a) 54 58 57 63 79

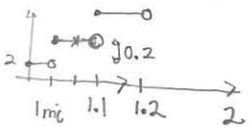
fn.

2.2 (5th P.167) # 17 22 23 24 25 33 46 47 49 53 55 57 63 65 67 69 71 89 (try ⌊x⌋ fn)

(6th P.160) Concept 4, 26 28 41 43 46 49 52 54 56 59 62 64 66 67 82 (try ⌊x⌋ 84

graph

$1 < x < 2$



$C(x) = 2 + 0.2\sqrt{10(x-1)}$

1.1

$0.1 \times 10 = 1$

$0.05 \times 10 = 0.5$

2.3 (new split) (P.169) 6th max/min → 6th ed: 6 8 7 22 ~~36~~ 40 42 50

14 16 by hand in class

talk in class

(40) $U(x) = x\sqrt{x - x^2}$

D: $0 \le x \le 1$

R:

(42) $V(x) = \frac{1-x^2}{x^3}$

2.4 (2.3 5th P.179) # 3 9 19 23 27 & instantaneous 29 35 37 39

6th (P. 177) CR 3, 4 | 8 12 16 20 & instantaneous f'(a) 22 27 31

rates

2.5 (2.4 5th P.190) # 1~47 odd ; 53~57 odd ; 61~73 odd ; 75 & 77

CW 2~48, 71 even

*Use graph paper
Show parent function

transform
6th P.187 CW Concept 1~4

6 ~ 68 even, 76~88 even

HW # 5 ~ 67 odd

75 ~ 87 odd, 90~93

2.6 (5th 2.7 P219) # 3 5 9 12 16 20 21 25 37 39 43 47 49 51~54 61 (Bonus 64, 65, 66) CW: 4 10 27 38 40 62

30 42 (41) 47 51 54 55~58 64 Bon 68 69 70 CW: 7 13 31 41 43 66

6th (P.196) # 8 9 14 15 19 23 25

combinations

2.7 (5th 2.8 P230) # 4 6 7 10 13 18 19 30 38 40 46 47 53 67 70 71. Bonus 83. CW 14 20 29 54 68

33 44 42 55 57 63 77 79 82. 93 17 24 34 64 78 & Prove (b,a) y=x (a,b)

6th (2.7 P204) # 7 10 12 13 18 21 23

one to one, inverse

Justin +2
Amanda +2

2.1 (P.149) #
6th

2.5 5th (P200) #11 15 25 33 37 39 43 49 57 60 65 67 { Graph ~~At~~ else only $x^{\div}$ + − }

69
3.4 6th (P.229) # 16 20 30 38 42 43 48 54 61 64 ~~76~~ 76
quadratic

revenue 77

2.6 5th Modeling (Pg 210) # 10 13 17 21 25 27 32 35.
(Pg 213) 6th focus (P.219) # 10 13 17 ~~1~~9 23 28 31

Practice Pg 211 area 19, 23 revenue/line 27, 28 minimize time 34
Pg 220 21 22 77 (3.1) 78 (3.1) 30

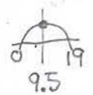
Pg220

(21) $x + y = 19$

xy as big

$f(x) = x(19-x)$

maximize



$x = 9.5$

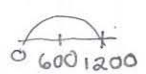
$y = 10.5$

max =

(22) $2x + y = 2400$

$A(x) = xy$

$= x(2400 - 2x)$



$x = 600$

$y = 1200$

$A = 720{,}000$

(77) Stadium Revenue

$R(x) = x N(x)$

revenue price

$N(10) = 27{,}000$

$\frac{\Delta N}{\Delta x} = \frac{+3000}{-1}$

$N(x) = -3000(x-10) + 27{,}000$

(78) $R(x) = N(x)(x-10)$

cost \$6 to make

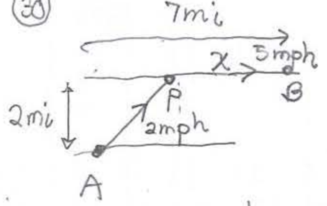
20 /wk at price \$10

$N(10) = 20$

$\frac{\Delta N}{\Delta x} = \frac{-2}{+1}$

$N(x) = -2(x-10) + 20$

(30)



a) $T(x) = \frac{d}{2} + \frac{x}{5}$

$= \frac{\sqrt{(7-x)^2 + 2^2}}{2} + \frac{x}{5}$

b) max T

Stewart 5th ed.

2.1 (P. 155) #3, 8, 11, 14, 15, 23, 27, 30 & $f'(a)$
47, 51, 52, 57, 69,

2.2 (P. 167) # 17, 22, 23, 24, 25, 33, 46, 47, 49, 53,
55, 57, 63, 65, 67, 69, 71, 89 (try int fn.)

2.3 (P. 179) # 3, 9, 19, 23, 27 & instantaneous, 29, 35, 37, 39

2.5 (P. 200) # 11, 15, 25, 33, 37, 39, 43, 49, 57, 69, 65, 67 { Graph / else only x÷ +−

2.6 (P. 210) # 10, 13, 17, 21, 25, 27, 32, 35.

2.4 (P. 190) # 1~47 odd; 53~57 odd; 61~73 odd, 75~77 every

2.7 (P. 219) # 3, 5, 9, ~~10~~, 12, 16, 20, 21, 25, ~~27~~, 37, ~~38~~, 39, ~~40~~
43, 47, 49, 51~54, 61, ~~62~~, (64, 65, 66) Bonus

CW: 4, 10, 27, 38, 40, 62

2.8 (P. 230) # 4, 6, 7, 10, 13, 18, 19, 30, 38, 40, 46, 47, 53, 67, 70
71. Bonus 83.
CW: 14, 20, 29, 54, 68

2.1 what is a function?

- See SAT pg. 1

ex1 machine diagram

$f(x) = x^2$ $\quad f: x \longmapsto x^2$

$f(\sqrt{5}) = 5$ $\quad \sqrt{5} \longmapsto 5$

$\sqrt{5} \longrightarrow \boxed{\text{square}} \longrightarrow 5$

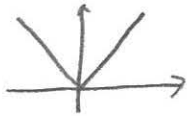
ex2 $f(x) = 3x^2 + x - 5$

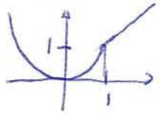
$f(\frac{1}{2}) = 3 \cdot \frac{1}{4} + \frac{1}{2} - 5 = -\frac{15}{4}$

ex3 Piecewise-defined function

$y = \begin{cases} x^2 & x < 1 \\ x + 0 & x \geq 1 \end{cases}$

a) $f(x) = |x| = \begin{cases} x & x \geq 0 \\ -x & x \leq 0 \end{cases}$





b) $C(x) = \begin{cases} 39 & 0 \leq x \leq 400 \\ 39 + 0.2(x - 400) & x > 400 \end{cases}$

\$39, first 400 minutes free
20¢ for each minute more

SAT 2.
$S = \begin{cases} 800 & R \geq 43 \\ 800 - 10(43 - R) & R < 43 \end{cases}$

- minus 10 per point below
- linear

ex4 $f(x) = 2x^2 + 3x - 1$

d)  a a+h

slope of secant line from a to $a+h$

$\frac{f(a+h) - f(a)}{h} = \frac{2(a+h)^2 + 3(a+h) - 1 - (2a^2 + 3a - 1)}{h}$

$= \frac{4ah + 2h^2 + 3h}{h} = 4a + 2h + 3$

$f'(a) = \lim_{h \to 0} \sim = 4a + 3$

H.A. $y = 0$
V.A. $x = 0, x = 1$

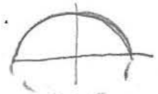
Domain & Range - See SAT pg 3

ex6 a) $f(x) = \frac{1}{x^2 - x} = \frac{1}{x(x-1)}$ domain $x \neq 0,\ x \neq 1$

CALC / CHECK → Range $y > 0 \cup y \leq -4$

b) $g(x) = \sqrt{9 - x^2}$ $\quad 9 - x^2 \geq 0$ $\quad 0 \leq y \leq 3$

$(3 - x)(3 + x) \geq 0$



$x^2 + y^2 = 3^2$
$y > 0$

$\boxed{-3 \leq x \leq 3}$

c) $h(t) = \frac{t}{\sqrt{t+1}}$ $\quad t + 1 \geq 0$ $\quad \boxed{t > -1}$

R

$t \uparrow \Rightarrow f_n \approx \sqrt{t} \uparrow$
$t \to -1 \Rightarrow -\frac{1}{0} = -\infty$

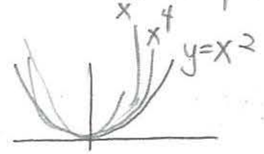
function:
- Verbally: y varies as the square of x
- algebraically: $f(x) = x^2$
- visually: 
- numerically:

x	y
1	1
2	4
3	9

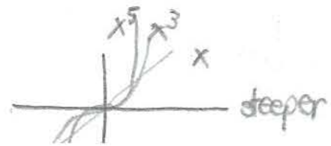
2.2 Function Graphs

ex2 Graphing calc

a) $y = x^n$ $\quad n = 2, 4, 6$
$[-2, 2] \times [-1, 3]$



b) $y = x^n$ $\quad n = 1, 3, 5$
$[-2, 2] \times [-2, 2]$



HW 1

Due: 9/7/2010 (+20 slip)

20% Syllabus slip

Total: 132 pts

miss -1

miss vocab -1

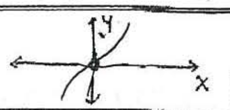
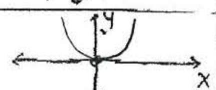
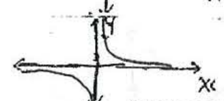
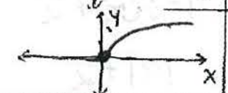
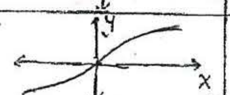
Name:

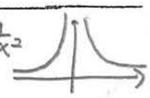
AP Calculus AB

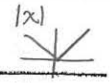
per 2

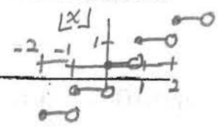
Library of Functions

② Sketch each function and fill in the blanks. (Intercept) (54)

Name	Function	Sketch	Domain	Range	One point on curve	Asymptotes (write equations or "none")
cubic poly	A) $y = x^3$ $x^5 \cdot x^7$		$\mathbb{R}$	$\mathbb{R}$	$(0,0)$	none
quadratic parabola	B) $y = x^2$ x^4 x^6		$\mathbb{R}$	$\{y \geq 0\}$	$(0,0)$	none
rectang. hyperbola rational	C) $y = \frac{1}{x}$		$\{x \neq 0\}$	$\{y \neq 0\}$	$(1,1)$	$x=0$ $y=0$
exp. growth	D) $y = \pi^x$		$\mathbb{R}$	$\{y > 0\}$	$(0,1)$	$y=0$
exp. decay	E) $y = (\frac{1}{2})^x$		$\mathbb{R}$	$\{y > 0\}$	$(0,1)$	$y=0$
root	F) $y = \sqrt{x}$		$\{x \geq 0\}$	$\{y \geq 0\}$	$(0,0)$	none
cubed root	G) $y = \sqrt[3]{x}$		$\mathbb{R}$	$\mathbb{R}$	$(0,0)$	none
log. growth	H) $\pi^y = x$ $y = \log_\pi x$		$\{x > 0\}$	$\mathbb{R}$	$(1,0)$	$x=0$
log. decay	I) $y = \log_{\frac{1}{2}} x$		$\{x > 0\}$	$\mathbb{R}$	$(1,0)$	$x=0$

$\frac{1}{x^2}$ 

$|x|$ 

$\lfloor x \rfloor$ 

5)
a) E This function is the inverse of I)
b) H This function is the inverse of D)
c) F This function is the inverse of B)
d) C This function is the inverse of C)
e) A This function is the inverse of G)

(6) Function Symmetry

odd $f(-x) = -f(x)$

even $f(-x) = f(x)$

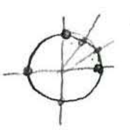
odd + odd = odd   odd × odd = even

even + even = even   even × even = even

odd + even = neither   even × odd = odd

Ⓠ $f(x) = x^3 \sin x - \frac{x^2}{\cos x} + 1$ is odd, even, neither?

Symmetry									
	radians	0	$\pi/6$	$\pi/4$	$\pi/3$	$\pi/2$	π	$3\pi/2$	2π
	θ degrees	0°	30°	45°	60°	90°	180°	270°	360°
odd	$\sin\theta$	0	$\frac{1}{2}$	$\sqrt{2}/2$	$\sqrt{3}/2$	1	0	-1	0
even	$\cos\theta$	1	$\frac{\sqrt{3}}{2}$	$\sqrt{2}/2$	$\frac{1}{2}$	0	-1	0	1
odd	$\tan\theta$	0	$\frac{1}{\sqrt{3}} = \sqrt{3}/3$	1	$\sqrt{3}$	dne	0	dne	0
even	$\frac{1}{\cos\theta}$ $\sec\theta$	1	$\frac{2}{\sqrt{3}} = \frac{2\sqrt{3}}{3}$	$\sqrt{2}$	2	dne	-1	dne	1



Functions

ⓐ $f(x) = \frac{x-3}{2}$, $g(x) = \sqrt{x}$

① $(f+g)(x) = \frac{x-3}{2} + \sqrt{x}$

4) $(f \cdot g)(x) = \frac{(x-3)}{2}\sqrt{x}$

$g^3(x) = [g(x)]^3 = (\sqrt{x})^3 = x^{3/2}$

$(g \circ f)(x) = g(f(x)) = \sqrt{\frac{x-3}{2}}$

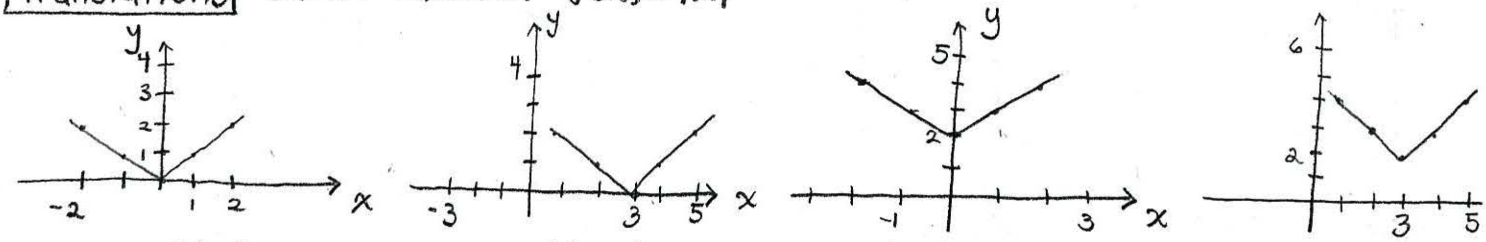
$(f \circ g)(x) = f(g(x)) = \frac{\sqrt{x} - 3}{2}$

$\left(\frac{f}{g}\right)(x) = \frac{x-3}{2\sqrt{x}}$

$y = \cancel{Af(Bx+C)} + D = -Af(-B(x - \frac{C}{B})) + D$

ⓐ $\boxed{\text{Translations}}$ Parent Function $f(x) = |x|$

8)



$y = f(x)$ — Specific: $y = |x|$

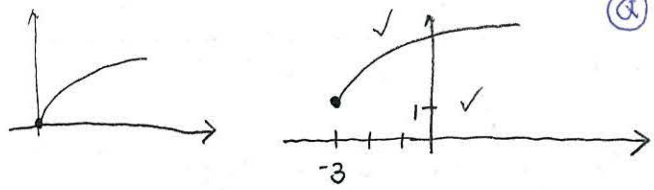
$y = f(x-3)$ — $y = |x-3|$

$y = f(x) + 2$ — $y = |x| + 2$

$y = f(x-3) + 2$ — $y = |x-3| + 2$

ⓐ Sketch the graph of $y = \sqrt{x+3} + 1$ without using a calculator

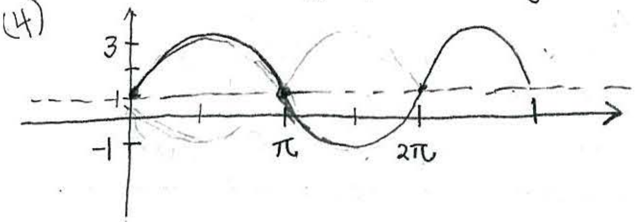
(3)



ⓐ Is $f(x) = x^3 \sin x - \frac{x^2}{\cos x} + 1$ odd, even, or neither? even

ⓐ Sketch the graph of $y = -2\sin(x - \pi) + 1$ without using a calculator.

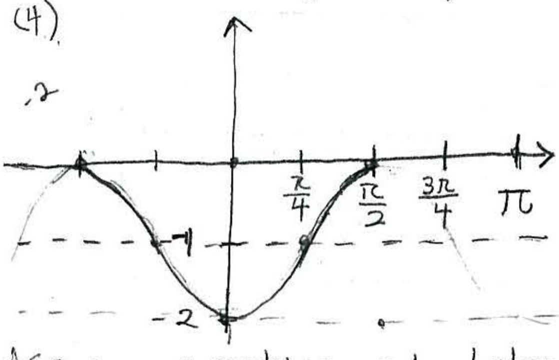
(4)



1. π
1. period
1. flip
1. Amplitude

③ Sketch the graph of $y = \cos(\underset{\text{① shrink}}{2}(x + \underset{\text{② left}}{\tfrac{\pi}{2}})) - 1$ without a calculator.

(4)

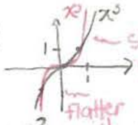


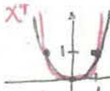
1 start
1 period
1 down
1 shape

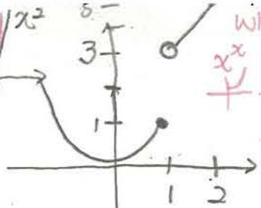
★ Bring graphing calculator. Height = ____ cm. Foot Length = ____ cm

(2) Regression & Calc Wksht → Quiz

$\boxed{2.2}$ $\boxed{ex3}$ Library & CALC 1.9







which is bigger? $x^x > 1.1^x > x^{100}$ x^{99}

new 2.3 (2) ∀ see SAT 19

$\boxed{ex5}$ 4

$$f(x) = \begin{cases} x^2 & x \le 1 \\ 2x+1 & x > 1 \end{cases}$$

which is bigger? $x^{\frac{1}{2}}$ $x^{1.1} > \log x$

CALC

$Y_1 = (X \le 1)X^2 + (X > 1)(2X+1)$

2nd (test)

$\lim_{x \to \infty} \frac{x^{0.1}}{\ln x} = \infty$

$\boxed{ex7}$ 6

$f(x) = \lfloor x \rfloor = \text{int}(x) = \text{floor}(x)$

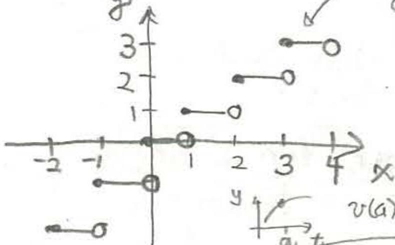
MATH NUM abs( ) int( )

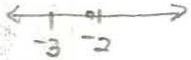
$f(1) = 1$

$f(1.5) = 1$

$f(7.8) = 7$

$f(-2.1) = -3$





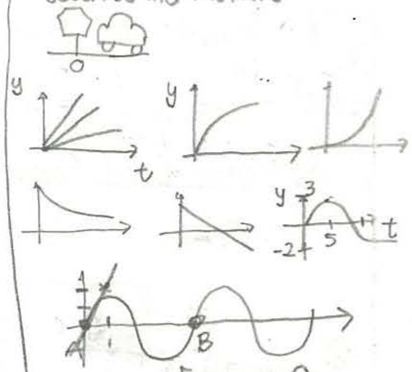
t changes $= \lim_{t \to a} \frac{y(t) - y(a)}{t - a}$

$v(a) = \lim_{h \to 0} \frac{y(a+h) - y(a)}{h}$ h changes

$$f(x) = \lfloor x \rfloor = \begin{cases} \vdots & \\ -1 & -1 \le x < 0 \\ 0 & 0 \le x < 1 \\ 1 & 1 \le x < 2 \\ 2 & 2 \le x < 3 \\ \vdots & \end{cases}$$

$\text{ceil}(x) = \lceil x \rceil$ $\lceil 2.1 \rceil = 3$ $\lceil -1.5 \rceil = -1$

Describe the motion



$v_{Avg} = 0$

$v_{inst}(A) = 2$

$\boxed{ex8}$ 7

$C(t)$ = cost of phone call for t minutes

69¢ 1st minute

58¢ each extra minute time over

dom = $(0, \infty)$

range = $0.69 + 0.58k$

$k \in \mathbb{N} \cup \{0\}$

✓Match

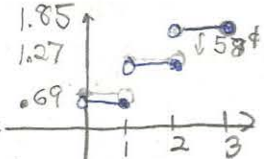
v_{avg} — $\frac{\Delta y}{\Delta t} = m_{sec}$

v_{inst} $a, h \to 0$, $b \to a$ — $\frac{dy}{dt} = m_{tan}$

$v(u) = \lim_{t \to u} \frac{y(t) - y(u)}{t - u}$

$v(t) = \lim_{u \to t} \frac{y(u) - y(t)}{u - t}$

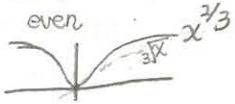
$v(t) = \lim_{u \to 0} \frac{v(t+u) - v(t)}{u}$



$C(t) = 0.69 + 0.58\lceil t - 1 \rceil$

Show TI ex4 CALC $12x^2 + 4x^3 - 3x^4 - \pi$

even



b) $D = \mathbb{R}$

$R = \{y \ge 0\}$

c) increase $[0, \infty)$

2.4 ex5 2.3 $\boxed{ex2}$ a) $f(x) = x^{2/3}$ Sketch

$\boxed{2.3}$ 2.4 ~~function symmetry SAT P4~5~~ (2.5)

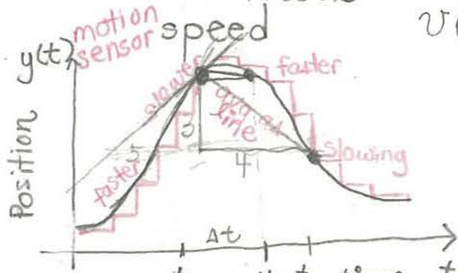
Rate of Change of y wrt x ~ $\frac{\Delta y}{\Delta x}$

PreCal Ch1 P3

- Speed = how fast position changes wrt time $\frac{\Delta y}{\Delta t} = \bar{v}$

Average $= \frac{y(t_f) - y(t_i)}{t_f - t_i}$ = slope of secant line at $[t_i, t_f]$

- Instantaneous speed



$v(t_0) = \lim_{\Delta t \to 0} \frac{\Delta y}{\Delta t} = \lim_{\Delta t \to 0} \frac{y(t+\Delta t) - y(t_0)}{\Delta t}$

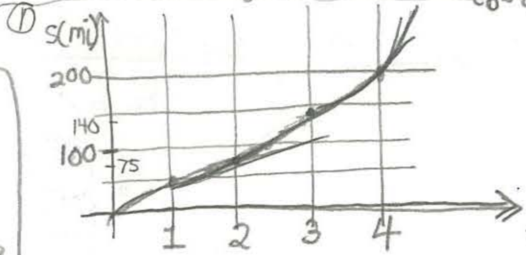
$= y'(t) = \lim_{u \to t} \frac{y(u) - y(t)}{u - t}$

= slope of tangent line at t

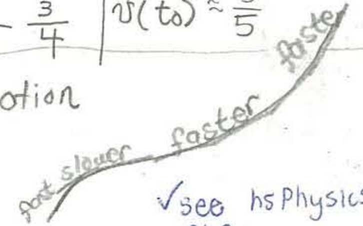
= limit of slope of secant line

$v_{avg\ t_0 \to t_f} = -\frac{3}{4}$ | $v(t_0) \approx \frac{3}{5}$

(1)



motion



✓see hs Physics PPT Ch 2 slides 4, 5, 7

line avg = instantan.

$v_{avg\ 1 \sim 4\,PM} = \frac{s(4) - s(1)}{4 - 1} = \frac{200 - 50}{3} \frac{mi}{h} = 50\ mi/h$

$v_{avg\ 2 \sim 3\,PM} = \frac{s(3) - s(2)}{3 - 2} = \frac{140 - 75}{1} = 65\ mi/h$

where? endpt singular stationary

$v(2)$ = slope of tangent line = 50 mph

HW — Physics example on rates

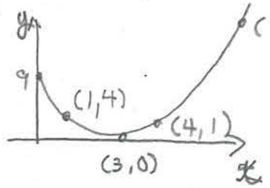
&SAT P1

2.3 Local (relative) max : $f(a)$ is a on I $f(x) \le f(a)\ \forall x \in I$

$f(b)$ is a local/relative min on I : $f(x) \ge f(b)\ \forall x \in I$

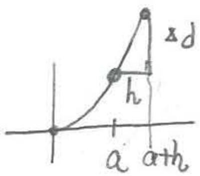
(TI- MAX function)

ex3 (ex1) $f(x) = (x-3)^2$



Avg rate of change on $[4,7]$

$$\frac{\Delta f}{\Delta x} = \frac{f(7)-f(4)}{7-4} = \frac{16-1}{3} = 5$$



ex4 (ex2) $d(t) = 16t^2$ (ft) t(sec)

avg Rate of change of position wrt time

a) avg speed on $[1,5] = \frac{\Delta d}{\Delta t} = \frac{d(5)-d(1)}{5-1} = \frac{400-16}{4} = 96$ ft/s

b) avg speed on $[a, a+h]$

$$= \frac{d(a+h)-d(a)}{(a+h)-a} = \frac{16(a+h)^2 - 16a^2}{h} = \frac{32ah + 16h^2}{h}$$

($a^2+2ah+h^2$)

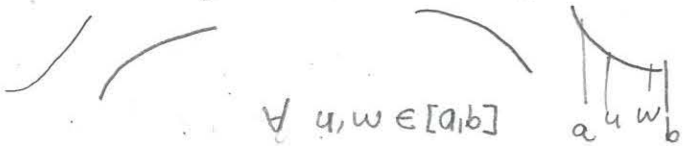
$= 32a + 16h$

c) inst. speed at a

$$v(a) = \lim_{h\to 0} \frac{d(a+h)-d(a)}{h} = \lim_{h\to 0}(32a+16h) = 32a$$

2.3

Strictly Monotonically Increasing | Decreasing on $[a,b]$



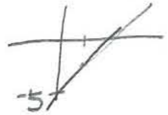
$\forall\, u, w \in [a,b]$

$f(u) < f(w)$ if $u < w$ | $f(u) > f(w)$ if $u > w$

$f'(u) > 0 \;\; \forall u \in [a,b]$ slope | $f'(u) < 0 \;\; \forall u \in [a,b]$ slope

new edition Ch2 Focus 3.1

ex6 (ex4) $f(x) = 3x - 5$



{slope of secant line on $[3,7]$ — $\frac{f(7)-f(3)}{7-3}$
{avg rate of change on $[a, a+h]$ is 3

{inst rate of change
{slope of tangent Line — at $x = a$

$$f'(a) = \lim_{\Delta x \to 0} \frac{\Delta f}{\Delta x} = \lim_{h\to 0} \frac{f(a+h)-f(a)}{h}$$

$$= \lim_{h\to 0} \frac{3a+3h-5-3a+5}{h} = \lim_{h\to 0}\frac{3h}{h} = \boxed{3}$$

★ $\boxed{\text{avg} = \text{inst rate for a line}}$ ★

secant slope = tangent slope

HW ~ fill in chart 1st Page (Library of Functions)

2.5 (3.1) Quadratic Functions – Review SAT Pg 9 — $a(x-h)^2 + k = y$ Vertex (h,k)

ex3 $f(x) = -x^2 + x + 2$ — Standard ax^2+bx+c

a) ① vertex ~~standard~~ form by completing the square OR $-\frac{b}{2a}$

$$f(x) = -\left(x^2 - x + \tfrac{1}{4}\right) + 2 + \tfrac{1}{4}$$

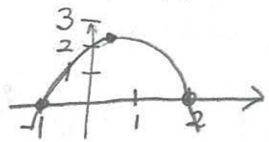
$2\cdot(\frac{1}{2})$

$$= \boxed{-\left(x-\tfrac{1}{2}\right)^2 + \tfrac{9}{4}}$$

② Check $V\left(-\frac{b}{2a}, f\left(-\frac{b}{2a}\right)\right)$ — $h = -\frac{b}{2a} = -\frac{1}{2(-1)} = \frac{1}{2}$

✓ $\left(\frac{1}{2}, \frac{9}{4}\right)$ — $k = -\left(\frac{1}{4}\right) + \frac{1}{2} + 2 = \frac{1}{4} + 2 = \frac{9}{4}$

b) ④ Sketch



③ $f(x) = -x^2 + x + 2 = \boxed{(-x+2)(x+1)}$ factored form

$-1 \quad 2$
$1 \quad 1$

OR Quadratic formula r_1, r_2 — $a(x-r_1)(x-r_2)$

⑤ Max or Min & Where?

Max is at $x = \frac{1}{2}$, max value $= \frac{9}{4}$

2.4 Precalculus 13.3 Do examples as HW

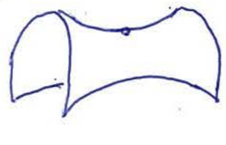
Matching

2.3

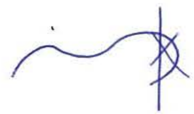
- Upper Bound
  - Least Upper Bound $\begin{cases} \text{maximum (achieve)} \\ \text{supremum (asymptote)} \end{cases}$ $f(x) \le M \ \forall x \in I$

- Lower Bound
  - Greatest Lower Bound $\begin{cases} \text{minimum} \\ \text{infimum} \end{cases}$ $f(x) \ge M \ \forall x \in I$

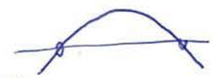
- Saddle Point   both a max & min



function
vertical line test A C



horizontal line test B D



B $x_1 \neq x_2 \Rightarrow f(x_1) \neq f(x_2)$

D $x_1 = x_2 \Leftarrow f(x_1) = f(x_2)$

(one to one fn)
horiz line test

~~$f(x_1) = f(x_2) \Rightarrow x_1 = x_2$~~

~~$f(x_1) \neq f(x_2) \Leftarrow x_1 \neq x_2$~~

not unequal

function
A $x_1 = x_2 \Rightarrow f(x_1) = f(x_2)$

C $x_1 \neq x_2 \Leftarrow f(x_1) \neq f(x_2)$

(2.7) one-to-one function E $x_1 \neq x_2 \Leftrightarrow f(x_1) \neq f(x_2)$

- Prologue example
- Prologue P4 amoeba
- P 70 #63 Bob & Jim

Contrapositive

$D \not\Leftrightarrow R$
cloud

$R \Rightarrow C$
rain cloud

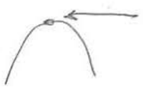
$R^c \Leftarrow C^c$

Demo

ex5 $M(s) = -\frac{1}{28}s^2 + 3s - 31$, $15 \le s \le 70$

gas mileage (miles per gal) — speed (mph)

a) Max gas mileage = 32 mi/gal
when speed = 42 mph



$-\frac{b}{2a} = -\frac{3}{2(-\frac{1}{28})} = 14 \times 3 = 42$

max mileage $= M(42) = 32$

$= -\frac{1}{28} 42 \cdot 42 + 3 \cdot 42 - 31$
$= 42(3 - \frac{3}{2}) - 31$
$= 42 \cdot \frac{3}{2} - 31 = 21 \cdot 3 - 31$
$= 63 - 31$
$= 32$

b) Get in standard form fast

$M(s) = -\frac{1}{28}(s - 42)^2 + 32$

c) Get in standard form by completing square

$M(s) = -\frac{1}{28}(s^2 - 84s + 42^2) - 31 + \frac{42^2}{28}$

± doesn't matter → except that

3×28

2×42

$= \boxed{-\frac{1}{28}(s - 42)^2 + 32}$

-31 $\quad \frac{(7 \times 6)(7 \times 6)}{28} = 9 \times 7 = 63$

d) factored form

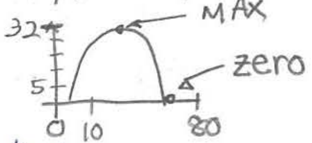
ANS entry (save variable)

$= 42 \mp 14\sqrt{32/7}$

$r_1, r_2 = \frac{-3 \pm \sqrt{9 - 4(-\frac{1}{28})(-31)}}{2(-\frac{1}{28})} = 12.07,\ 71.93$

$M(s) = -\frac{1}{28}(s - r_1)(s - r_2)$

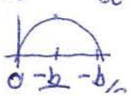
SHOW CALC Check



TRACE

Value input

Note: $ax^2 + bx + 0 \rightarrow ax(x + \frac{b}{a})$ ← $c = 0$ use factored form



ex7 (skip) max value of

$l(t) = -0.0113t^3 + 0.0681t^2 + 0.198t + 99.1$

(5.1514, 100.38241)

Regression - extrapolation
- interpolation

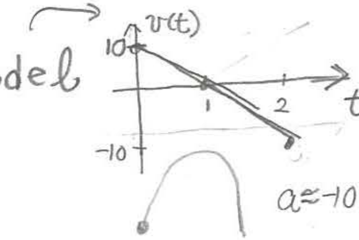
Predict at later time

2.6 Model

Pg 213 Focus

$f(t)$ is an algebraic model from geometry

$V(r) = \frac{1}{3}\pi r^2$



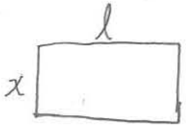
$a \approx -10\ m/s^2$

• Modeling from data - Linear regression Later

predict

cw

ex2 (Pg 215 ex2) 6th



140 feet of fencing

$2x + 2l = 140$

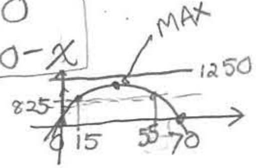
$l = 70 - x$

a) function for area

$A = xl$

$A(x) = x(70 - x)$

CALC



b) what range of widths make area $\ge 825\ ft^2$

$x(70 - x) \ge 825$

$x^2 - 70x + 825 \le 0$

-15

-55

$(x - 15)(x - 55) \le 0$

$\boxed{15 \le x \le 55}$

c) can area be $1250\ ft^2$?

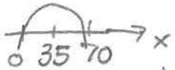
$x(70 - x) = 1250$

$x^2 - 70x + 1250 = 0$

no

d) Dimensions of largest area?

$A(x) = x(70-x)$



Logically Square $-(x-35)^2+1225$

$\cancel{A(35) = 35 \cdot 25 = 875}$

$\cancel{(30+5)(20+5)}$

$\cancel{\boxed{35\text{ft} \times 25\text{ ft}} = 875\text{ft}^2}$

$35 \times 35 = 1225$

ex3 (6th 3.1 ex6) CW

ticket price

$A(t) = -1000(t-14)+9500$

attendance $= \boxed{-1000t + 23500}$

$A(14) = 9500$

$\frac{\Delta A}{\Delta t} = \frac{+1000}{-1}$

OR logically

$A(t) = 9500 + 1000(14-t)$

add; for every dollar bellow

a) Revenue

price × attendance

$R(t) = t \cdot A(t) = t(23500 - 1000t)$

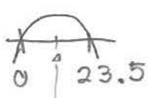
$= -1000t^2 + 23500t$

b) When will there be no attendance, no revenue?

- $A(t) = 0 \Rightarrow 23500 = 1000t$

$t = \$23.5$

- $R(t) = 0$



c) Find price that Maximizes the Revenue

$t = \boxed{\$11.75}$

A hockey team plays in an arena w/a seating capacity of 15000 ppl with ticket price \$14, avg attendance at games is 9500. A survey shows for each dollar the ticket price is lowered, avg attendance increases by 1000. → Line slope

$A(14) = 9500, \ \frac{\Delta A}{\Delta t} = -1000$

Practice Pg. 211

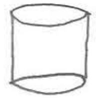
Area problem: 19 23

Revenue/Line word prob: 27 28

minimize time: 34

(6th pg 217 ex3) CW

ex4 CALC What radius minimizes the amount of metal in a can that holds 1 L of ~~water~~ oil? (1000cm^3)



Objective: surface area

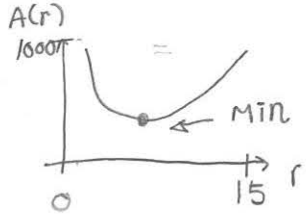
$A(r) = 2\pi r^2 + 2\pi r h$

$\pi r^2 h = 1000\text{cm}^3$

$h = \frac{1000}{\pi r^2}$

minimize $A(r) = 2\pi r^2 + 2\pi r\left(\frac{1000}{\pi r^2}\right)$

$= 2\pi r^2 + \frac{2000}{r}$



Min at $r = \boxed{5.4\text{ cm}}$

$A = 554\text{cm}^2$

SAT

50 Questions

$R \geq 44 \Rightarrow S = 800$

ex For each point below 44 for raw score R, the scaled score decreases by 10 points. $0 \leq R \leq 44$

Write $S(R) = 10(R-44)+800 = 800 - 10(44-R)$ Logically

$800 \quad R \geq 44$

$\frac{\Delta S}{\Delta R} = \frac{-10}{-1} = 10$

$S(44) = 800$

2.5 Transformations

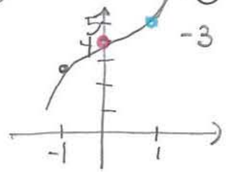
① - important in Calculus. Mem lib, building blocks. Physics waves time & space, circular motion, signal cell phone, - video sig processing - coding animation. Later rotations too

- symmetry even/odd SAT P4~5

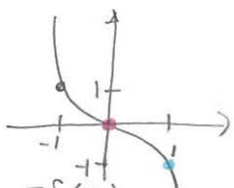
vertical (outside) + k
- $\boxed{f(x)}$ y

② $f(x) = x^3$ (on board) 3 min write 7 minutes

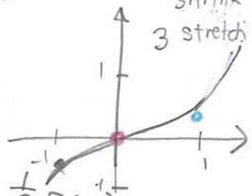
$g(x) = f(x) + 3$ up, -3 down



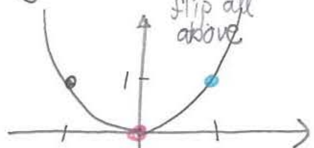
$g(x) = -f(x)$ flip across x axis



$g(x) = \frac{1}{3} f(x)$ shrink, 3 stretch



$g(x) = |f(x)|$ flip all above



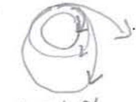
x	$f(x)$	$f(x)+3$	$-f(x)$	$\frac{1}{3}f(x)$	$\lvert f(x)\rvert$
-1	-1	2	1	$-\frac{1}{3}$	1
0	0	3	0	0	0
1	1	4	-1	$\frac{1}{3}$	1

(11.1, 11.19)
CISE SHM animation?

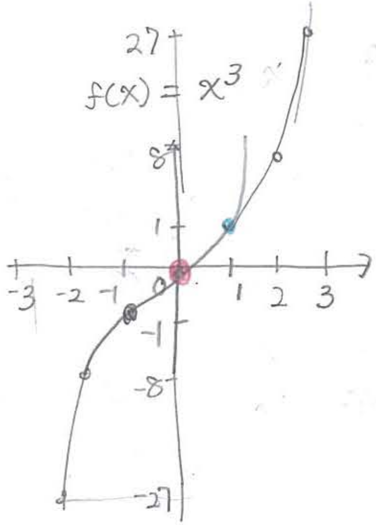
- Rotations GUI button to rotate
- Java
- SciFair? design

Load → [ ] matrix → Pixels → export to JPG

- Satellite launch simulator

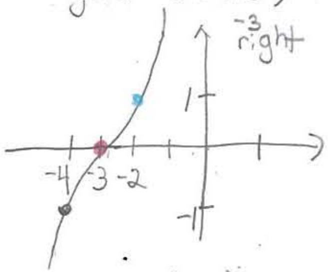


Horizontal inside ( )



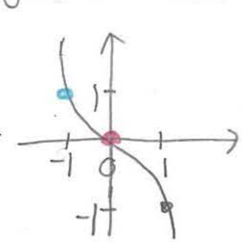
Thinking backwards

$g(x) = f(x+3)$ left, -3 right



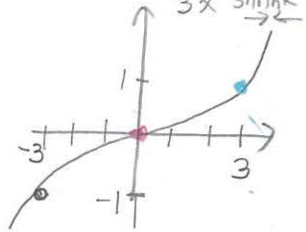
$g(0) = f(3)$
$g(-4) = f(-1)$
$g(-3) = f(0)$
$g(-2) = f(1)$

$g(x) = f(-x)$ left/right flip across y axis



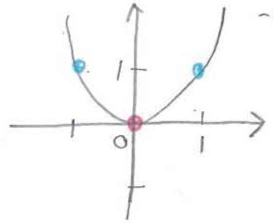
$g(-1) = f(1)$
$g(0) = f(0)$
$g(1) = f(-1)$

$g(x) = f(\frac{1}{3}x)$ stretch ↔, 3x shrink



$g(-3) = f(-1)$
$g(0) = f(0)$
$g(3) = f(1)$

$g(x) = f(|x|)$
- flip right to the left
- lose the left



$g(-1) = f(1)$ ← right side
$g(0) = f(0)$ } same
$g(1) = f(1)$

- book examples on notes P4
- my SAT examples
- Textbook cw
- Combined Transformations

Translations

when is $g(\ ) = 1$?
when $f(1)$

$g(x) = f(\underbrace{x+3}_{\tilde{x}})$

$x = \tilde{x} - 3$
new old left

Reflections Scaling

$g(x) = f(\overbrace{3x-6}^{\tilde{x}}) = f(3(x-2))$
1st to x coord.

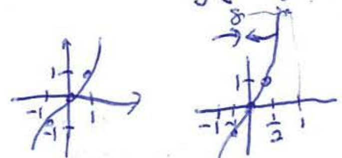
old $\frac{\tilde{x}+6}{3} = x$ new 2nd

$x = \frac{\tilde{x}}{3} + 2$

Absolute Value

Note $(2x)^3 = 8x^3$

$f(2x) = 8f(x)$



2.5 6th ed. Symmetry even odd See SAT P4~5

☆(2.4) Transformations ★ & Library

outside → vertical
$r(x) \pm 3$
inside: horizontal

- see SAT pg. 5
- make page for chart (tell students)

③ ① $\boxed{\text{Translations}}$

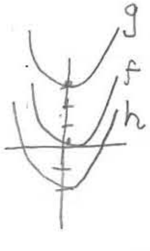
me board

$\boxed{\text{ex1}}$ $f(x) = x^2$

write in HW

a) $g(x) = x^2 + 3 = f(x) + 3$

b) $h(x) = x^2 - 2 = f(x) - 2$

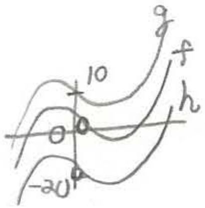


$\boxed{\text{ex2}}$ $f(x) = x^3 - 9x = x(x-3)(x+3)$

write in HW

a) $g(x) = x^3 - 9x + 10 = f(x) + 10$

b) $h(x) = x^3 - 9x - 20 = f(x) - 20$

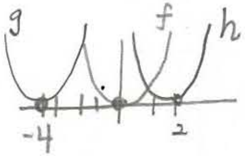


$\boxed{\text{ex3}}$ $f(x) = x^2$

write in HW.

a) $g(x) = (x+4)^2 = f(x+4)$

b) $h(x) = (x-2)^2 = f(x-2)$

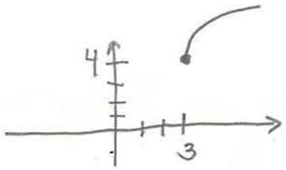


★$\boxed{\text{ex4}}$ Combining $p(x) = \sqrt{x}$

$f(x) = \sqrt{x-3} + 4$

$= p(x-3) + 4$

write in HW



wRite the function

$r(x) = x^3$

① move ~~left~~ right 2 $(x-2)^3 = h$
flip down ↓ $-(x-2)^3 = -h = g$
down 1 $-(x-2)^3 - 1 = g - 1 = l$
Stretch ↕ 3 $-3(x-2)^3 - 3 = 3l$

② stretch 2 ↕ $2x^3$
up 5 → flip down $-2x^3$
flip left $-2(-x)^3 + 5$
move right 3 $+2(x-3)^3 + 5$
stretch 2 ↔ $2(\frac{1}{2}x - 3)^3 + 5$

$-2x^3 + 5$

$h(x) = r(x-3) = (x-3)^3$

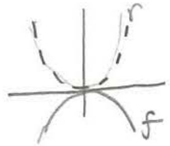
$g(x) = h(x) + 1 = (x-3)^3 + 1$

$l(x) = 2g(x) = 2(x-3)^3 + 2$

$m(x) = -2(x-3)^3 - 2 = -l(x)$

② $\boxed{\text{Reflections}}$ - See SAT pg 5

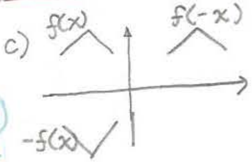
a) $f(x) = -x^2$
$= -\boxed{r(x)}$



b) $g(x) = \sqrt{-x}$
$= r(-x)$ ← write in HW ★

parent fn.
$r(x) = \sqrt{x}$

c)



CW ⑤ SAT P6

P5 ⑪

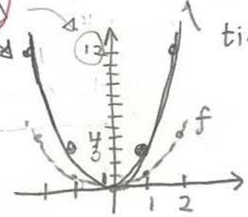
③ $\boxed{\text{Stretch/Shrink}}$ - Scaling see SAT Pg. 5

$\boxed{\text{ex6}}$ $f(x) = x^2$ $4x^2$

a) $g(x) = 3x^2$
$= 3f(x)$

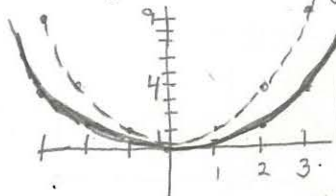
times y-value

show axes

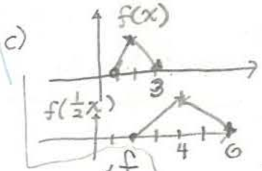


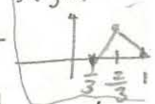
x	f	3f

b) $h(x) = \frac{1}{3}x^2$ $(2x)^2$
$= \frac{1}{3}f(x)$



c)



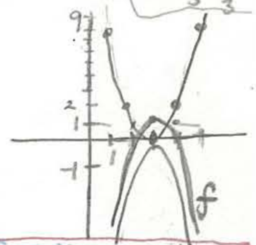


CW: # 2~38 even

$\boxed{\text{ex7}}$ ⑧ ③ Combining
$f(x) = 1 - 2(x-3)^2$
$= -2r(x-3) + 1$

★ Parent fn
$r(x) = x^2$

$Af(B(x-h)) + k$



vertically order of ops
Stretch before translate vertically
Stretch horiz before translate horiz
opposite
Thinking backwards

Is any order OK? No!

r(x)
move right 3
move up 1
stretch ↕ 2
flip ↷

OK
r(x)
move right
stretch & flip ②
up ①

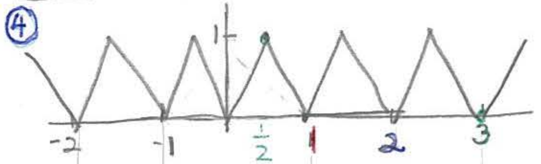
OK
r(x)
Stretch & flip ②
up 1
move right

Horiz Stretch/shrink

HW show scale

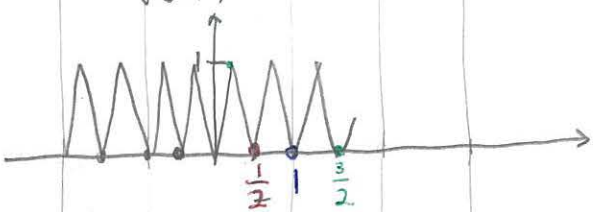
$\boxed{ex8}$ 4

$f(x)$



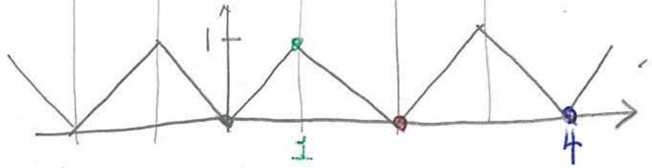
1-periodic

$f(2x)$ ← shrink 2. Each $x \cdot \frac{1}{2}$, y value same



$\frac{1}{2}$ periodic ☆ $\boxed{\frac{P}{B}}$

$f(\frac{1}{2}x)$

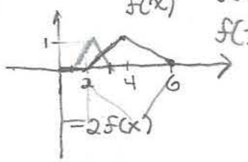


Period = $\frac{1}{1/2} = 2$ ...

☆(about y-axis)

$f(2x)$

$f(\frac{1}{2}x)$



* Periodic means (p)

$f(x+p) = f(x)$

- $g(x) = f(Bx)$

$g(x+T) = g(x)$ find T

$f(B(x+T)) = f(Bx)$

$f(Bx + \boxed{BT}) = f(Bx)$

when $BT = p$

$\boxed{T = \frac{P}{B}}$

ⓐ $f(x) = \sqrt{x}$

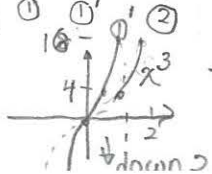
left 2	$\sqrt{x+2}$
up 3	$+\sqrt{x+2}+3$
flip down	$-\sqrt{x+2}-3$
→← 3	$-\sqrt{3x+2}-3$
↕5	$-5\sqrt{3x+2}-15$
flip left	$-5\sqrt{-3x+2}-15$

ⓐ $16x^3 - 2$

$2[(2x)^3 - 1]$

$2f(2x) - 2$

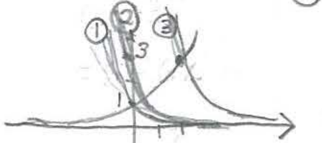
① ①' ②



ⓐ $g(x) = 3e^{-2x+4} + 2$

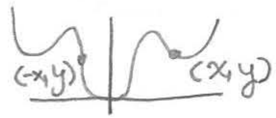
$= 3f(-2(x-2)) + 2$

② ① ③ ④



even

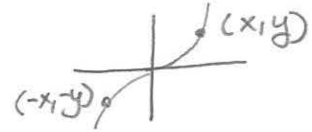
$f(-x) = f(x)$



symm about y-axis

odd

$f(-x) = -f(x)$



symm about origin

$\boxed{ex9}$ a) $f(x) = x^5 + x$
$=$ odd + odd
$=$ odd

b) $g(x) = 1 - x^4$
ev ev
$=$ even

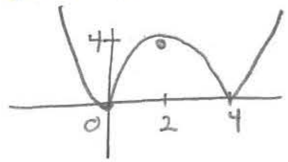
c) $h(x) = 2x - x^2$
$=$ odd + even
$=$ neither

$h(-x) = -2x - x^2$
$\neq h(x)$
$\neq -h(x)$

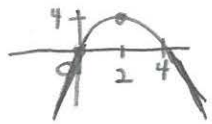
Class Practice - & give answers

6 $\boxed{\#2 \sim 48, 71 \text{ even}}$ Pg 190 *Use graph paper

71b 5 $g(x) = |4x - x^2|$



$f(x) = 4x - x^2$
$= x(4-x)$



next: Multiple horizontal transformations

9a (1) factor out the const in front of x

(2) Outside in. (Scale before translate in each direction)

$y = f(-Bx + C) = f(\underbrace{-B}_{}(x - \frac{C}{B}))$

① flip stretch ② left/Right

ⓐ x^2 $(-2x+1)^2$

What happens if

OR

① left C

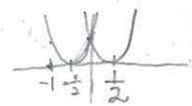
② Shrink/stretch/flip?

$f(x+C) = g(x)$ $(x+1)^2$

$g(-Bx) = f(-Bx+C)$ $(-2x+1)^2$

ex OR $[-2(x-\frac{1}{2})]^2 = f(-2(x-\frac{1}{2}))$

left, then shrink



Combining Transformations

Ⓠ Like Quiz

⑦ demo $r(x) = x^3$

	move right 2	$r(x-2) = (x-2)^3$
$h(x)$	flip down ↓	$-r(x-2) = -(x-2)^3$
$h(x)-1$, $g(x)$	down 1	$-r(x-2)-1 = -(x-2)^3-1$
$3g(x)$	Stretch ↕ 3	$-3r(x-2)-3 = -3(x-2)^3-3$

& mention (8, 9a 9b)

① vertical $\underset{①}{3}f(x)\underset{②}{+4}$ order of ops $\underset{②}{3}[f(x)\underset{①}{+4}]$

Horiz-Op order $f(\underset{①}{3}(x\underset{②}{-4}))$ $f(\underset{②}{3}x\underset{①}{+4})$

Ⓠ ② cw $r(x) = x^3$

Stretch 2 ↕	$2r(x) = 2x^3$
flip down	$-2r(x) = -2x^3$
up 5	$-2r(x)+5 = -2x^3+5$
flip left	$-2r(-x)+5 = -2(-x)^3+5 = 2x^3+5$
move right 3	$-2r(-(x-3))+5 = 2(x-3)^3+5$
Stretch 2 ⟷	$-2r(-(\frac{x}{2}-3))+5 = 2(\frac{x}{2}-3)^3+5$

Ⓠ $f(x) = \sqrt{x}$

left 2	$f(x+2) = \sqrt{x+2}$
up 3	$f(x+2)+3 = \sqrt{x+2}+3$
flip down	$-[f(x+2)+3] = -\sqrt{x+2}-3$
→← 3	$-f(3x+2)-3 = -\sqrt{3x+2}-3$
↕ 5	$-5f(3x+2)-15 = -5\sqrt{3x+2}-15$
flip left	$-5f(-3x+2)-15 = -5\sqrt{-3x+2}-15$

(8cw P4 order OK?)

$m_1 - m_2 - t_1 - t_2$
$m_1 - m_2 - t_2 - t_1$
$m_1 - t_1 - m_2 - t_2$

(3 possibilities) ×2 = 6 ways

See P4b 9a

MULTIPLE TRANSFORMATIONS

⑨b Continue to end of sheet & $y = \sin(-2x-2\pi)$ Then 2.6

Parent $y = f(x)$

1b Sketch

$$y = \underset{3}{A}f(\underset{4}{B}(x-\underset{5}{h}))+\underset{6}{k}$$

①′ ②′ Horizontal

① vertical ②

OK ① 2 1′ 2′
1 1′ 2 2′
1′ 1 2′ 2 etc

★ Horizontal & Vertical are independent!

④ • Alternatively

$f(\overset{4}{B}x - \overset{2}{M})$ ② ①

Horizontal: always opposite arithmetic order of operations

$\overset{3}{A}[f(x) + \overset{2}{N}]$ ② ①

Vertical: always same as order of ops.

see P5

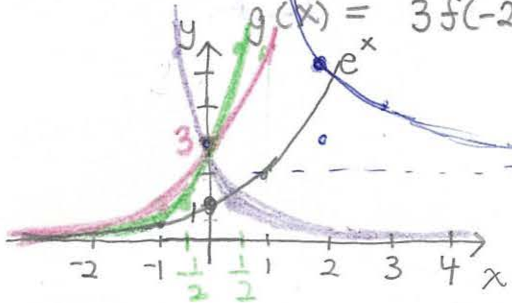
⑤ Ⓠ Sketch $g(x) = 3e^{-2x+4}+2$

Like Quiz 5 boxes

$f(x) = e^x$

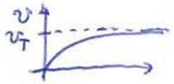
$g(x) = 3f(-2x+4)+2 = \boxed{3}f(-2(x-2))+2$

① shrink ①′ ②′ ③ right 2 up 2



$10(1-e^{-\frac{1}{2}t})$

Physics air drag



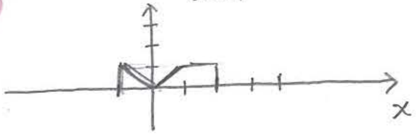
$v(t) = v_T(1-e^{-\frac{k}{m}t})$

$(f = kv)$

Q Like Quiz

6

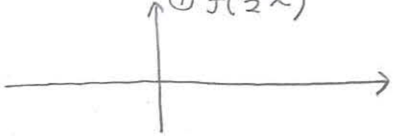
$f(x)$



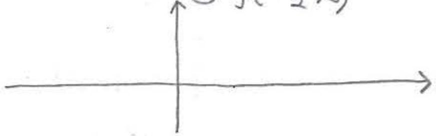
Sketch $y = -2f(-\frac{x}{2}+1) - 2$

$= -2f(-\frac{1}{2}(x-2)) - 2$

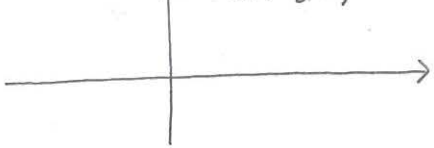
① $f(\frac{1}{2}x)$



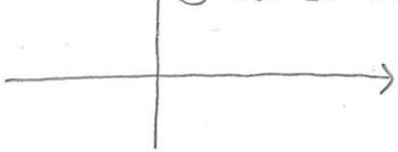
② $f(-\frac{1}{2}x)$



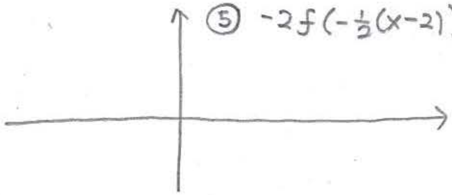
③ $-2f(-\frac{1}{2}x)$



④ $-2f(-\frac{1}{2}(x-2))$

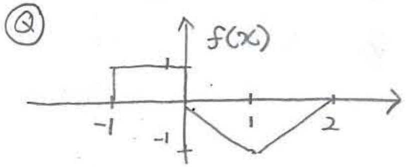


⑤ $-2f(-\frac{1}{2}(x-2))-2$



Q

$f(x)$



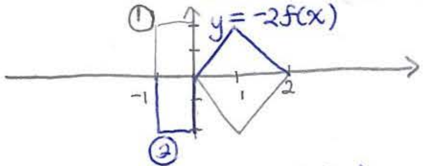
Sketch

$y = -2f(-\frac{1}{2}x - 1) - 2$

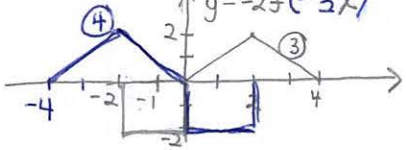
$= -2f(-\frac{1}{2}(x+2)) - 2$

② ① ④ ③ ⑤ ⑥

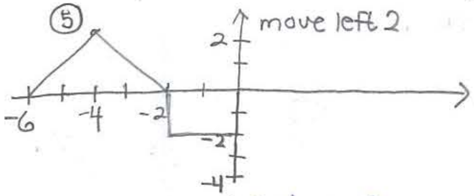
① ② $y=-2f(x)$



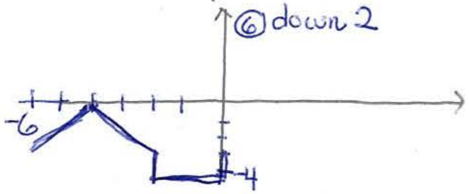
④ ③ $y=-2f(-\frac{1}{2}x)$

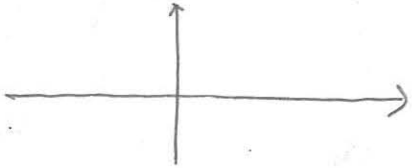


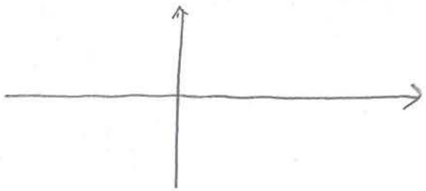
⑤ move left 2



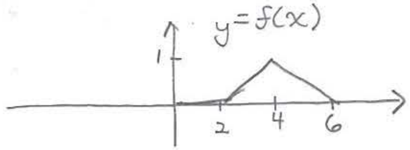
⑥ down 2







Q $y=f(x)$



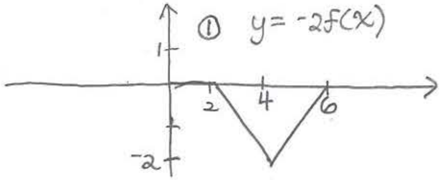
Sketch

$y = -2f(-2x+4) + 1$

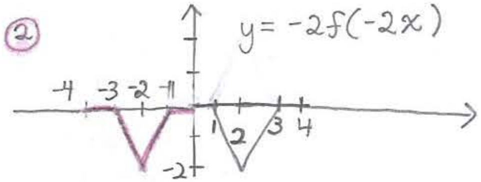
$= -2f(-2(x-2)) + 1$

① ② ③ ④

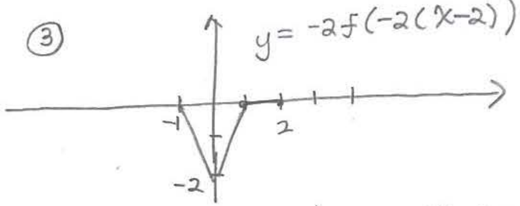
① $y=-2f(x)$



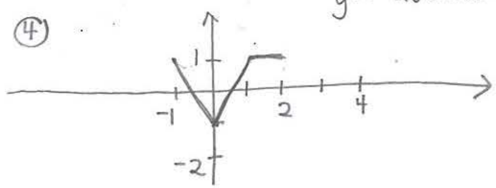
② $y=-2f(-2x)$



③ $y=-2f(-2(x-2))$



④ $y=-2f(-2(x-2))+1$

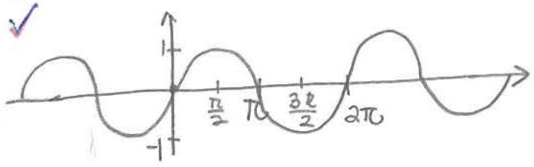


(See ex7 on P.4)

ex9 p4 odd/even

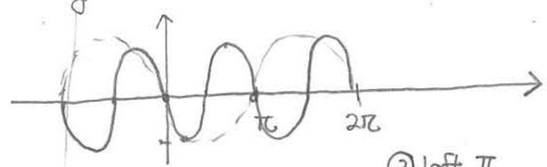
Q

4 $f(x) = \sin x$ ✓



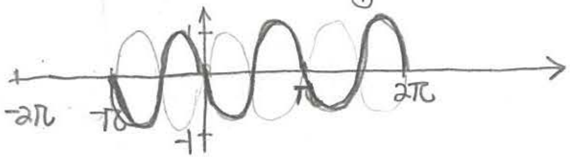
method 1

$y = \sin(\overset{②}{-2}x - \overset{①}{2\pi})$

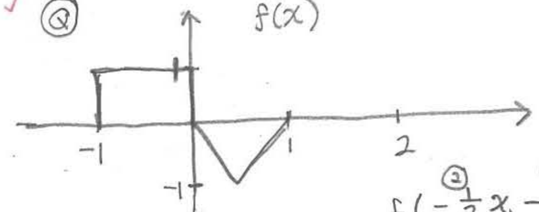


method 2

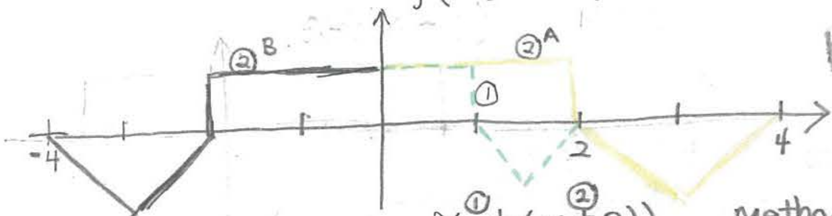
OR $y = \sin(-2(x + \pi))$ ② left π ①



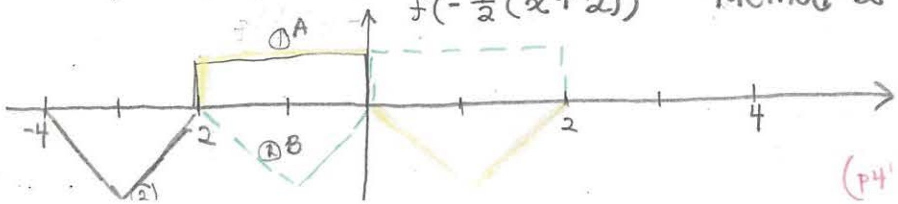
✓ Q $f(x)$



$f(\overset{②}{-\frac{1}{2}}x \overset{①}{-1})$ Method 1



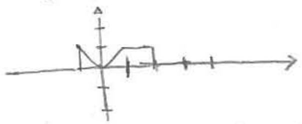
$f(\overset{①}{-\frac{1}{2}}(x \overset{②}{+} 2))$ Method 2



(p4' ⑤)

Q Like Quiz

$f(x)$

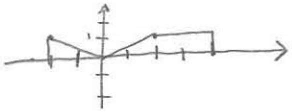


Graph $-2f(-\frac{x}{2} + 1) - 2$

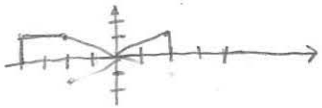
$y = -2f(-\frac{1}{2}(x-2)) - 2$

① $f(+\frac{1}{2}x)$

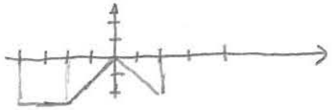
$-2x - 2\pi$. shrink



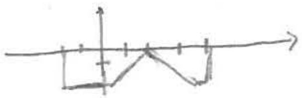
② $f(-\frac{1}{2}x)$



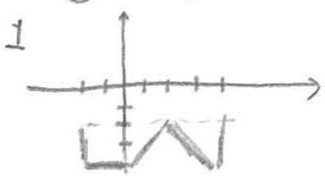
③ $-2f(-\frac{1}{2}x)$



④ $-2f(-\frac{1}{2}(x-2))$



⑤ $-2f(-\frac{1}{2}(x-2)) - 2$



Suggestion

$$y = \overset{①}{A} f(\overset{②'}{B}x + \overset{①'}{C}) + \overset{②}{D}$$

$$= \underset{①}{A} f(\underset{①'}{B}(x + \underset{②'}{\tfrac{C}{B}})) + \underset{②}{D}$$

10 CW/HW 2.5

$\boxed{2.7}$ Algebra of Combining Functions

2.6

f has domain A
g has domain B $\Rightarrow$ $(f \pm_{\div}^{\times} g)(x)$ has domain $A \cap B$, $g(x) \neq 0$

ex1

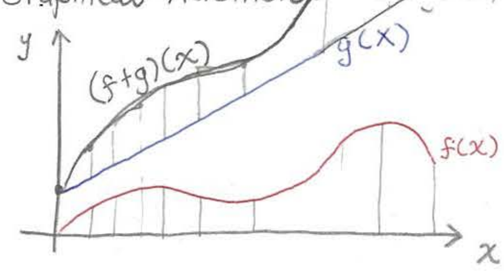
		Domain
	$f(x) = \frac{1}{x-2}$	$\{x \neq 2\}$
	$g(x) = \sqrt{x}$	$\{x \geq 0\}$
a)	$f+g = \frac{1}{x-2} + \sqrt{x}$	$[0, 2) \cup (2, \infty)$
	$f-g = \frac{1}{x-2} - \sqrt{x}$	"
	$fg = \frac{\sqrt{x}}{x-2}$	"
	$\frac{f}{g} = \frac{1}{(x-2)\sqrt{x}}$	$(0, 2) \cup (2, \infty)$

b) at $x = 4$

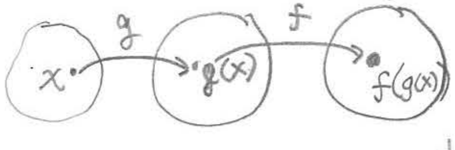
$(f+g)(4) = \frac{5}{2}$ $(fg)(4) = 1$ $f^2(5) = \frac{1}{9}$

$(f-g)(4) = -\frac{3}{2}$ $(\frac{f}{g})(4) = \frac{1}{4}$

ex2 Graphical Addition



ex4 Composition $(f\circ g)(x) = f(g(x)) \neq (g\circ f)(x)$



	domain	
$f(x)=\sqrt{x}$	$\{x\geq 0\}$	
$g(x)=\sqrt{2-x}$	$2-x\geq 0$	$\boxed{\{x\leq 2\}}$

a) $f\circ g = \sqrt{\sqrt{2-x}}$ $\boxed{x\leq 2}$ $(-\infty,2]$

$\sqrt{(\sqrt{x})^2}=|x|$

b) $g\circ f = \sqrt{2-\sqrt{x}}$ $x\geq 0$ & $\sqrt{x}\leq 2$, $x\leq 4$ $\boxed{[0,4]}$

c) $f\circ f = \sqrt{\sqrt{x}}$ $\boxed{x\geq 0}$ $[0,\infty)$

$= \sqrt[4]{x}$

d) $(g\circ g)(x) = \sqrt{2-\sqrt{2-x}}$

$x\leq 2$ &

$\sqrt{2-x}\leq 2$ → $\boxed{[-2,2]}$

positive though $|2-x|\leq 4$

$-4\leq x-2 \leq \cancel{4}$

$-2\leq x \leq \cancel{6}$

$2-x\leq 4$

$-2\leq x$

$f(g(x))$

any even = even

even any/odd = even (not neither)

odd odd odd

ex5 $f(x)=\frac{x}{x+1}$ $g(x)=x^{10}$ $h(x)=x+3$

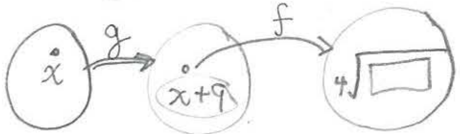
$(f\circ g\circ h)(x) = \frac{(x+3)^{10}}{(x+3)^{10}+1}$

$\frac{f(a+h)-f(a)}{h}$ $\frac{f(b)-f(a)}{b-a}$

Calculus

ex6 * Decomposing into simpler functions

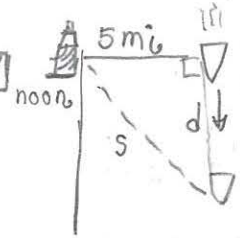
$F(x)=\sqrt[4]{x+9}$ $F=f\circ g$. Find f & g



$\boxed{g(x)=x+9 \quad f(x)=\sqrt[4]{x}}$

$F(x) = f(g(x))$
$= f(x+9)$
$= \sqrt[4]{\boxed{x+9}}$

ex7



20 mph

$\frac{d}{dx}\left(\sqrt{2(5x^2+x+1)^3-7(5x^2+x+1)^2+1}\right)$

$f(g(h(x)))$

$h(x)=5x^2+x+1$
$g(x)=2x^3-7x+1$
$f(x)=\sqrt{x}$

$\frac{d}{dx}(\ ) = \frac{1}{\sqrt{\ }}[6(\ )^2-14(\ )]\cdot(10x+1)$

dist. traveled since noon

a) $s=f(d)=\sqrt{5^2+d^2}$ Related Rates

b) $d=g(t)$ $d(t) = 20t$

miles, time since noon hourse

c) $s(d(t))$ $f\circ g = \sqrt{25+(20t)^2} = s(t)$

dist. from lighthouse to boat as fn of time since noon,

2.7. Pullback $f(x)=\frac{5x}{2-3x}$

$y(2-3x)=5x$ → $x=\frac{2y}{5+3y}$

$2y=x(5+3y)$

$f^{-1}(y)=\frac{2y}{5+3y}$

About variables See SAT Pg 4

a) $f(g(x))=4x^2-8x$
$f(x)=x^2-4$
$g(x)=?$
$\pm 2(x-1)$

a) $g(x)=3x+2$
$g(f(x))=x$ also inverse
$f(2)=?$
0
$f(x)=\frac{x-2}{3}$

(2.8) 2.7 One-to-One Function & Inverse

See p.2! under 2.3

$x_1 \neq x_2 \overset{\text{invertible}}{\Rightarrow} f(x_1) \neq f(x_2)$

$\overset{\text{fn.}}{\Leftarrow}$

Proof : Assume $f(x_1) = f(x_2)$. Then $x_1 = x_2$

cat 4 legs
$A \Rightarrow B$
$B^c \Rightarrow A^c$

ex2 Horizontal Line Test

Is $g(x) = x^2$ one-to-one?

no!

$g(-1) = 1$
$g(1) = 1$

[y fn of x
x not fn of y

ex3 Show a fn is one-to-one

★ $f(x) = 3x + 4$

Logic

$f(x_1) = f(x_2)$

$3x_1 + \cancel{4} = 3x_2 + \cancel{4}$

$x_1 = x_2$

Therefore f is one-to-one

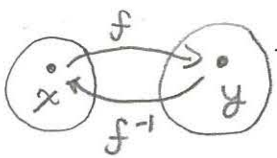
$g(x) = x^2$
$g(x_1) = g(x_2)$
$x_1^2 = x_2^2$
$x_1 = \pm x_2$
$x_1 \neq x_2$
NOT fn.

Calculus

Prove it's always increasing (or decreasing) strictly monotonic

Definition

$y = f(x)$
$x = f^{-1}(y)$

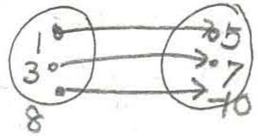


circles
$f^{-1}(f(x)) = x$
$f(f^{-1}(y)) = y$

ex4 $f(1) = 5$
$f(3) = 7$
$f(8) = -10$

$\Rightarrow$

$f^{-1}(5) = \underline{1}$
$f^{-1}(7) = \underline{3}$
$f^{-1}(-10) = \underline{8}$

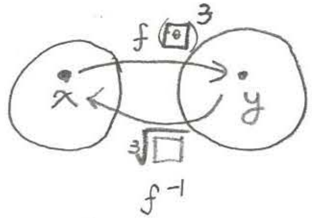


-10 is the image of 8
Pullback of -10 is 8

Inverse fn property

• After 2.7, 1.9 CALC
• Go to notes P2b
3 both sides
Ch2 focus
21, 22, 30
77, 78

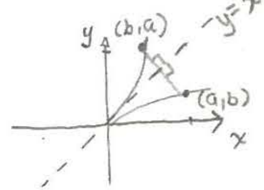
$y = f(x) = x^3$ $\qquad x = \sqrt[3]{y}$



f & f^{-1} undo each other

$$\boxed{\begin{aligned} f(f^{-1}(y)) &= y \\ f^{-1}(f(x)) &= x \end{aligned}}$$

$\Leftrightarrow$ f & f^{-1} are inverse functions



ex5 Verify inverses $f(x) = x^3$
$g(x) = x^{\frac{1}{3}}$

$f(g(x)) = (x^{\frac{1}{3}})^3 = x$
$g(f(x)) = (x^3)^{\frac{1}{3}} = x$

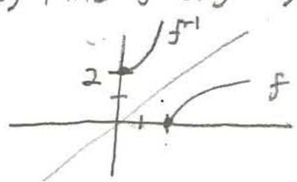
Bonus: inverse is reflected across y=x
Show $m_\perp$

How to find inverse
1. $y = f(x)$
1. $x =$ solve for x $f^{-1}(x)$ Find the x that gave y
2. $f^{-1}(y) =$ expression
3. $f^{-1}(z) = \sim z$ dummy variable $\quad u \rightarrow \boxed{f^{-1}} \rightarrow f^{-1}(u)$ Rule
u u
x x

ex6 $f(x) = \dfrac{x^5 - 3}{2}$
y

① $x = \sqrt[5]{2y+3}$
② $f^{-1}(y)$
③ $f^{-1}(x) = (2x+3)^{\frac{1}{5}}$

Graph of inverse
a) $f(x) = \sqrt{x-2}$ sketch
b) Sketch f^{-1}
c) Find $f^{-1}(x) = x^2 + 2$



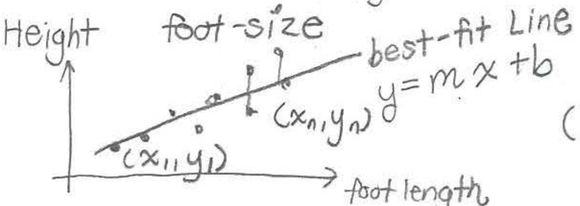
• Library
• SAT Pg 2

• Ch2 FOCUS Word Probs P3

Ch1 FOCUS

Linear Regression (Pg. 239) & prediction

Pg 130



(20cm, 158cm)

$$\hat{m}, \hat{b} = \underset{m,b}{\arg\min} \underbrace{\sum_{i=1}^{n} (y_i - (mx_i + b))^2}_{\text{sum of least squares}}$$

$\hat{m} =$ formula in calculus - see CalcC Bonus Worksheet

$\hat{b} =$

strong

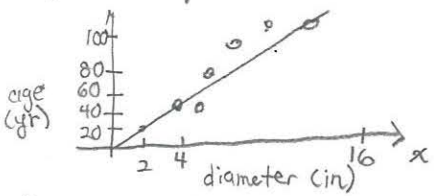
$-1 \le r \le 1$ $r \approx 0$ uncorrelated

correlation coefficient r indicates how good a fit

Pg 244

③ inch d	years age of tree
2.5	15
4	24
6	32
8	56
9	49
9.5	76
12.5	90
15.5	89

a) scatter plot



b) Draw Line

$y = \underbrace{6.451}_{+1 in = +6 yrs.} x - \underbrace{0.1523}_{0 yrs \approx 0 in.}$

c) $y(18) =$ 116 years

d) $r =$

$\boxed{HW}$ answer a, b, c, d for x = foot length of classmates, y = height

F6 cleanup "clear all"
Catalog → "DelVar" a x ← F4

$\boxed{TI-89}$ Intro Linear Least Squares - Calculus will derive

- Home, clear A to Z
  F6 → 2: New Prob
  clears all (lists, data!)

F1 format grid on

- APPS → data/matrix editor
  New: data/main
  Variable: tree (Pg 239)

c1	c2
diameter	age

Pg 244 (P 136)

F6 clear column

- F2 Plot Setup
  F1 define
  Plot1 { as scatter, mark □, x c1, y c2 }
  enter, enter

*Useful - Also show stats for L_1 μ σ

F5 Calc
{ LinReg, x c1, y c2, store reg as Y1 }

$y = ax + b$

⇒ $a = 6.451$
$b = -0.152279$
corr = 0.9503

$\boxed{Y=}$ { y1, Plot 1 } $\boxed{Graph}$ window $\Delta x = 2$
x: 0~20

$\boxed{TI-84}$

- STAT Edit

L3	L4
diam	age

- STAT PLOT
  { Plot1 On, Scatter, XList (2^{nd} 3 → $\boxed{L3}$), YList L4, Mark □ }
  Quit
- Home
  { STAT → CALC
  * $\boxed{\text{LinReg}(ax+b)\ L_3, L_4, Y_1}$
  VARS → Y-Vars (store) function Y_1 }
- { • Y= has Line
  • Window
  • Trace }

Largest feet: Brahim Takioullah
[38.1cm Long left foot
246 cm (>8ft) tall

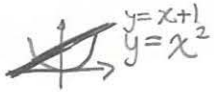
Solve ◆ EE → KEY of shortcuts
◆ 7 → Log

F2 Solve($x^2 - 1 = 0$, x)

84 - Graphically

1.9 Graphing Calculator

① Basic



Bold, Select
Window $[-5,5]^2$
Zoom Square ...

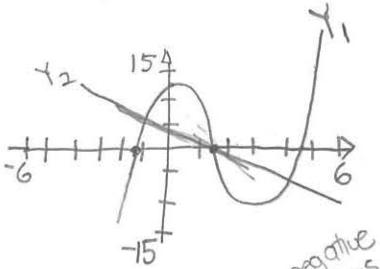
F4 DelVar a

$(y = x^3 - 5x^2 \geq -8)$

② You try
$\begin{cases} y_1 = x^3 - 5x^2 + 8 \\ y_2 = 2.0 - 1.4x \end{cases}$
window

· F1 format grid on

Jump to Y_1, Y_2 ⇕



(– negative
– minus
different on 89)

Zeros

Intersection
#1

try: #2

Y_1 Max

try: Min

★ Do not set x to value or cannot graph

Now ① Solve $x^3 - 5x^2 \geq -8$

Ans: zeros, intervals

Y_1 $\int f(x)dx$

$f'(1) =$

try $f'(4) =$

② solve $x^3 - 5x^2 + 8 = 2.0 - 1.4x$
intersection

OR $Y_3 = x^3 - 5x^2 + 8 - 2 + 1.4x \overset{\text{zeroes}}{=} 0$

③ Solve $(x^2-1=0, x)$ expand$((x-1)(x+1))$ factor$(\cdots)$

④ Solve $\begin{cases} x+y+z=1 \\ 2x+3y-z=3 \\ -5x+2y+3z=3 \end{cases}$

Go over Later too

$$\underbrace{\begin{bmatrix} 1 & 1 & 1 \\ 2 & 3 & -1 \\ -5 & 2 & 3 \end{bmatrix}}_{B} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 1 \\ 3 \\ 3 \end{bmatrix}$$

matrix

try → $A = \begin{bmatrix} 1 & 1 & 1 & 1 \\ 2 & 3 & -1 & 3 \\ -5 & 2 & 3 & 3 \end{bmatrix}$

rref

$\det B =$

$B^T =$

$B^{-1} =$

$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = B^{-1} \begin{bmatrix} 1 \\ 3 \\ 3 \end{bmatrix}$

③ Graphing a Circle

$x^2 + y^2 = 1$

$y^2 = 1 - x^2$

$y_1 = \pm\sqrt{1-x^2}$

y_2

Zoom fit box or square?

Note - can even write programs for TI Pac man?

⑥ Linear Regression - See Ch2 notes at end

- average height → 1-var STATS
  standard deviation

TI-89 TI-84

clearall: vars

clear one var: Home F4 DelVar a done

⑤ TI-89

◆ EE → KEY of shortcuts

$\log_2 x = \frac{\ln x}{\ln 2}$ or $\log(x, 2)$

ans Solve $(x^2 - 1 = 0, x)$

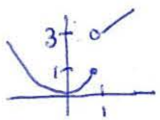
clear x if accidentally set to value:

• $-\frac{7}{3} \to a$ store

$\frac{-\frac{7}{3}}{2(-\frac{7}{3})+7} - \frac{-\frac{7}{3}+1}{-\frac{7}{3}+3} \overset{?}{=} 1$

$$\vec{u} \times \vec{v} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 3 & 2 & 1 \\ 1 & 2 & 3 \end{vmatrix} = \langle 4, -8, 4 \rangle$$

$$y = \begin{cases} x^2 & x \le 1 \\ 2x+1 & x > 1 \end{cases}$$



CALC

$Y_1 = (X \le 1)\,X\text{^}2 + (X>1)(2X+1)$

2nd (Test)

estimate $\sqrt{2} \approx 1.414$ 0.005

$x = \sqrt{2}$ $1 \leq x < 2$

solve $x^2 = 2$

1 1.1 1.4 1.5 2

x	x^2
1.1	1.21
1.2	1.44
1.3	
1.4	1.96
1.5	2.25
1.6	
1.7	
1.8	
1.9	

1.4 1.5

x	x^2
1.4	
1.41	1.9881
1.42	2.0164
1.43	

estimate $1.415 \approx \sqrt{2}$

$\sin(\frac{\pi}{3}) \simeq 0.866025$?

$\sin(\frac{\pi}{3}) = \frac{\sqrt{3}}{2}$

estimate $\frac{\sqrt{3}}{2} = x$

$\sqrt{3} = 2x$

$3 = 4x^2$

solve $x^2 = \frac{3}{4}$

0 1

x	x^2
0.1	
0.2	
0.3	
⋮	
0.8	0.64
0.9	0.81
1.0	

past 0.75

x	x^2
0.81	
0.82	
⋮	
0.86	0.7396
0.87	0.7569

0.75

estimate $\left|\boxed{0.865} - \frac{\sqrt{3}}{2}\right| < 0.005$

ε