James Stewart Precalculus 6th edition

1.1 (P.10) # 2, 36, 46, 54, 58, 60, 64, 70, 72c
old → 5, 40, 49, 57, 61, 63, 67, 74, 75c
new

1.2
old (P21) # 1~8, 18, 26, 40, 44, 50, 52, 70, (80), 85, 87, 92, 93
new (P21) 7~14, 24, 31, 48, 69, 50, 59, 56, 60, 75, 77

1.3
old (P. 31) # 26, 30, 40, 54, 58, 64, 68, 88, 102
6th (Pg 32) Concept 1~6, 39, 53, 49, 73, 79, 85, 91, 113, 125

Quiz Thurs 1.1 - 1.4 (110)

$[-5,3) \cup (2,7) = [-5,7)$

$[-5,3) \cap (2,7) = (2,3)$

$A = \{2n : n \in \mathbb{Z}\}$ (0, ±2, 4, 6, 8, ...)

$B = \{3m : m \in \mathbb{Z}\}$ (0, ±3, ±6, ±9, ±12, ...)

$A \cup B = \{0, \pm 2, \pm 3, \pm 4, \pm 6, \ldots\}$

$A \cap B = \{6n : n \in \mathbb{Z}\}$

1.4 Summary ① $\dfrac{\frac{a}{b}+\frac{e}{f}}{\frac{c}{d}+\frac{g}{h}}$ ← same den, same den → flip.

② Use LCD of all fractions

③ factor out the least power in num & denomin

④ Rationalize $(\sqrt{x}+5)$ ← $\dfrac{\sqrt{x}-5}{\sqrt{x}-5}$ conjugate radical

ex Linear Regression

1.4
old (P.41) #4, 6, 8, 15, 26, 28, 50, (55, 59), (63), 65, 71, 77, 81, 93
6th (P.41) # 8, 9, 13, 21, 33, 36, 57, 60, 62, 66, 69, 74, 80, 85, 90, 101

isolate function, radical, abs value, quadratic

1.5
old (P.55) #(3), 12, 20, 30, 34, 36, 44, 45, 49, 60, 62, (65), 69, 71, 73, (76, 84, 85, 93, 96, 113), 80, 82, 101

6th (P.54) 10, 18, 25, 36, 39, 42, 50, 55, 59, 72, 75, 83, 79, 82, 83, 86, 93, 95, 101, 105, 89, 82, 111, 123

1.6
5th 49, 55, 61, 72, 75, 81
6th (P.67) # 51, 57, 64, 74, 79, 86

1.7
5th (P.84) # 25, 32, 38, 57, 59, 61, 71, 75, 78, 80, 86, 90, 105, 111
6th (P.86) # 31, 38, 44, 67, 70, 71, 83, 85, 90, 91, 97, 101, 103, 104, 108, 111, 120, 122, 123, 124

1.8
5th (P97) # 8, 22, 26, 33, 64, 73, 77, 78, 80, 85, 94, 96
6th (P.92) # 14, 22, 27, 31, 40, 44, 46, 64, 79, 81, 82, 83, 84, 85, 99, 107, 109, 117, 122

1.9
5th (109) #20, 30, 54
Graph TI89 6th (104) CR 3, 4, 28, 33, 63 (Ex)
+ Regression for HW data

1.10
5th (120) # 11, 14, 28, 32, 45, 69
6th (115) 7, 17, 32, 36, 50, 60, 63, 66, 73 (76)

1.11
5th (127) 13, 14, 17, 18, 24, 35, 42
6th (121) CR 1~4, 17, 18, 24, 23, 30, 42, 47

1.6 (P.69) #49, 53, 61, 72, 75, 81

1.7 (P.84) #25, 32, 38, 57, 59, 61, 71, 75
78, 80, 86, 90, 105, 111

1.8 (P.97) #8, 22, 26, 33, 64, 73,
77, 78, 80, 85, 94
96

1.10 (120) #11, 14, 28, 32, 45, 69

1.11 (127) 13, 14, 17, 18, 24, 35, 42

1.9 (109) #20, 30, 54
Graphically

Precalculus 2011

T	Th	F
1/4 Q&A 1.9 (2.5) 2.6 - practice Library	- Quiz 2.1 ~ 2.5 ~~2.4~~	2.4 transform. - Library & practice
Quiz • Library • 2.6 prob from hw/ex • 2.4 transform 2.7 Combine fn. 2.8 inverse	Chapter Review	• ~~Quiz~~ • 3.1 7 practice • Linear Regression (Ch. Review)
Quiz 2.4, 2.7, 2.8 - Library - multiple transform	3.3	3.4 ~ 3.5

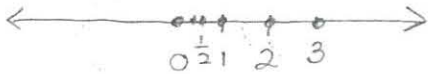
3.2

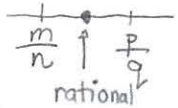
Precalculus

1.1 (P.10) #2, 36, 46, 54, 58, 60, 64, 70, 72c
1.2 (P.21) #1~8, 18, 26, 40, 44, 50, 52, 70, 80, 86, 87
1.3 (P.31) #26, 30, 40, 54, 58, 64, 68, 88, 102.
1.4 (P.41) #4, 6, 8, 15, 26, 28, 50, 55, 59, 63, 65, 71, 77, 81, 93,
1.5 (P.55) #3, 12, 20, 30, 34, 36, 44, 45, 49, 60, 65, 69, 71, 73, 80, 82
101, 62
+ 76, 84, 85, 93, 96, 113
- isolate fraction, radical, abs value
- quadratic pattern

- Syllabus
- Attendance
- Book - sign out
- Classwork

1.1 Real Numbers



What	Example / Reasoning
$\mathbb{N} = \{1, 2, 3, \ldots\}$ natural numbers	counting
whole numbers $= \mathbb{N} \cup \{0\} = \{0, 1, 2, \ldots\}$	no books
Integers $\mathbb{Z} = \{\ldots -3, -2, -1, 0, 1, 2, \ldots\}$	3-2 books
Rational $\mathbb{Q} = \{\frac{m}{n} : m, n \in \mathbb{Z}\}$ $= \{$all fractions$\}$ $= \{$terminating or repeating decimals$\}$	
Irrational ← (fills up gaps) $\mathbb{Q}^c = \{$numbers that cannot be put in fraction form$\}$ $= \{$nonterminating and nonrepeating decimals$\}$	① $2.3\overline{45} = 2.345\,4545\ldots = x \in \mathbb{Q}$

logic: $[(A \cup B)^c = A^c \cap B^c]$

$1000x = 2345.4545\ldots$
$10x = 23.4545\ldots$
$990x = 2322$
$x = \frac{1161}{495} = \frac{129}{55}$

② $2.345 = 2 + \frac{345}{1000} \in \mathbb{Q}$

③ $1.01011011101111\ldots \in \mathbb{Q}^c$

④ $e \approx 2.718\ldots \in \mathbb{Q}^c$
$\pi \approx 3.14 =$ circumference / diameter

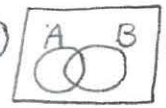
Logic

Sets A, B
$A = \{1, 2, 3\}$ raining
$B = \{-2, 2, 5\}$ cloudy

$2 \in A$ $5 \notin A$
$\emptyset$ null set $\{\}$ empty

① A or B: $A \cup B$ — A ∪ union or
A and B: $A \cap B$ — ∩ intersection and

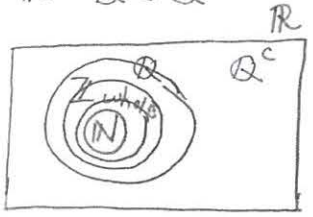
② Contrapositive
$A \Rightarrow B$ means $B^c \Rightarrow A^c$
raining, cloudy — not cloudy, not raining
$\not\Leftarrow$

③ 

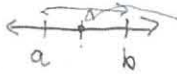
$(A \cup B)^c = A^c \cap B^c$
c complement, not

④ $A \Leftrightarrow B$ iff
$A \overset{\text{only if}}{\Rightarrow} B$
$A \overset{\text{if}}{\Leftarrow} B$

real numbers
$\mathbb{R} = \mathbb{Q} \cup \mathbb{Q}^c$



- all #s measured, all on the line
- tree height
- Why is $2 < 3$? Pie
- Why's $-5 < -2$? debt / missing / took away



midpoint $= \frac{a+b}{2}$
$= a + \frac{b-a}{2}$

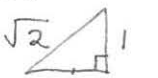
Fundamental Thm of Arithmetic
- any integer has unique prime factorization

$\mathbb{Z}$
- Prime: 2, 3, 5, 7, 11, 13, 17, 19, 23, 29, ...
- Composite: $100 = 2^2 \cdot 5^2$

Project?
- how? cryptology? how comp find primes?

Questions
- Is the sum of two real numbers real? yes
  - integ: integ? yes $1+2=3$
  - rational: rational? yes $\frac{m}{n} + \frac{p}{q} = \frac{mq+pn}{nq}$
  - irrational: irrational? no $(3-\pi)+\pi = 3$
  - rational + integer = rational? yes $1 + 3/2 = 5/2$
  - natural + integer = natural? no $3 - 5 = -2$

⑤ $\sqrt{\text{nonperfect square}} \in \mathbb{Q}^c$
$\sqrt{3}$, $\sqrt{5}$, $\sqrt{\frac{89}{11}}$



If $\sqrt{5} = \frac{m}{n}$
$n\sqrt{5} = m$
$\sqrt{5}$ even ↯

Ch 1 P1

See P2

fractions P. 5

$\frac{ac}{bc} = \frac{a}{b}$ $(c \neq 0)$

$\frac{a}{b} = \frac{c}{d} \Rightarrow ad = bc$ cross multiply

ex3 LCD least common denominator

$$\frac{5}{36} + \frac{7}{120} = \frac{5(2\cdot 5)}{2^2 3^2 (10)} + \frac{7(3)}{2^3 3\cdot 5 (3)} = \frac{71}{360}$$

36 → 3, 12 → 3, 4, 2

triangle inequality

$|a+b| \leq |a| + |b|$

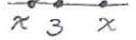
$|1 + (-1)| \leq |1| + |1|$

$0 \qquad\qquad 2$

$|-1-1| \leq |-1| + |-1|$

$2 \qquad\qquad 2$

$(x-3)$ $\begin{cases} >0 & \text{if } x>3 \\ =0 & \text{if } x=3 \\ <0 & \text{if } x<3 \end{cases}$



Graph P. 7

Interval notation set notation

$(2, \pi] = \{x \in \mathbb{R} : 2 < x \leq \pi\}$

$(-\infty, e] = \{x \in \mathbb{R} : x \leq e\}$

ex5 closed $[a,b]$, open (a,b) ← no max or min (yes upper bound, lower bound)

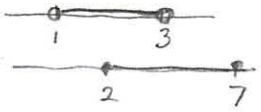
$[1.5, 4) = \{x \in \mathbb{R} : 1.5 \leq x < 4\}$

ex CW

$= [-2,-1] \cup (1,2] = \{x \in \mathbb{R} : -2 \leq x \leq -1 \text{ OR } 1 < x \leq 2\}$

ex6 $(1,3) \cap [2,7] = [2,3) = \{x \in \mathbb{R} : 2 \leq x < 3\}$

on test show final interval shaded



$(1,3) \cup [2,7] = (1,7] = \{x \in \mathbb{R} : 1 < x \leq 7\}$

classwork

ex a) $|x| + |-x| = 2|x|$

b) $|-x| - |x| = 0$

c) $|x-y| - |y-x| = 0$

ex Simplify $0 < x < 2$ $\quad 2x > 0$

$$\frac{|2x|}{|x| + |x-2|} = \frac{|2x|}{|(x) + |x-2|} = \frac{2x}{x - (x-2)} = \frac{2x}{2}$$

$|x| > 0$, $|x-2| < 0$, $|2x| > 0$

$= x$

ex Simplify $0 < x < 1$ $\quad x+7 > 0$

$$\frac{|x+7|}{|x| + |x-1|} = \frac{|x+7|}{|x| + |x-1|} = \frac{x+7}{x-(x-1)} = x+7$$

ex $d(5,-2) = 7$

ex $\frac{2}{\frac{1}{50} + \frac{1}{40}} = \frac{2}{\frac{4}{50\cdot 4} + \frac{5}{40\cdot 5}} = \frac{2}{\frac{9}{200}} = \frac{400}{9} = 44\frac{4}{9}$

$10\cdot 5 \quad 10\cdot 4$

ex incorrect step in "proof" that $2=1$

$a = b > 0$

$a^2 = ab$

$a^2 - b^2 = ab - b^2$

$(a+b)(a-b) = b(a-b)$ ← Assumes $a-b \neq 0$, $a \neq b$ →←

$a+b = b$

$b+b = b$

$2b = b$

$2 = 1$

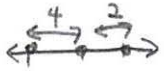
anchor

Absolute Value

$$|a| = \begin{cases} a, & a \geq 0 \\ -a, & a < 0 \end{cases}$$

Pg. 9 Properties

- $|a| \geq 0$
- $|a| = |-a|$
- $|ab| = |a||b|$
- $\left|\frac{a}{b}\right| = \frac{|a|}{|b|}$
- distance from origin



$|0| = 0$

$|-4| = 4 = -(-4)$

$|2| = 2$

manufacture

Target 2 cm, uncertain ±0.1 cm

$1.9 \leq r \leq 2.1$

$|r-2| \leq 0.1$

★ Distance Idea

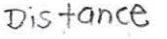
Note

- $|r| = 0.3$ $\quad r = \pm 0.3$
- $|r| < 0.3$ $\quad -0.3 < r < 0.3$
- $|r| > 0.3$ $\quad r < -0.3$ OR $r > 0.3$

$|r-2| = 0.3$ etc

$<$ $>$

Distance



$d(a,b) = d(b,a) = |a-b| = |b-a| \geq 0$

$d(-\pi, -2) = |-\pi + 2| = |-2 + \pi|$

$= \pi - 2$

1.2 exponents & radicals

	Why? Reasoning
$a^n = a \cdots a$	defn.
$a^m a^n = a^{m+n}$	$a^2 a^3 = (a\cdot a)(a\cdot aa)$
$a^0 \equiv 1$	$a^0 a^2 = a^{0+2} = a^2 \Rightarrow a^0 = 1$ (want rule; mult. ID; itself)
$a^{-n} = \frac{1}{a^n}$	$a^{-2}(a^2) = a^{-2+2} = a^0 = 1 \Rightarrow a^{-2} = \frac{1}{a^2}$ (mult. inverse)

Laws P.14

$(a^m)^n = a^{mn}$

$(ab)^n = a^n b^n$

$\left(\frac{a}{b}\right)^n = a^n/b^n$

$\left(\frac{a}{b}\right)^{-n} = \left(\frac{b}{a}\right)^n$

$\frac{a^{-n}}{b^{-m}} = \frac{b^m}{a^n}$

ex2 c) $(-2)^{-3} = \frac{1}{(-2)^3} = \frac{1}{-8}$

ex3 b) $y^4 y^{-7} = y^{4-7} = y^{-3}$

cw ⓒ $\frac{c^9}{c^5} = c^{9-5} = c^4$

ex4 a) $(2a^3b^2)(3ab^4)^3 = 54\,a^{3+3}b^{2+12} = 54a^6b^{14}$ (2·27)

ⓑ $\left(\frac{x}{y}\right)^3\left(\frac{y^3x}{z}\right)^4 = \frac{x^{3+4}y^{8-3}}{z^4} = \frac{x^7y^5}{z^4}$

cw → ex5 eliminate negative exponents

a) $\frac{6st^{-4}}{2s^{-2}t^2} = \frac{3s^3}{t^6}$ b) $\left(\frac{y}{3z^3}\right)^{-2} = \frac{9z^6}{y^2}$

① ⑤

Scientific Notation ⑥

$40,000 = 4\times10^4$

$0.00125 = 1.25\times10^{-3}$

ex6 a) $327900 = 3.279\times10^5$ CALC 3.279 EE 5 (5)

b) $0.000627 = 6.27\times10^{-4}$

ex7 $\frac{(0.000046)(1.697\times10^{22})}{2.91\times10^{18}} = \frac{4.6\times1.697}{2.91}\times10^{-4+22+18} \cong 2.7\times10^{36}$

Why?

Radical (rational exp)

$a^{\frac{1}{n}} \equiv \sqrt[n]{a}$

Pg. 18–19

② $\sqrt[n]{ab} = \sqrt[n]{a}\sqrt[n]{b}$ $(ab)^{\frac{1}{n}} = a^{\frac{1}{n}}b^{\frac{1}{n}}$

$\sqrt[m]{\sqrt[n]{a}} = \sqrt[mn]{a}$ $\left(a^{\frac{1}{n}}\right)^{\frac{1}{m}} = a^{\frac{1}{mn}}$

$\left(\sqrt[n]{x}\right)^n = x$

③ $\sqrt[n]{x^n} = \begin{cases} |x|, & n \text{ even} \\ x, & n \text{ odd} \end{cases}$

CW

$\sqrt[4]{16} = 2$

$\sqrt[4]{-16} = \pm 2i$

$\sqrt[3]{27} = 3$

$\sqrt[3]{-27} = -3$

$\sqrt[6]{a^6} = |a|$

$\sqrt[11]{x^{11}} = x$ ③

$\left(\sqrt[n]{a}\right)^6 = a$ ④ $a \geq 0$ or else undef.

even $\sqrt{2a}$

odd $\sqrt{}$ all OK

$\sqrt[3]{-27} = -3$

$3e^{i60^\circ} = 3\left(\frac{1}{2} + i\frac{\sqrt{3}}{2}\right)$

$3\left(\frac{1}{2} - i\frac{\sqrt{3}}{2}\right)$

$\left(a^{\frac{1}{n}}\right)^n = a^1$

□□□…□ (n) by defn □ = $\sqrt[n]{a}$

$(ab)^{\frac{1}{n}} = a^{\frac{1}{n}}b^{\frac{1}{n}}$ $\left(a^{\frac{1}{n}}\right)^{\frac{1}{m}} = a^{\frac{1}{nm}}$

$\sqrt{x^2} = |x| < 1 \Rightarrow -1 < x < 1$ ★ $x^2 - 1 < 0 \Rightarrow x^2 < 1$ easier

even n^{th} root ⇒ defined positive ⇒ doesn't always exist in $\mathbb{R}$

$\sqrt{4}$ is the # s.t. $\# \cdot \# = 4$

$2\cdot 2 = 4$

$(-2)(-2) = 4$ ⇒ $\sqrt{4}$ is (2) or −2

Principal Root

$\sqrt{4}$ when thinking $\mathbb{R}$

$\sqrt{-4}$ no $\mathbb{R}$, $r\cdot r = -4 \Rightarrow \pm 2i$

$\sqrt[3]{-8} \Leftarrow (-2)(-2)(-2) = -8$ ← unique for odd n^{th} root & always exists

$= -2$ actually there's 3 rd roots $(2\cdot 2\cdot 2 = +8)$

$\left(\pm\frac{\sqrt{2}}{2} \pm i\frac{\sqrt{2}}{2}\right)\sqrt{2}$

$\sqrt{(4)^2} = 4$ $\sqrt[3]{(-2)^3} = \sqrt[3]{-8} = -2$

$\sqrt{(-4)^2} = 4$ $(\sqrt[4]{-2})^4 = \sqrt[4]{2}^4 i^4 = 4$ DeMoivre $(2e^{i\pi})^{\frac{1}{4}} = (\sqrt[4]{2})^{\frac{1}{4}} e^{i\frac{\pi+2k\pi}{4}} = -2$

$\sqrt{(-2)^2} = 2$ $(\sqrt{-2})^2 = (\pm\sqrt{2}\,i)^2 = -2$ ~~or 2~~

$\sqrt[4]{-2} = \sqrt[4]{2}e^{i\frac{\pi}{4} + k\frac{\pi}{2}}$ ★ $i^{\pi+4k}$

✓ ex8 b) $\sqrt[4]{81x^8y^4} = \sqrt[4]{81}\,x^{8/4}\sqrt[4]{y^4} = 3x^2|y|$

ex9 Combining Radicals (largest squares)

✓a) $\sqrt{32} + \sqrt{200} = \sqrt{16\cdot2} + \sqrt{100\cdot2} = (4+10)\sqrt{2}$

b) $\sqrt{25b} - \sqrt{b^3} = (5-b)\sqrt{b}$ $b > 0$

③ ex10 ✓b) $8^{2/3} = (\sqrt[3]{8})^2 = 4$ (write as integer or)

c) $125^{-1/3} = \frac{1}{\sqrt[3]{125}} = \frac{1}{5}$

d) $\frac{1}{\sqrt[3]{x^4}} = x^{-4/3}$

ex11 a) $a^{1/3}a^{7/3} = a^{8/3}$

b) $\frac{a^{2/5}a^{7/5}}{a^{3/15}} = a^{6/5}$

c) $(2a^3b^4)^{3/2} = 2^{3/2}a^{9/2}b^6 = 2\sqrt{2}\,a^{9/2}b^6$

d) $\left(\frac{2x^{3/4}}{y^{1/3}}\right)^3\left(\frac{y^4}{x^{-1/2}}\right) = 8x^{\frac{9}{4}+\frac{1}{2}}y^{4-1} = 8x^{\frac{11}{4}}y^3$

Radicals

ex12

a) $(2\sqrt{x})(3\sqrt[3]{x}) = 6x^{\frac{1}{2}+\frac{1}{3}} = 6x^{5/6}$

b) $\sqrt{x\sqrt{x}} = (x x^{\frac{1}{2}})^{\frac{1}{2}} = (x^{\frac{3}{2}})^{\frac{1}{2}} = x^{\frac{3}{4}}$

Rationalizing

$\frac{1}{\sqrt{a}} \cdot \frac{\sqrt{a}}{\sqrt{a}}$

$\frac{1}{\sqrt[n]{a^m}} \cdot \frac{\sqrt[n]{a^{n-m}}}{\sqrt[n]{a^{n-m}}} = \frac{\sqrt[n]{a^{n-m}}}{a}$

$a^{\frac{m}{n}+(\frac{n-m}{n})}$

ex13

b) $\frac{1}{\sqrt[3]{x^2}} \cdot \frac{\sqrt[3]{x}}{\sqrt[3]{x}} = \frac{\sqrt[3]{x}}{\sqrt[3]{x^3}} = \frac{\sqrt[3]{x}}{x}$

Get 3

c) $\sqrt[7]{\frac{1}{a^2}} = \frac{1}{\sqrt[7]{a^2}} \cdot \frac{\sqrt[7]{a^5}}{\sqrt[7]{a^5}} = \frac{\sqrt[7]{a^5}}{a}$

Get 7

Q) $\sqrt[8]{\frac{x^8}{y^4}} = \frac{|x|}{\sqrt[8]{y^4}} \cdot \frac{\sqrt[8]{y^4}}{\sqrt[8]{y^4}} = \frac{|x|\sqrt[8]{y^4}}{|y|}$

$\sqrt[8]{\frac{x^8}{y^{5-4}}}$ → $\sqrt[8]{y^8}$

formulas cw

Factor completely to linear or irreducible quadratics

$5x^3-7x^2-7x+2$

$= (x-2)(5x^3+3x-1)$

$x^5-x^3+x^2-6x-3$

$= (x^2-3)(x^3+2x+1)$

Rationalize the denomin.

why? $= \sqrt{x}-1$

Q) $\frac{x-1}{\sqrt{x}+1} = \frac{[(\sqrt{x}+1)(\sqrt{x}-1)]}{[\sqrt{x}+1]}$

$\frac{x-1}{\sqrt{x}+1} \cdot \frac{\sqrt{x}-1}{\sqrt{x}-1} = \sqrt{x}-1$

1.3 Algebraic Expressions

monomials ax^n — n = degree, $a \in \mathbb{R}$, a = leading coefficient, $n \in \{0, 1, 2, 3, \ldots\}$

binomial $2x^3+1$

polynomial $a_n x^n + a_{n-1}x^{n-1} + \ldots + a_1 x + a_0$ (term)

$a_0, \ldots a_n \in \mathbb{R}$, $n \in \mathbb{N}$

Shortcut

$5x^3-7x^2-7x+2$

Hint Plug 2 ⇒ $5\cdot 8 - 28 - 14 + 2 = 0$

$= (x-2)(5x^2+3x-1)$

e-book P34 6th ed nice problems

FOiL $(a+b)(c+d) = ac+ad+bc+bd$

FORMULAS

$(A \pm B)^2 = A^2 \pm 2AB + B^2$

$(A+B)^3 = A^3 + 3A^2B + 3AB^2 + B^3$

$(A-B)^3 = A^3 - 3A^2B + 3AB^2 - B^3$

$(A+B)(A-B) = A^2 - B^2$

chll Binomial expansion & Pascal's triangle

$A^3 - B^3 = (A-B)(A^2+AB+B^2)$ — see ex2b backward & shortcut

$A^3 + B^3 = (A+B)(A^2-AB+B^2)$

$(A-2B)^5 = \binom{5}{0}A^5(-2B)^0 + \binom{5}{1}A^4(-2B)^1 + \binom{5}{2}A^3(-2B)^2 + \binom{5}{3}A^2(-2B)^3 + \binom{5}{4}A^1(-2B)^4 + \binom{5}{5}A^0(-2B)^5$

$= A^5 - 10A^4 + 40A^3B^2 - 80A^2B^3 + 80AB^4 - 32B^5$

a) $(3x+5)^2 = 9x^2+30x+25$

ex3 b) $(x^2-2)^3 = (x^2)^3 + 3(x^2)^2(-2) + 3x^2(-2)^2 + (-2)^3$

$= x^6 - 6x^4 + 12x^2 - 8$

c) $(2x-\sqrt{y})(2x+\sqrt{y})$

$= 4x^2 - y \quad (y>0)$

ex2b $(2x^3-9x^2+6x-1)(x^2-x+2) = 2x^5-11x^4+19x^3-25x^2+13x-2$

$(x^2-3)(x^3+2x+1) = x^5+(-3+2)x^3+(1)x^2-6x+$

$= x^5 - x^3 + x^2 - 6x - 3$

g) $(1+\sqrt{x})(2-3\sqrt{x}) = 2-3|x|-\sqrt{x}$

Factor

ex4 b) $8x^4y^2 + 6x^3y^3 - 2xy^4$

$= 2xy^2(4x^3+3x^2y-y^2)$

c) $(2x+4)(x-3) - 5(x-3) = (x-3)[2x+4-5]$

ex6 $6x^2+7x-5 = (2x-1)(3x+5)$

$2 \cdot 3$, $-1 \cdot 5$

$x^2+7x+12$: $1 \cdot 1$, $3 \cdot 4$

$= (x+3)(x+4)$

ex8 a) $4x^2-25 = (2x-5)(2x+5)$

b) $(x+y)^2 - z^2 = (x+y-z)(x+y+z)$

ex9 b) $x^6+8 = (x^2)^3+2^3 = (x^2+2)(x^4-2x^2+4)$

ex10 b) $4x^2-4xy+y^2 = (2x-y)^2$

ex11b $x^5y^2 - xy^6 = xy^2(x^4-y^4)$

$= xy^2(x^2-y^2)(x^2+y^2)$

$= xy^2(x-y)(x+y)(x^2+y^2)$

ex12b ★ Factor out Smallest exp.

a) $3x^{3/2} - 9x^{1/2} + 6x^{-1/2}$

$= 3x^{-1/2}(x^{2/2} - 3x^1 + 2) = 3x^{-1/2}(x-2)(x-1)$

ex13 Grouping b) $x^3-2x^2-3x+6 = x^2(x-2) - 3(x-2)$

$= (x-2)[x^2-3] = (x-2)(x-\sqrt{3})(x+\sqrt{3})$

1.4 Rational Expressions

fractional $\frac{\sqrt{x}+3}{x+1}$ rational $\frac{x+3}{x^2+4}$ ← 2 polynomials

$y=f(x)$

- exclude $\div 0$
- exclude $\sqrt{-}$, $\sqrt[4]{-}$, $\sqrt[n]{-}$ (n even)

domain $= \{x\in\mathbb{R}: f(x)\in\mathbb{R}\}$ = where f is defined and real

range $= \{y\in\mathbb{R}: y=f(x),\ x\in D\}$ set of all values mapped from domain

- $x^n \geq 0$ n even
- $\sqrt[n]{x} \geq 0$ n even
- $|x| \geq 0$
- $-1\leq \sin x\leq 1$
- $-1\leq \cos x\leq 1$

odd x^n, x^3, $\sqrt[3]{x}$ range $\mathbb{R}$

ex

f: Dom $\longrightarrow$ Range

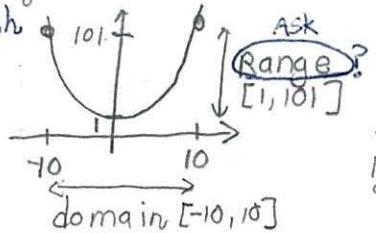
$x \longmapsto f(x)$

$f: [-10,10] \longrightarrow \mathbb{R}$

$x \longmapsto x^2+1$

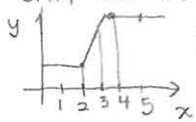
$y=f(x)=x^2+1$

Graph



@ * function as map

SAT, Fourier, Laplace transforms

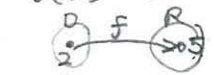


for what x is $f(x+1)>f(x)$

$x=2$
$x=3$
$x=4$ no
$x=1$ no

$x\in(2,4)$

$f(2)=5$



2 is pre-image / pull back of 5 is inverse

Image of 2 is 5

cw ex domain of $f(x)=\frac{x+2}{x-2}$? $\{x\in\mathbb{R}: x\neq 2\}$

ex domain of $g(x)=\sqrt{(x+5)(x-1)}$ $\{x\in\mathbb{R}: x\leq -5 \text{ or } x\geq 1\}$

≥ 0

$(-\infty,-5]\cup[1,\infty)$

Simplifying

$\frac{A\cancel{k}}{B\cancel{k}}=\frac{A}{B}$ $(C\neq 0)$

$\frac{A}{B}\cdot\frac{C}{D}=\frac{AC}{BD}$

$\frac{A}{B}\div\frac{C}{D}=\frac{A}{B}\cdot\frac{D}{C}$

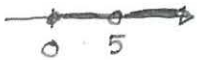
$\frac{A}{C}+\frac{B}{C}=\frac{A+B}{C}$

$\star\ \frac{\frac{ae}{bf}}{\frac{ce}{df}}=\frac{a}{b}\cdot\frac{d}{c}$

$\frac{\frac{a}{b}}{c}=\frac{a}{bc}$

ex1c domain of $\frac{\sqrt{x}}{x-5}$? $\{x\in\mathbb{R}: 0\leq x<5 \text{ or } 5<x<\infty\}$

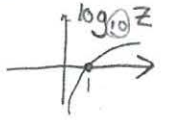
$x\geq 0$ and $x\neq 5$ $= \{x\in\mathbb{R}: x\geq 0 \text{ and } x\neq 5\}$



$= [0,\infty)\setminus\{5\} = [0,5)\cup(5,\infty)$

$[0,\infty)\cap\{5\}^c$

@ domain of $f(x)=\log(\sqrt{x^2-1})$? $(-\infty,-1)\cup(1,\infty)$



$b>1$

$\log_{10}1=0$

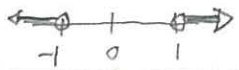
≥ 0

$\neq 0$

$x^2-1>0$ ⇝ $(x-1)(x+1)>0$

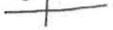
$\sqrt{x^2}>\sqrt{1}$

$|x|>1$

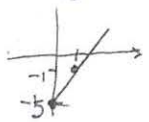


@ range? $f(x)=|-x^2-8|=|x^2+8|=x^2+8$

$[8,\infty)$



@ range? $f(x)=4x-5$, domain $0\leq x\leq 10$



$f(0)=-5$

$f(10)=35$

$[-5,35]$

✓

Ex4 $\frac{x-4}{x^2-4}\div\frac{x^2-3x-4}{x^2+5x+6}$

✓ $\frac{\cancel{x-4}}{(x-2)(x+1)}\cdot\frac{(x+2)(x+3)}{\cancel{(x-4)}(x+1)}=\frac{x+3}{(x-2)(x+1)}$

ex5b $\frac{1}{x^2-1}-\frac{2}{(x+1)^2}=\frac{1(x+1)}{(x-1)(x+1)^2}-\frac{2(x-1)}{(x+1)^2(x-1)}$

LCD

$=\frac{-x+3}{(x-1)(x+1)^2}$

Physics

Verify

- $T=2\pi\sqrt{\frac{L}{g}}$ $\quad\sqrt{\frac{m}{m/s^2}}=s$
- $\frac{kg\cdot m/s^2}{m/s}=\frac{kg}{s}$ $\quad\frac{\frac{ab}{cd}}{\frac{ae}{df}}=\frac{b}{c}\frac{f}{e}$

Compound fractions

ex6 Simplify to $\frac{\square}{\square}$ $\quad \frac{\frac{x}{y}+1}{1-\frac{y}{x}}$

① $\frac{\frac{x+y}{y}}{\frac{x-y}{x}} = \frac{x+y}{y}\cdot\frac{x}{x-y}$

② Use LCD of all fractions

$$\frac{\frac{x}{y}+1}{1-\frac{y}{x}}\cdot\frac{xy}{xy} = \frac{x^2+xy}{xy-y^2} = \frac{x(x+y)}{y(x-y)}$$

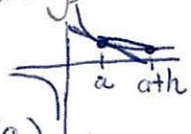
ex7 $\left(\frac{\frac{1}{a+h}-\frac{1}{a}}{h}\right)$ 2.4 rates Intro derivative

$f(x)=\frac{1}{x}$

$\left.\frac{df}{dx}\right|_{x=a} = \lim_{h\to 0}\frac{f(a+h)-f(a)}{h}$

$f'(a)$

$f'(a)$



Sketch

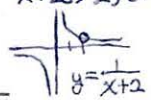
① derivative = slope of tangent line

② Slope of secant = $\frac{f(a+h)-f(a)}{h} = \frac{\text{rise}}{\text{run}}$

③ Take limit as $h\to 0$

$\lim_{x\to 2}(x+3) = 5$

$\lim_{x\to 2}\frac{(x-2)}{x^2-4} \overset{\frac{0}{0}}{=} \lim_{x\to 2}\frac{(x-2)}{(x-2)(x+2)} = \frac{1}{4}$



CW & Sketch meaning. Then intro D_x

$$\frac{f(a+h)-f(a)}{h} = \frac{\frac{1}{a+h}-\frac{1}{a}}{h} = \frac{a-(a+h)}{a(a+h)} \quad h\neq 0$$

$$= \frac{a-(a+h)}{ha(a+h)} = \frac{-h}{ah(a+h)} = \frac{-1}{a(a+h)}$$

$$\lim_{h\to 0}\frac{-1}{a(a+h)} = \boxed{-\frac{1}{a^2}}$$

Shortcut $D_x x^{-1}\big|_{x=a} = -1x^{-2}\big|_{x=a} = -\frac{1}{a^2}$ ← fraction not rational

ex8 Simplify $\frac{(1+x^2)^{1/2} - x^2(1+x^2)^{-1/2}}{1+x^2}$

①
$$= \frac{\frac{(1+x^2)^{1/2+\frac{1}{2}}}{(1+x^2)^{1/2}} - \frac{x^2}{(1+x^2)^{1/2}}}{1+x^2} = \frac{(1+x^2)-x^2}{(1+x^2)^{1+1/2}} = \frac{1}{(1+x^2)^{3/2}}$$

② Factor out smallest power

$$\frac{(1+x^2)^{-1/2}\left[(1+x^2)^{1/2+\frac{1}{2}} - x^2\right]}{(1+x^2)} = \frac{1}{(1+x^2)^{3/2}}$$

Summary ① $\frac{\frac{a}{b}+\frac{c}{b}}{\frac{c}{d}+\frac{a}{b}}$ ← same den ② LCD of all fractions ← same den ③ Factor out least power in num & den ④ Rationalize $\frac{(\sqrt{x}+5)}{(\sqrt{x}-5)}\frac{(\sqrt{x}-5)}{(\sqrt{x}-5)}$

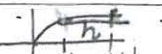
(80) CW $\frac{3(1+x)^{1/3} - x(1+x)^{-2/3}}{(1+x)^{2/3}} = \frac{d}{dx}\left[\frac{3x}{(1+x)^{1/3}}\right]$

$= \frac{(1+x)^{-2/3}\left[3(1+x)-x\right]}{(1+x)^{2/3}} = \frac{3+2x}{(1+x)^{4/3}}$

Rationalize $(2-\sqrt{3})(2+\sqrt{3})$ ← conjugate radical

ex10 CW

$f(x) = \sqrt{x}$



$f'(4) = ?$ $\lim_{h\to 0}\frac{\sqrt{4+h}-\sqrt{4}}{h}$

$= \lim_{h\to 0}\frac{\sqrt{4+h}-2}{h} \overset{\frac{0}{0}}{=} \cdot\frac{\sqrt{4+h}+2}{\sqrt{4+h}+2}$

$= \lim_{h\to 0}\frac{(4+h)-4}{h(\sqrt{4+h}+2)} \overset{h\neq 0}{=} \lim_{h\to 0}\frac{1}{\sqrt{4+h}+2}$

$= \frac{1}{\sqrt{4}+2} = \frac{1}{4}$ $\qquad \frac{1}{2\sqrt{x}}\Big|_{x=4}$

1.5 Equations - lots technique here

expression in x & y

$\frac{x^2+1}{3x-2}+y$ ← Give x, y, get new number

equation

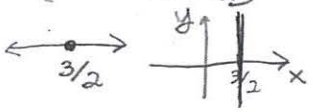
Root / Zero - just Move

Soln/Root Solve

$2x=3$ $\quad 2x-3=0$ $\quad 2(x-3/2)=0$

all x that make it true

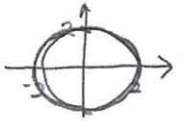
$\{x\in\mathbb{R}: x=3/2\}$



Graph

$x^2+y^2=4$

$\{(x,y)\in\mathbb{R}^2 : x^2+y^2=4\}$



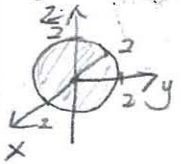
curve

Maybe NOT function

y is not a fn of x

$x^2+y^2+z^2=4$

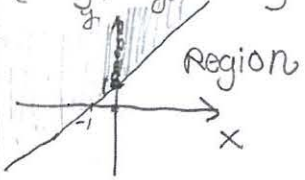
$\{(x,y,z)\in\mathbb{R}^3 : \text{sphere}\}$



inequality

$y \geq x+1$

$\{(x,y);\ y\geq x+1\}$



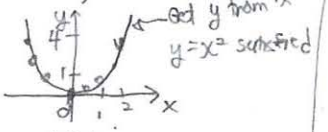
Function

$y=f(x)$

y is a fn of x

$x \longmapsto f(x)$

$f(x)=x^2$



Get y from x

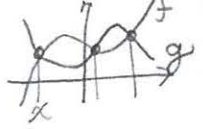
$y=x^2$ surfaced

x	f(x)
0	0
±1	1

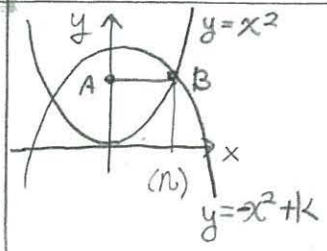
x makes y-values same

intersection

$f(x)=g(x)$



cw ②



- segment $\overline{AB}$ length is a positive integer
- What could k be? (Find k in terms of n)

$n^2 = -n^2 + k$

$2n^2 = k$

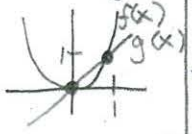
n	k
1	2
2	8
3	18
4	32

④

$f(x) = x^2$

$g(x) = x$

$f(x) = g(x)$



$\{(0,0),(1,1)\}$

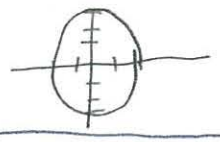
$x = 0, 1$

Graph the soln set for

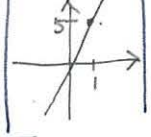
$x + y \le 5$

$y \le -x + 5$

$\left(\frac{x}{2}\right)^2 + \left(\frac{y}{3}\right)^2 = 1$

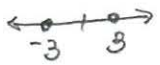


$5x = y$



$x^2 = 9$

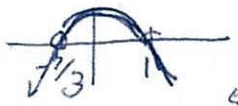
$\{-3, 3\}$



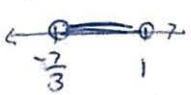
$-(3x^2 + 4x - 7) \ge 0$

3×-1, 1×7

$-(3x+7)(x-1) \ge 0$



$\left(-\frac{7}{3}, 1\right)$



ex1 Solve a Linear Eqn.

Combine Like terms

$7x - 4 = 3x + 8$

$4x = 12$

$x = 3$

Check ?

$7(3) - 4 = 3(3) + 8$

$17 \neq 17$

ex2 Solve for h in terms of others

3

$A = 2lw + 2wh + 2lh$

$h = \frac{A - 2lw}{2(w+l)}$

Note: Linear Factor $(3x+1)$ $(x-2)$

Quadratic { reducible $x^2+x-2 = (x-1)(x+2)$, $(x-\pi)(x+\sqrt{3})$ (real roots)
irreducible x^2+2x+5 $B^2-4AC < 0$
$= (\ i\ )(\ i\ )$ cannot factor without getting i

Completely factored into linear & irreducible quadratics

$(3x+1)(x-1)(x+2)(x^2+2x+5)$

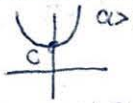
Quadratic functions (Parabolas)

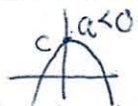
$y = f(x) = ax^2 + bx + c$

$= a(x-r_1)(x-r_2)$ ← factored form shows x-cross

$= a(x-h)^2 + k$ ← vertex form

roots zeros $r_1, r_2 = \frac{-b \pm \sqrt{b^2-4ac}}{2a}$ Quadratic formula





r_1, r_2 not $\mathbb{R} \Leftrightarrow$ "irreducible"

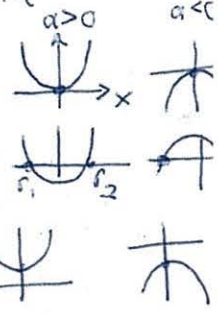
★ 3 points in sketch

Discriminant $D = b^2 - 4ac$

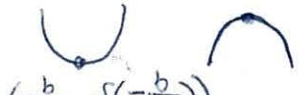
$= 0 \Rightarrow r_1 = r_2$ real

$> 0 \Rightarrow r_1 \neq r_2$ real

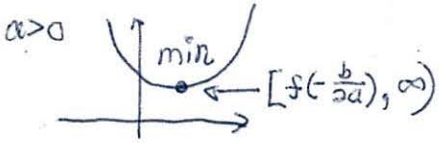
$< 0 \Rightarrow r_1$ & r_2 complex conjugates (irreducible)



Vertex $\left(-\frac{b}{2a}, f\left(-\frac{b}{2a}\right)\right)$



Range



$r_1 + r_2 = -\frac{b}{a}$

$r_1 \cdot r_2 = (A+B)(A-B) = A^2 - B^2 = \left(-\frac{b}{2a}\right)^2 - \left(\frac{b^2-4ac}{4a^2}\right) = \frac{c}{a}$

Similar ex'2'

Solve by ✓ Factoring Method

$3x^2 + 10x = 8$

$3x^2 + 10x - 8 = 0$

3×4, 1×-2

$(3x-2)(x+4) = 0$

$x = -4$ or $2/3$

$\sqrt{x^2-1} > 0$ ⟶ $x^2 - 1 > 0$ ⟶ ~~$x^2 > 1$~~ ⟶ ~~$\sqrt{x^2} > \sqrt{1}$~~ ⟶ ~~$x > 1$~~ ⟶ ~~$|x| > 1$~~ ⟶ $x < -1$ or $x > 1$

Solving Simple Quadratics / Method of taking Square Roots

ex5 b) $(x-4)^2 = 5$ ✓ already ~~factored~~ sqr = #

$\sqrt{(x-4)^2} = \sqrt{5}$

$|x-4| = \sqrt{5}$

$x - 4 = \pm\sqrt{5}$

$x = 4 \pm \sqrt{5}$

cw $(x-3)^2 = -28$

$x - 3 = \pm\sqrt{-28} = \pm\sqrt{i^2 2^2 7}$

$= \pm 2i\sqrt{7}$

$x = 3 \pm 2i\sqrt{7}$

complex conjugates

Method of completing the Square

$(x \pm d)^2 = x^2 \pm 2xd + d^2$

ex6 a) $x^2 - 8x + 13 = 0$ ✓

$x^2 - 2\cdot 4 x + 4^2 = -13 + 4^2$

$(x-4)^2 = 3$

$x - 4 = \pm\sqrt{3}$

$x = 4 \pm \sqrt{3}$

$(x-4-\sqrt{3})(x-4+\sqrt{3})$ simplest factors

cw b) $3x^2 - 12x + 6 = 0$

$3x^2 - 12x = -6$

$3(x^2 - 4x) = -6$ ($2\cdot 2$)

$3(x^2 - 2\cdot 2x + 2^2) = -6 + 12$

$3(x-2)^2 = 6$

$(x-2)^2 = 2$

$x = 2 \pm \sqrt{2}$

$3x^2 - 11x + 6 = 0$

$3(x^2 - \frac{11}{3}x + \frac{121}{36}) = -6 + \frac{121}{12}$ ($2\cdot\frac{11}{6}$)

$3(x - \frac{11}{6})^2 = +\frac{49}{12}$

$(x - \frac{11}{6})^2 = \frac{49}{36}$

$x = \frac{11}{6} \pm \frac{7}{6} = \frac{18}{6}, \frac{4}{6}$

$= 3, \frac{2}{3}$

Solve / Derive Quadratic Formula

J1

Solve $ax^2 + bx + c = 0$ by Completing the Square

$a(x^2 + \frac{b}{a}x) = -c$ ($2\cdot\frac{b}{2a}$)

$(x^2 + \frac{b}{a}x + (\frac{b}{2a})^2) = -\frac{c}{a} + \frac{b^2}{4a^2}$

$(x + (\frac{b}{2a}))^2 = -\frac{c}{a} + \frac{b^2}{4a^2}$

$x = -\frac{b}{2a} \pm \sqrt{-\frac{c\cdot 4}{a\cdot 4} + \frac{b^2}{4\cdot a^2}}$

$= -\frac{b}{2a} \pm \sqrt{\frac{b^2 - 4ca}{4a^2}} = \frac{-b}{2a} \pm \frac{\sqrt{b^2-4ac}}{|2a|}$

$|2a| = -b \pm \sqrt{b^2-4ac}$

$2a$

ex7 Method of cw Quadratic formula — irreducible

a) $x^2 + 2x + 2 = 0$

$x = \frac{-2 \pm \sqrt{4 - 4(1)(2)}}{2(1)} = \frac{-2 \pm \sqrt{-4}}{2} = \frac{-2 \pm 2i}{2}$

$= -1 \pm i$

irreducible?

ex8 Using the discriminant. How many solns? Sketch ∪ —

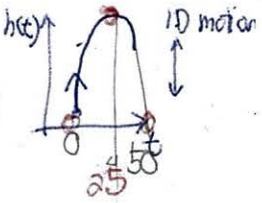
cw a) $x^2 + 4x - 1 = 0$ b) $4x^2 - 12x + 9 = 0$ c) $\frac{1}{3}x^2 - 2x + 4 = 0$

$D = 4^2 - 4(1)(-1)$ $D = (-12)^2 - 4\cdot 4\cdot 9 = 0$ $D = (-2)^2 - 4(\frac{1}{3})4$

$= 20 > 0$ $= -\frac{4}{3} < 0$

no ∪ no — yes ∪

ex9 next time. Projectile



Sketch h 3pts

cw no calc $h(t) = -16t^2 + v_0 t$, $v_0 = 800$ ft/s

$= -16t^2 + 800t = -16t(t-50)$

a) When fall to ground level?

$t = 50$s

$\frac{16}{2} = 8$

$16 \times 0.5 = 800$

b) When is the height 6400ft?

$-16t^2 + 800t = 6400$

$-t^2 + 50t = 400$

$-t^2 + 50t - 400 = 0$

1×-10, -1×40

$(t-10)(-t+40) = 0$

$(t-10)(t-40) = 0$

$t = 10, 40$ s

d) highest point?

$t = 25 \Rightarrow h(25) = -16(25)^2 + 800\cdot 25 = 10{,}000$

$= +16(25)(-25) = 10000$ ✓

$= +16\cdot\frac{1}{4}\cdot\frac{1}{4}\cdot 10^4$

$16\times\frac{1}{4}\times\frac{1}{4}\times 10000$ $800\times\frac{1}{4}\times 100$ 0.35

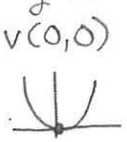
OR $t = -\frac{b}{2a} = -\frac{800}{2(-16)} = 25$

Derive Location of Vertex

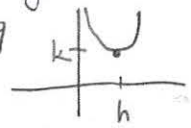
J2

$y=x^2$

$V(0,0)$



$y=a(x-h)^2+k$



$V(h,k)$

$y=ax^2+bx+c$

$=a\left(x^2+\frac{b}{a}x\right)+c$

$=a\left(x^2+2\left(\frac{b}{2a}\right)x+\left(\frac{b}{2a}\right)^2\right)+\left[c-\frac{b^2}{4a}\right]$

$=a\left(x+\frac{b}{2a}\right)^2+\left[c-\frac{b^2}{4a}\right]$

$=a(x-h)^2+k$

$V\left(-\frac{b}{2a}, f\left(-\frac{b}{2a}\right)\right)$

Check $c-\frac{b^2}{4a^2} \stackrel{?}{=} a\left(-\frac{b}{2a}\right)^2+b\left(-\frac{b}{2a}\right)+C$

$= +\frac{b^2}{4a} - \frac{2b^2}{2\cdot 2a} + C$

$\stackrel{\checkmark}{=} -\frac{b^2}{4a} + C$

Other types - later

ex10

fractional eqn.

- put fractions together

$\left(\frac{3}{x}+\frac{5}{x+2}=2\right)$ ✓

$\frac{3x+6+5x}{x(x+2)}=2$

$8x+6=2x^2+4x$

$2x^2-4x-6=0$

$1 \quad 2 \quad -3 \quad -2$

$(2x+2)(x-3)=0$

$x=-1, 3$

Condition $x\neq 0, -2$

$2+\frac{5x}{x-6}=\frac{30}{x-6}$ Inconsistent eqn.

cw

Condition $x\neq 6$

$\frac{2x-12+5x}{x-6}=\frac{30}{x-6}$

$7x-12=30$

$7x=42$

extraneous → $x=6$ ✗

no solution

Eqn w/ radical

isolate $\sqrt{\ }$

cw with hints

ex11

$2x=1-\sqrt{2-x}$ ← $(2-x)\geq 0$

$(2x-1)^2=(-\sqrt{2-x})^2$

$(2x-1)^2=|2-x|$

$4x^2-4x+1=2-x$

$4x^2-3x-1=0$

$4 \quad -4 \quad -1 \quad 1$

$(4x+1)(x-1)=0$

$x=1, -\frac{1}{4}$

~~$x\leq 2$~~

$1-2x\geq 0$

$2x\leq 1$

$x\leq \frac{1}{2}$

Check: ✱

$2 \stackrel{?}{=} |1-\sqrt{2\cdot 1}|$

$2\neq 0$

$-\frac{1}{2} \stackrel{?}{=} 1-\sqrt{x}$

$=1-\sqrt{\frac{9}{4}}$

$=1-\frac{3}{2}$

$=-\frac{1}{2}$ ✓

Quadratic in disguise

ex12

$x^4-8x^2+8=0$

$(x^2)^2-8(x^2)+8=0$

$x^2=\frac{8\pm\sqrt{64-32}}{2}=4\pm\frac{\sqrt{32}}{2}=4\pm\frac{\sqrt{16\cdot 2}}{2}=4\pm 2\sqrt{2}$

$>0 \quad <2$

$x=\pm\sqrt{4\pm 2\sqrt{2}}$

four roots

fractional powers

quad disguise

ex13

$x^{1/3}+x^{1/6}-2=0$

~~$x^{1/6}(x^2+1-2x^6)$~~

$(x^{\frac{1}{6}})^2+(x^{1/6})-2=0$

$1 \quad 1 \quad -2 \quad -1$

$(x^{\frac{1}{6}}+2)(x^{\frac{1}{6}}-1)=0$

$x^{\frac{1}{6}}=\overset{\text{not principal}}{-2}$ or 1

$x=64$ or 1

Check: $\sqrt[3]{64}+\sqrt[6]{64}-2 \stackrel{?}{=} 0$

$4+2-2\neq 0$

$4\neq 0$

$1+1-2=$ ✓✓

Absolute Value

(isolate & remove)

ex14

$|2x-5|-3=0$

$|2x-5|=3$

$2x-5=\pm 3$

$2x=5\pm 3$

$x=\frac{8}{2}$ or $\frac{2}{2}$

$=4, 1$

Check.

no restrictions

$|-x^2-1|=5$

$x^2+1=5$

$x^2=4$

$x=\pm 2$

(a) $|x-3|+2=0$

$|x-3|=-2$

no soln

② $|2x+5| - 4 = 3x$ ← $(3x+4 \geq 0,\ x \geq -4/3)$

$|2x+5| = 3x+4$

$2x+5 = 3x+4$ or $2x+5 = -3x-4$

$1 = x$ | $5x = -9$

$\boxed{x=1} \geq -4/3$ ✓ | $x = -9/5 \geq -1.3$?

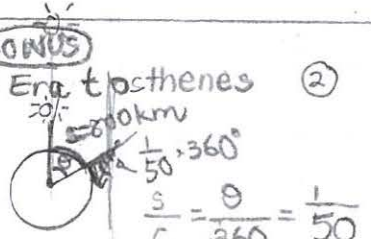
? | -1.8 No!

"extraneous solution"

1.6 Modeling with Equations

- Define x
- set up eqn in x
- Solve & check

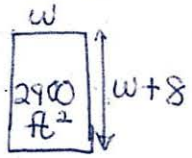
BONUS ① Eratosthenes ②



$\frac{s}{C} = \frac{\theta}{360} = \frac{1}{50}$

$C = 5 \cdot 50 \approx 40{,}000$ km

ex4 rectangle



dimensions?

$w(w+8) = 2900$

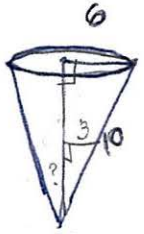
$w^2 + 8w - 2900 = 0$

$(w-50)(w+58) = 0$

$\boxed{w = 50 \text{ ft}}$ or -58

$\boxed{l = 58 \text{ ft}}$

Like ex5 similar triangle



- Cone tank. Radius 6 feet, height 8 feet
- fills with water $V(h) = ?$
- [water height when radius 3 feet?]

$\frac{3}{h} = \frac{6}{8} \Rightarrow h = 4$

$V(h) = \frac{1}{3}\pi r^2 h = \frac{1}{3}\pi \frac{9}{16} h^3 = \frac{3}{16}\pi h^3$

$\frac{r}{h} = \frac{6}{8} \Rightarrow r = \frac{3}{4}h$

ex6 mixtures

5% OJ (900 gal soda). Need add how many gal pure OJ = 10% pure OJ?

$0.05 \times 900 + g = 0.10(900+g) \Rightarrow 45+g = 90+0.1g$

pure OJ | total

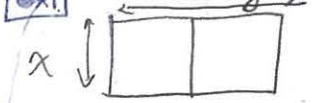
$0.9g = 45$

$g = 50$ gallons

distance = speed × time

total energy = rate × time

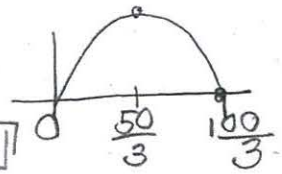
ex3 VPR 9.64



- 100 m of wire fence
- build identical adjacent pens
- maximize area
- dimensions?

$3x + 2y = 100 \Rightarrow y = 50 - \frac{3}{2}x$

$A(x) = xy = x(50 - \frac{3}{2}x)$



$x = \boxed{\frac{50}{3} \text{ m}}$ $y = 50 - 25 = \boxed{25 \text{ m}}$

≈ 16.67 m

Ch0 Anton

open box. Volume 80 m³

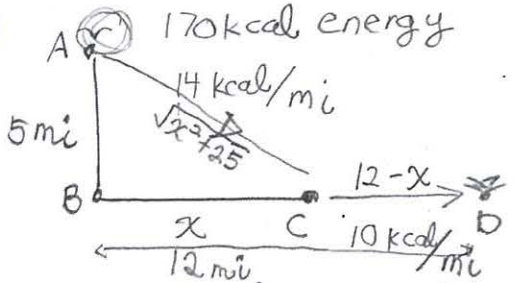
$5/m^2$ for base (square base)

$3/m^2$ for sides

Cost as fn. of base length x

$C(x) = 5x^2 + 3\left(\frac{80}{x^2}\right) \cdot x \cdot 4$ (height)

ex1 170 kcal energy



a) use 170 kcal in flight. where's C?

$\frac{14 \text{ kcal}}{\text{mi}} \times \sqrt{x^2+25} + \frac{10 \text{ kcal}}{\text{min}} \times (12-x) = 170$

$(14\sqrt{x^2+25})^2 = (50+10x)^2$

$196(x^2+25) = 2500 + 1000x + 100x^2$

$96x^2 - 1000x + 2400 = 0$

$12x^2 - 125x + 300 = 0$ (3, 4 × −20, −15)

$(3x-20)(4x-15) = 0$

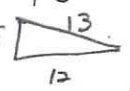
$x = \frac{20}{3}, \frac{15}{4}$

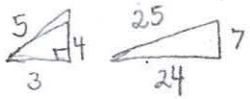
$= \boxed{6\frac{2}{3} \text{ or } 3\frac{3}{4} \text{ mi}}$

enough energy

b) A → D directly?

$14 \times \left(\frac{\sqrt{5^2+12^2}}{13}\right) = \boxed{182 \text{ kcal}}$ No





1.7 ex4 $x(x-1)^2(x-3) < 0$

$x - c$ { >0 when $x > c$; <0 when $x < c$

* Fill SAT variation problems

$|x-2| = 3 \quad x-2 = \pm 3$

$|x-2| \leq 3$ ← No ±!

1.7 Inequalities · Constraints on time, cost of manufacture
· Time in the air

Word prob/reality: Make a measurement $\pm \frac{1}{3}$, error $\geq \frac{1}{3}$ unacceptable range

If $a \leq b$

$a \pm c \leq b \pm c$

$ca \leq cb \ (c>0)$
$\geq \ (c<0)$

$\frac{1}{a} \geq \frac{1}{b}$

- $a \leq b, \ c \leq d$
  $\Rightarrow a+c \leq b+d$
- $|a+b| \leq |a|+|b|$
- $|x-a| < c$ ⟺ $a-c < x < a+c$
  $|x-a| > c$ ⟺ $x < a-c$ or $x > a+c$

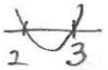
ex2 $4 \leq 3x-2 < 13$

$6 \leq 3x < 15$

$2 \leq x < 5$ ← Isolate x

ex3 $x^2 - 5x + 6 \leq 0$

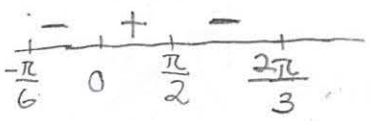
Factor $(x-2)(x-3) \leq 0$



$2 \leq x \leq 3$

BONUS ex $f(x) = \frac{\cos x}{\sin^{\frac{1}{3}} x}$ $\quad -\frac{\pi}{6} < x < \frac{2\pi}{3}$

a) $f(x) > 0$



b) $f(x) < 0$

(ex4 above) ex5 x in denominator → $\frac{\square}{\square} \leq 0$ This form, factored

$x < \frac{2}{x-1}$

($\times (x-1)^2$ both sides $\quad x(x-1)^2 < 2(x-1)$) OR

Nah.

$(x-1)(x(x-1)-2) < 0$

$(x-2)(x+1)$

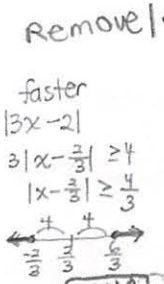
Absolute Value ex7

Remove $|\cdot|$

faster $|3x-2|$

$3\left|x-\frac{2}{3}\right| \geq 4$

$\left|x-\frac{2}{3}\right| \geq \frac{4}{3}$



ex10

$\frac{450-54x+50x-0.1x^2}{x} < 0$

$\frac{-0.1x^2-4x+450}{x} < 0$

$\frac{x^2+40x-4500}{x} > 0$

$\frac{(x+90)(x-50)}{x} > 0$

$-\frac{(x--90)(x-50)}{x} < 0$

1.8 Cartesian, Euclidean, Rectangular Coordinates

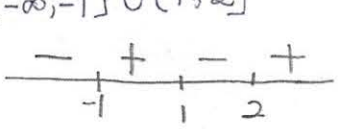
ex1

$d(\vec{a},\vec{b}) = \sqrt{(a_1-b_1)^2 + (a_2-b_2)^2 + (a_3-b_3)^2}$

ex5 $x - \frac{2}{x-1} < 0$

$\frac{x^2-x-2}{(x-1)} < 0$

$\frac{(x-2)(x+1)}{(x-1)} < 0$



$(-\infty,-1] \cup (1,2]$

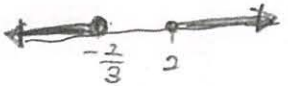
$x - C$ { >0 $x>C$; $=0$ $x=C$; <0 $x<C$

$|-3x+2| \geq 4$

$-3x+2 \leq -4$ or $-3x+2 \geq 4$

$-3x \leq -6$ $\quad -3x \geq 2$

$\{x:\ x \geq +2$ OR $x \leq -\frac{2}{3}\}$



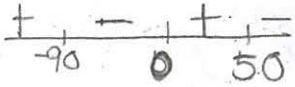
$(-\infty, -\frac{2}{3}] \cup [2, \infty)$

Bus \$450, shared among x students = ?

· Tickets \$50 − 10¢ for each in group

· Ticket < \$54

$\frac{450}{x} + (50 - 0.10x) < 54$

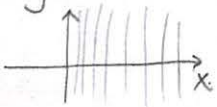


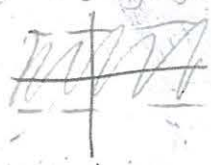
$\frac{(90+x)(50-x)}{x} < 0$

$\{-90 < x < 0\} \cup \{50 < x\}$

$x \geq 51$ people

a) $\{(x,y): x \geq 0\}$ b) $\{(x,y): y=1\}$ c) $\{(x,y): |y| < 1\}$





$-1 < y < 1$

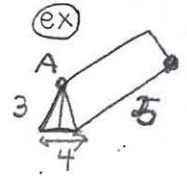
HW (Prove) Pythag & 3D dist formula

distance

$d(\vec{a},\vec{b}) = \sqrt{(a_1-b_1)^2+(a_2-b_2)^2+(a_3-b_3)^2}$

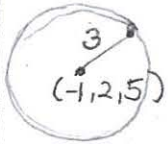
$= \sqrt{(\Delta x)^2+(\Delta y)^2+(\Delta z)^2}$

(ex)



$d(A,B) = \sqrt{9+4+25} = \sqrt{38}$

(ex)



$\sqrt{(x+1)^2+(y-2)^2+(z-5)^2} = 3$

$(x-2)^2+(y+1)^2 = 25$

midpoint formula $\left(\frac{a_1+b_1}{2}, \frac{a_2+b_2}{2}\right)$

Quiz Defn of sphere. distance to equation

set of all points r $\{(x,y,z) \mid d(P,C) = r\}$ from (h,k,l)

$\sqrt{(x-h)^2+(y-k)^2+(z-l)^2} = r$

ex1 Eqn of circle with P(1,8) and Q(5,-6) as endpts of diameter

$C\left(\frac{1+5}{2}, \frac{8-6}{2}\right) = C(3,1)$

$d = \sqrt{4^2+14^2} = \sqrt{212} = 2\sqrt{53}$

$r = \sqrt{53}$

$\sqrt{(x-3)^2+(y-1)^2} = \sqrt{53}$

$\boxed{(x-3)^2+(y-1)^2 = 53}$

1.8 ★ Symmetry ★ - See SAT Pg 4 & its examples ↖ Later in Chapter 2, Now just P90 & ex 11-12

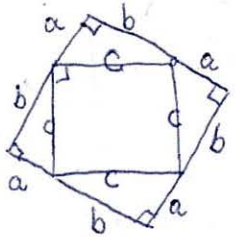
✓ Show complete the square

$2x^2-4x+2y^2-12y+2=0$

center:

radius:

circle $(x-1)^2+(y-3)^2 = 9$



arith $\frac{a+b}{2} \geq$ geo $\sqrt{ab}$ mean a,b positive

↑ when $a=b$

iff $(a+b)^2 \geq 4ab$

$a^2-2ab+b^2$

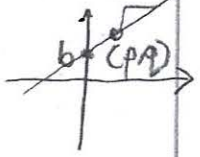
$(a-b)^2 \geq 0$

or $a<b$

$a^2<b^2$

1.10 Lines = $\{(x,y) : \frac{\Delta y}{\Delta x} = \text{constant}\}$? m slope

and thru point (h,k)

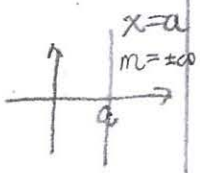


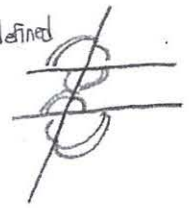
• Any (h,k)

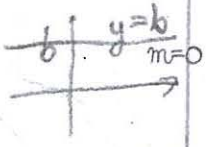
$\frac{y-k}{x-h} = m \Rightarrow \boxed{y = m(x-h)+k}$ pt-slope form

$= mx+(-mh+k)$

$\boxed{y = mx+b}$ slope-intercep form UNIQUE

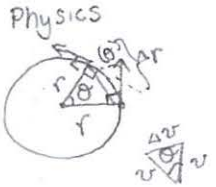






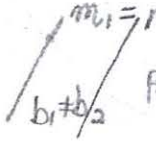
$mx-y = b$

$\boxed{Ax+By = C}$ standard form NOT Unique

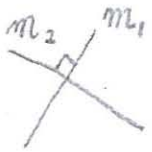


Prove similar

$3Ax+3By = 3C$



Parallel



$m_1 = -\frac{1}{m_2}$

Use

• simplest polynomial

• Locally linear

$f(x) \approx f'(a)(x-a)+f(a)$

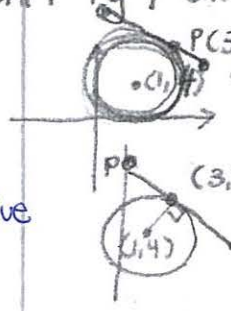
errors $\Delta V \approx V'(h)\Delta h$

• instantaneous (speed) velocity • RATES

• Stairs

★ See SAT Pg 9 examples

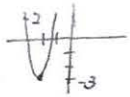
(a)

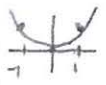


$\frac{3}{2} \perp -\frac{2}{3}$

Line tangent there $y = -\frac{2}{3}(x-3)+7$

(a) Parabola





$a<0$

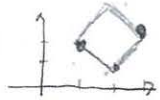
$-1<a<0$

$0<a<1$

$a=1$

$\boxed{a>1}$ ✓ steepness

(a)



square, diagonal endpts $(1,2), (5,8)$

Other diagonal intersects this diagonal at what point? (midpt! $(3,5)$)