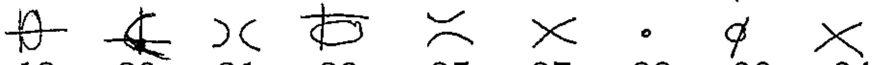
- Ellipse/Parabola/Hyperbola Worksheet.

Parabola	<u>10.1</u> (Pg. 751) #	1~6,	(12,	13,	15)	16	18,	(25,	(29,	32,	36,	41,	43,	43,	44,	(45,	49.
	<u>11.1</u> (P. 730)	5~10	16)	18)	20)	19	21	29)	34)	35	39	45		47	48	49)	53

Ellipse	<u>10.2</u> (Pg. 759) #	1~4,	8,	10,	19,	21,	23,	(24,	(31,	35,	38,	40,	49,	50,	54. 55
	<u>11.2</u> (P. 739)	5~8	12	13	23	25	27	28)	36)	40	42	44	53	54	*58 *59

Hyperbola	<u>10.3</u> (Pg. 768) #	1~4,	11,	16,	18,	20,	21,	(22,)	42	44,	45,	46.	Bonus 47,	48.
	<u>11.3</u> (P. 748)	5~8	16	19	20	recog 24	25	(26)	46	48	word 49	50	*51	52



Translate	<u>10.4</u> (Pg. 781) #	13~18,	19,	20,	21,	23,	25,	27,	29,	30.	34,	38.	*40
	<u>11.4</u> (P. 756)	17~22	24	complete 23	the sqr. 25	28	30	31	34	degenerate 33	37	projectile 42	*44

- We will have time for some problems on Thursday. **Quiz: 10.1~10.4**

<u>10.7</u> (Pg. 807) # 9, 10, 11, 12, 20, 21, 28, 29, 30, 35, 40, 45, 47~50, 62.
Parametric

Bonus # 53, 58, 61.

- <u>10.5</u> (Pg. 791) # 19 11.5 (P.764) #20 Rotation of Axes
  - a) Use the discriminant to determine if it's an ellipse, etc.
  - b)
    - 1) Derive [x; y] = [cos -sin; sin cos] [X ;Y]
    - and [X; Y] = [???] [x ;y]
    - 2) Use the magic formula to find phi
    - 2') Use the right triangle method like example 4 to fill in the matrix in #1
    - 3) Change the xy equation to XY equation by plugging #1 and combining like terms
  - c) Sketch the graph on the rotated axes.

$$11x^2-24xy+4y^2+20=0.$$

Polar Conics
10.6 (P. 800) # 9~14 with reasons, #19.
11.6 (P.771) # 11~16 " #34

- Barron's Model Test #3
    - Students who wrote it before and want a new test, please get a green book in class. Final is still based on Barron's.
- Show work for all 50 questions! (half credit if only letter answer)
- Please bring your SAT 2 Math Level 2 practice test book.
- Barron's Model Test #6 will be timed in class. The score will not affect your grade.
- If you have done Test #6 before, you will take another new test.
- Conic sections derivations worksheet.

Look through Barron's Model Test #3 to see if you have questions to ask tomorrow.
Final Ch. 5~11

- Half: Same as quiz questions
    - 10.7, 11.1~11.3, 11.6 based on HW
    - Not tested: 10.5, 10.6, 11.4, 11.5 (maybe as bonus)
- Half: Picked from two SAT 2 Practice Tests
    - Barron's Model Test # 6 and #3

at least +10% on final. If you forgot to take a worksheet, please print it out the bonus here.

10.1 (P.751) Parabola v(0,0)

#(1~6), 12, ⑬, 15, ⑱, 25, ㉙ 32, 36, ㊶, 42, 43, 44, 45
49

10.2 (P.759) Ellipse

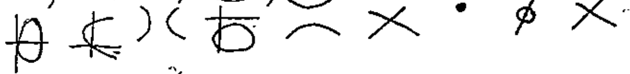
#1~4, ⑧, 10, ⑲, ㉑, ㉓, 24, 31, 35, 38, ㊵, 49, 50, 54

10.3 (P.768) Hyperbola

#1~4 recog, 11, 16, 18, 20, 21, 22 recog, 44, 45, 46 word, *47, 48

10.4 (P.781) Translated

⑥ 13~18 recog, 19, 20, 21, ㉓, ㉕, ㉗, 29, 30, 34 degenerate, 38 projectile



complete the sqr.

Just answer a&b

10.7 (P.807) Parametric

#10, 11, 12, ⑨, ⑳, 21, 28, 29, ㉚ plug x&y into 29, 35, 40, 45, CALC 47~50, 62

*53, 58, 61

10.5 (P.791) #19

a)

10.6 (P.800) #9~14, 19

WS - derive each derive mirror
- string
- parabola
- ineqlties

• ~~Lens~~ bonus - see where focus Light Collimator
• Degenerate cases SAT

BONUS:
• Kepler WS (make better than mine
• Architecture String?

*CALC
- derive comet trajectory by Newton's Laws!,
- reflective properties

10.3 hyp
10.4 translate
10.7 param ext Projectile
10.5 Rotate
10.6 polar
bonus quiz

11.1 Seq
11.2 arith
11.3 geo
11.4 finance
11.5 induction
11.6 binomial

SAT Prep

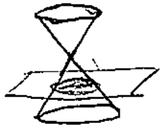
Tue	Th.	Fri
	• 11.1~11.4 forms Talk: • intro: notes P3 cw: WS P. 5~6 identify parabola, ellipse, hyperbola Talk: Parts of parabola, WS P.3 ellipse hyp. cw: WSP.5~6	- Derive eqns. cw: ws P.1~2 Complete Square
• finish examples on 11.1~11.4 • 11.5 Shift, complete sqr. ws P6 Derive	rotation ws P. 7 - Due: Textbook 11.1~11.4	
Quiz Ch 12 Ch 11 -Due: Guide	- Due: Barron's 3, 6	
		final

Conic Sections

optics
architecture

$$Ax^2+\underbrace{Bxy}_{\text{rotation}}+Cy^2+Dx+Ey+F=0$$

COMPLETE
THE
SQR.

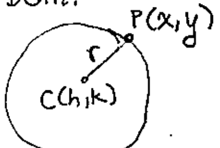


circle

- unit circle
- waves
- circular motion

$(x-h)^2+(y-k)^2=r^2$

Defn.



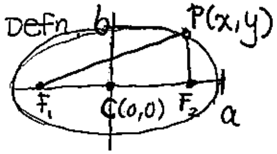
$\{(x,y): d(P,C)=r\}$

ellipse

- sound in room
- orbit around sun

$\left(\frac{x-h}{a}\right)^2+\left(\frac{y-k}{b}\right)^2=1$

Defn



$d(P,F_1)+d(P,F_2)=\text{constant}$

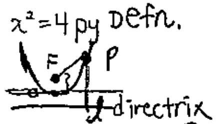
- reflective
- building sunburn
- headlight
- oven

parabola
- ~~mirror~~
- projectile motion

$y=ax^2+bx+c$

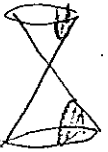
	p>0	p<0
$x^2=4py$	∪	∩
$y^2=4px$	(	)

$x^2=4py$ Defn.



$y=-p$

$\{(x,y): d(P,F)=d(P,\ell)\}$

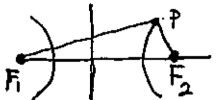


hyperbola

LORAN
navigation

- architecture
- cooling tower

$\left(\frac{x-h}{a}\right)^2 \pm \left(\frac{y-k}{b}\right)^2=1$



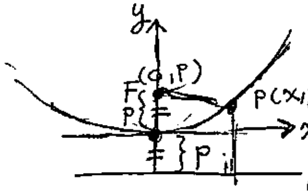
$\{P(x,y): d(P,F_1)-d(P,F_2)=2a\}$ constant

		A x^2	+ (Bxy)	+ Cy^2	+ Dx	+ Ey	+ F
circle	○	✓		✓	✓	✓	✓
ellipse	⬭ 0	✓		✓	✓	✓	✓
parabola	(	×		✓	✓	✓	✓
	∩	✓		×	✓	✓	✓
hyperbola	)(≍	✓		✓	✓	✓	✓

$\boxed{10.1}$ Parabola

WS P2

Ⓘ Derive the eqn. from geometry



$$d(P,\ell)=d(P,F)$$
$$y+p=\sqrt{(x-0)^2+(y-p)^2}$$
$$(y+p)^2=x^2+y^2-2py+p^2$$
$$\cancel{y^2}+2py+\cancel{p^2}=x^2+\cancel{y^2}-2py+\cancel{p^2}$$
$$4py=x^2$$

- Derive equations starting from Center at (0,0)
- $f(x,y)=c$

Translate right 2, up 3 $\Rightarrow f(\underset{\sim}{x-2},\underset{\sim}{y-3})=c$

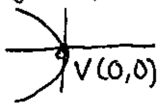
both values need to be bigger to get c right & up

Translate left 5, up 2

$f(x+5,y-2)=c$

UNLIKE
$y=g(x)+c$
$y=g(x-h)+c+k$
OR LIKE
$y-k=g(x-h)+c$

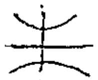
- $y^2=4px$ $p<0$. Translate so $V(2,-3)$



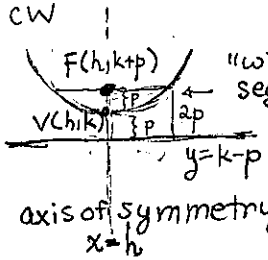
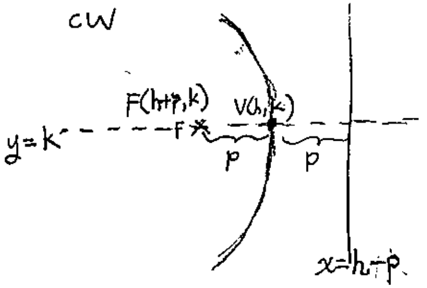
$f(x-2,y+3)=c$
$(y+3)^2=4p(x-2)$

$(y-k)^2=4p(x-h)$

Give WS

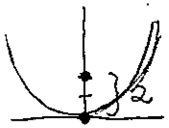


fill in info.
① Parabola sketch
② ellipse string
③ derive

WS P3	p>0	p<0
$(x-h)^2=4p(y-k)$	CW  "width" segment "Latus rectum" length focal diameter $\boxed{4p}$	HW
$(y-k)^2=4p(x-h)$	HW	CW 

ex1 $V(0,0)$ Parabola?

$F(0,2)$



Up

$(x-0)^2 = 4\overset{2}{p}(y-0)$

$x^2 = 8y$

Directrix $y=-2$

axis of symm : $x=0$

ex2 $y=-x^2 \Longrightarrow x^2=-y$

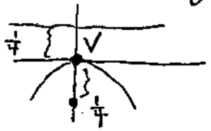
$F(0,-\frac{1}{4})$

$V(0,0)$

D $y=\frac{1}{4}$

Axis of symm $x=0$

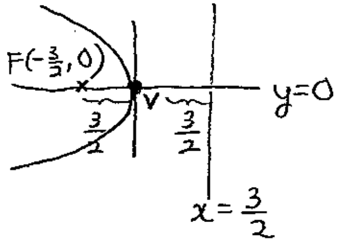
$x^2 = 4(-\frac{1}{4})y$ ↑ $(0,0)$



ex3 $6x+y^2=0 \rightarrow y^2=4px$ Left/right

a) $y^2=-6x$

$y^2 = 4(-\frac{3}{2})x$



axis of symm

b) CALC

$y=\pm\sqrt{-6x}$

ex4 $y=\frac{1}{2}x^2$

$x^2 = 2y$

$= 4(\frac{1}{2})y$ → $V(0,0)$

$F(0,\frac{1}{2})$

dir $y=-\frac{1}{2}$

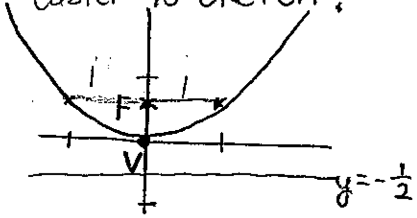
V, F, dir, axis of symm?

Length of latus rectum

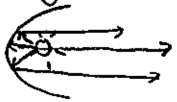
focal diameter

$= 4p = \boxed{2}$ ← Left/Right

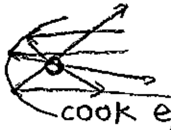
easier to sketch!



ex6 Apps. Light Reflective property

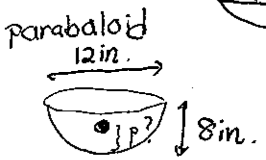


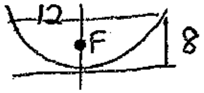
headlight /Flashlight mirror



cook egg

paraboloid





$x^2 = 4py$

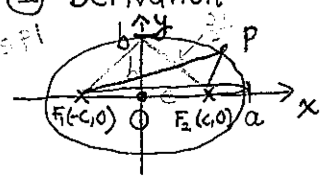
$6^2 = 4p8$

$9 = p8$

$\boxed{\frac{9}{8} = p}$

10.2 Ellipse

① Derivation



$b^2+c^2 = \left(\frac{2a}{2}\right)^2$

$c^2 = a^2-b^2$

$\left(\frac{x}{a}\right)^2 + \left(\frac{y}{b}\right)^2 = 1$

$\{P(x,y) : d(P,F_1)+d(P,F_2) = \underbrace{\text{constant}}\}$

$d(P,F_1)+d(P,F_2) = 2a$

$\sqrt{(x+c)^2+y^2} + \sqrt{(x-c)^2+y^2} = 2a$

$\sqrt{(x-c)^2+y^2} = 2a - \sqrt{(x+c)^2+y^2}$

$(x-c)^2+y^2 = 4a^2 - 4a\sqrt{(x+c)^2+y^2} + (x+c)^2+y^2$

$\cancel{x^2}-2xc+\cancel{c^2}+\cancel{y^2} = 4a^2 - 4a\sqrt{(x+c)^2+y^2}$

$+\cancel{x^2}+2xc+\cancel{c^2}+\cancel{y^2}$

$\cancel{4}a\sqrt{(x+c)^2+y^2} = \cancel{4}a^2+\cancel{4}xc$

$a^2[(x+c)^2+y^2] = a^4+\cancel{2a^2xc}+x^2c^2$

$a^2[x^2+\cancel{2xc}+c^2+y^2]$

$(a^2-c^2)x^2 + a^2y^2 = a^4-c^2a^2$

$= a^2(a^2-c^2)$

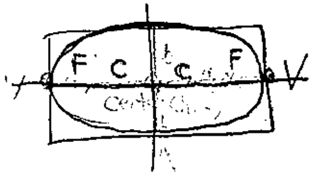
$\frac{x^2}{a^2} + \frac{y^2}{a^2-c^2} = 1$

$\left(\frac{x}{a}\right)^2 + \left(\frac{y}{b}\right)^2 = 1$

Parts of an ellipse WSP3

LONG - SHORT

Focus $c = \sqrt{a^2 - b^2}$



Axis

or 2b

$$\left(\frac{x-h}{a}\right)^2 + \left(\frac{y-k}{b}\right)^2 = 1$$

$C(h,k)$	$C(h,k)$
$F_1(h-c,k)$	$F_1(h,k+c)$
$F_2(h+c,k)$	$F_2(\ ,k-c)$
$V_1(h-a,k)$	V_1
$V_2(h+a,k)$	

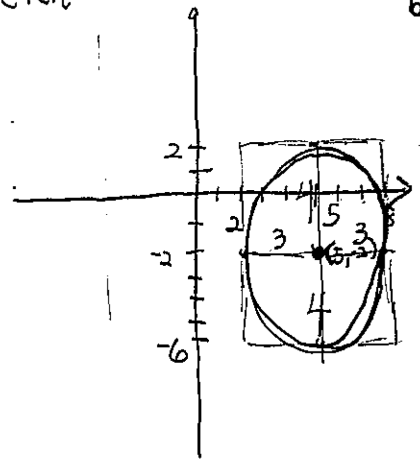
$e = 1$

$0 < e = \quad < 1$

ex1 $\left(\frac{x-5}{3}\right)^2 + \left(\frac{y+2}{4}\right)^2 = 1$

Sketch

a)



b) $C(5,-2)$

$V_1(5,2)$

$V_2(5,-2-4) = (5,-6)$

MAJOR axis Length : 8

MINOR : 6

$F_1(5,-2+\sqrt{7})$ $c = \sqrt{16-9} = \sqrt{7}$

$F_2(5,-2-\sqrt{7})$

$e = \frac{\sqrt{7}}{4}$

c) Sketch on CALC ✓

$$\begin{matrix} Y_1 \\ Y_2 \end{matrix} = \pm 4\sqrt{1 - \left(\frac{x-5}{3}\right)^2} - 2$$

ex2 Sketch

a)

$9x^2 + 16y^2 = 144$

$\frac{x^2}{16} + \frac{y^2}{9} = 1$

$\left(\frac{x}{4}\right)^2 + \left(\frac{y}{3}\right)^2 = 1$

b) MAJOR : 8
MINOR : 6

$V_1(4,0)$ $F_1(\sqrt{7},0)$

$V_2(-4,0)$ $F_2(-\sqrt{7},0)$

$c = \sqrt{16-9} = \sqrt{7}$

$e = \sqrt{7}/4$

ex3 ✓ $V(\pm 4,0)$ $\xrightarrow{eqn?}$

$F(\pm 2,0)$

~~$e(4,0)$~~

$\left(\frac{x}{4}\right)^2 + \left(\frac{y}{\sqrt{12}}\right)^2 = 1$

$c^2 = a^2 - b^2$

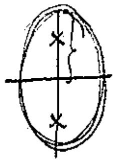
$4 = 16 - b^2$

ex4 ✓ $F(0, \pm 8)$

$e = \frac{4}{5}$

$\rightarrow$ eqn?

$\left(\frac{x}{6}\right)^2 + \left(\frac{y}{10}\right)^2 = 1$



$e = \frac{c}{b}$

$\frac{4}{5} = \frac{8}{b}$

$b = 10$

$a^2 = b^2 - a^2$

$64 = 100 - a^2$

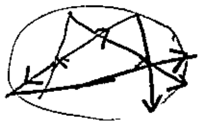
$a^2 = 36$

Apps

Whispering gallery

Reflecting property

Planet orbit



$e \approx 0$

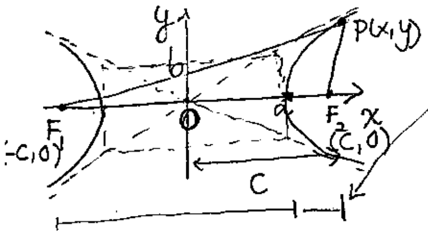
5

$\boxed{10.3}$ Hyperbola

Ⓘ Derivation

wsp2

① $\{P(x,y) \mid d(P,F_1)-d(P,F_2)=\text{constant}\}$



constant = $2a$

$$\sqrt{(x+c)^2+y^2}-\sqrt{(x-c)^2+y^2}=2a$$

$$\sqrt{(x+c)^2+y^2}=2a+\sqrt{(x-c)^2+y^2}$$

$$\cancel{x^2}+2xc+\cancel{c^2}+\cancel{y^2}=4a^2+4a\sqrt{(x-c)^2+y^2}+\cancel{x^2}-2xc+\cancel{c^2}+\cancel{y^2}$$

$$\cancel{4}a\sqrt{(x-c)^2+y^2}=\cancel{4}xc-\cancel{4}a^2$$

$$a^2(x^2-\cancel{2x}c+c^2+y^2)=x^2c^2-\cancel{2xca^2}+a^4$$

$$a^2(c^2-a^2)=x^2(c^2-a^2)-y^2a^2$$

$$\boxed{\frac{x^2}{a^2}-\frac{y^2}{c^2-a^2}=1}$$

Set $b^2=c^2-a^2$

★ $\boxed{c^2=a^2+b^2}$ Hyperbola: Sum. Not difference

② **Why 2 branches:**

$$\left(\frac{y}{b}\right)^2=\left(\frac{x}{a}\right)^2-1\geq 0 \Rightarrow \left(\frac{x}{a}\right)^2\geq 1$$

$$x^2\geq a^2 \quad \boxed{|x|\geq a}$$

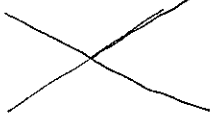
Branches along
x direction

Symmetrical across:

x-axis $(x,y)\longleftrightarrow(x,-y)$

y-axis $(-x,y)$

Asymptote far away



$$y=\pm b\sqrt{\left(\frac{x}{a}\right)^2-1}$$

no info:
(As $x\to\infty$
$y\to\infty$)

$$=\pm\frac{b}{a}x\sqrt{1-\left(\frac{a}{x}\right)^2}$$

$x\to\infty$

$$y\to\pm\frac{b}{a}x$$

Parts of a hyperbola

wsp3

$$\left(\frac{x-h}{a}\right)^2-\left(\frac{y-k}{b}\right)^2=1$$

TRANSVERSE AXIS

CONJUGATE AXIS

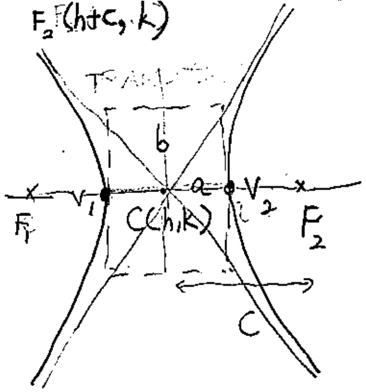
$\boxed{c^2=a^2+b^2}$

NOT long or short
NOW it's branches

$V_1(h-a,k)$
$V_2(h+a,k)$
$F_1(h-c,k)$
$F_2(h+c,k)$

$C(h,k)$

Asymptote
$y=\pm\frac{b}{a}(x-h)+k$



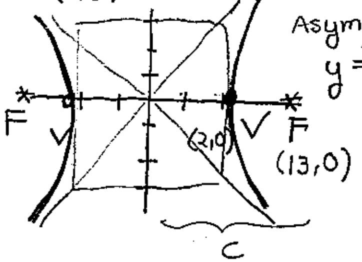
$$-\left(\frac{x-h}{a}\right)^2+\left(\frac{y-k}{b}\right)^2=1$$

CONJUGATE

TRANSVERSE

$C(h,k)$
$V_1(h,k+b)$
$V_2(h,k-b)$
$F_1(h,k+c)$
$F_2(h,k-c)$

③ $\left(\frac{x}{2}\right)^2-\left(\frac{y}{3}\right)^2=1$



Asymptote
$y=\pm\frac{3}{2}x$

$c=\sqrt{4+9}$

what if ... move all
left 5, up 3?

$$\left(\frac{x-5}{2}\right)^2-\left(\frac{y+3}{3}\right)^2=1$$

$\boxed{ex2}$ $x^2-9y^2+9=0$ Sketch & Label

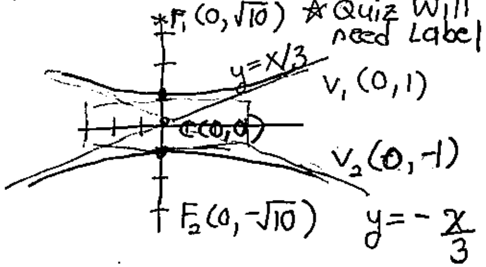
. Get in form

$$x^2-9y^2=-9$$

$$-\frac{x^2}{9}+y^2=1$$

$$-\left(\frac{x}{3}\right)^2+\left(\frac{y}{1}\right)^2=1$$

$$c^2=9+1$$



$\boxed{ex3}$ ✓ $V_1(\pm 3,0)$ Equation of hyperbola?

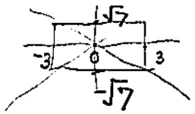
$F(\pm 4,0)$

Along x

$$c^2=a^2+b^2$$

$$16=9+b^2$$

$$b=\sqrt{7}$$

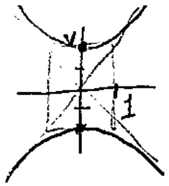


$$\left(\frac{x}{3}\right)^2-\left(\frac{y}{\sqrt{7}}\right)^2=1$$

$$\frac{x^2}{9}-\frac{y^2}{7}=1$$

$\boxed{ex4}$ ✓ $V(0,\pm 2)$ Hyperbola eqn?

$y=\pm 2x$

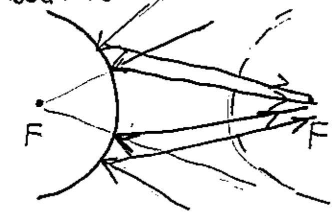


Along y: $-\left(\frac{x}{1}\right)^2+\left(\frac{y}{2}\right)^2=1$

✓ <u>APPS</u>: Reflection Property

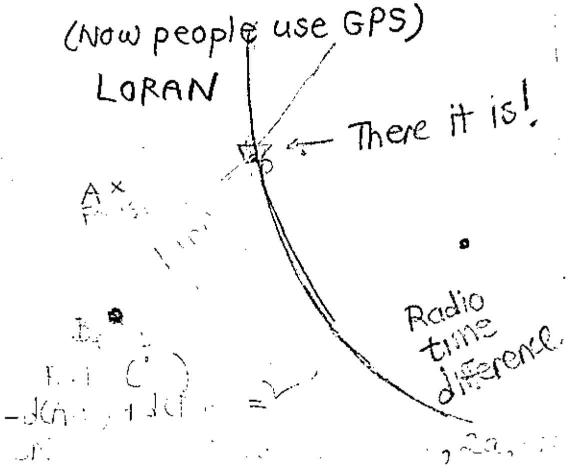
(Cassegrain-type telescope)

Light aimed at one focus bounces toward other focus



(Now people use GPS)

LORAN



$\boxed{10.4}$ Shifted Conics COMPLETE THE SQUARE

$\boxed{ex1}$ $y^2-4y=8x-28$ ← y^2 (no x^2) parabola Right

$$y^2-4y+4=8x-28+4$$

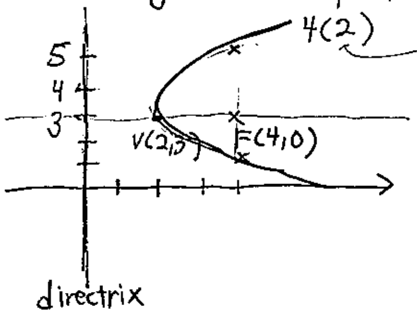
$$(y-2)^2=8(x-3)$$

$$y^2=4px$$

$$(y-h)^2=4p(x-k)$$

Lactus rectum half $=4$

Sketch & Label



can see x^2, y^2 & ± hyperbola

$\boxed{ex3}$ $9x^2-72x-16y^2-32y=16$

$$9(x^2-8x+16)-16(y^2+2y+1)=\cancel{16}+9\cdot 16-\cancel{16\times 1}$$

$$9(x-4)^2-16(y+1)^2=16(9)$$

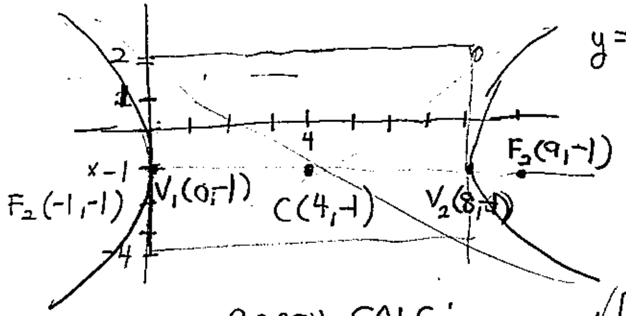
$$\frac{(x-4)^2}{16}-\frac{(y+1)^2}{9}=1$$

$$\left(\frac{x-4}{4}\right)^2-\left(\frac{y+1}{3}\right)^2=1$$

Sketch & Label

$c^2=16+9=25$

$y=\pm\frac{3}{4}(x-4)-1$



GRAPH CALC: $y=\pm\sqrt{\left(\frac{x-4}{4}\right)^2-1}\;)3-1$

General Eqn of a Shifted Conic

	Ax^2 +	(Bxy) +	Cy^2 +	Dx +	Ey +	$F = 0$	DEGENERATE CASE
Parabola	✓ : ∪ ∩		×	✓	✓	✓	ϕ since complete the square.
	×		✓ ⊂	×	×	×	
ellipse	✓ same sign ✓			✓	✓	✓	
hyperbola	✓ diff. sign ✓			✓	✓	✓	×

$\boxed{ex4}$ A Degenerate Conic

$9x^2 - y^2 + 18x + 6y = 0$

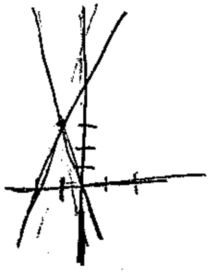
$\uparrow$ hyperbola : complete the square

$(x+1)^2 - \left(\frac{y-3}{3}\right)^2 = \boxed{0}$ NOT 1

$\left(\frac{y-3}{3}\right)^2 = (x+1)^2$

$y - 3 = 3(\pm(x+1))$

$y = \pm 3(x+1) + 3$



ellipse?

$4x^2 + y^2 - 8x + 2y + 6 = 0$

$\boxed{ex}$ $(x-1)^2 + \frac{(y+1)^2}{4} = \boxed{-\frac{1}{4}}$ NEVER TRUE NO SOLN

$\boxed{ex}$

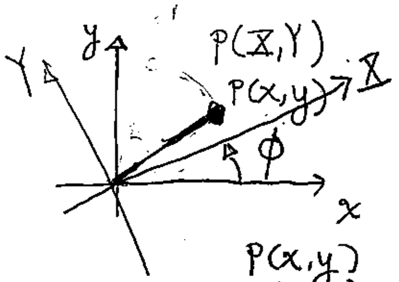
★ Derivations WS ★

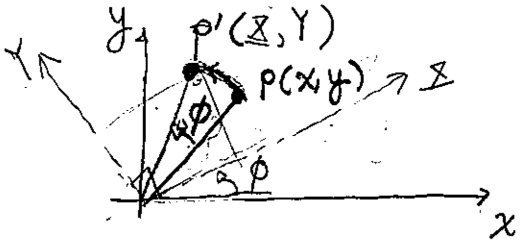
- Skip to 10.7 - basic parametric
- Come back to more
  - conic sections
  - polar
  - trig
  - matrix rotation/image → POSTER PROJECT!

what fun!

10.5 Rotation of Axes

friends: POLAR, TRIG, MATRIX!

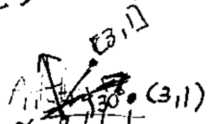




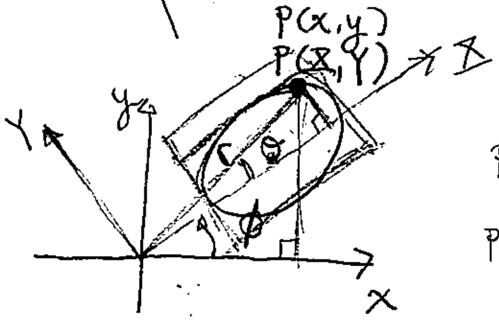
Rotation of axes

How to write P'(X,Y) as P'(x,y)

has values P(x,y)



$$\begin{bmatrix} x \\ y \end{bmatrix} \longrightarrow \begin{bmatrix} x' \\ y' \end{bmatrix} = \begin{bmatrix} \overset{\cos 30^\circ}{\cos\phi} & -\sin\phi \\ \sin\phi & \cos\phi \end{bmatrix} \overset{\begin{bmatrix} 3 \\ 1 \end{bmatrix}}{\begin{bmatrix} x \\ y \end{bmatrix}}$$



P(X,Y) $X = r\cos\theta$ $Y = r\sin\theta$

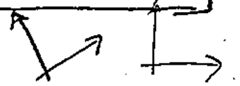
P(x,y) $x = r\cos(\theta+\phi)$ $y = r\sin(\theta+\phi) = r(\sin\theta\cos\phi + \cos\theta\sin\phi)$

$= r(\cos\theta\cos\phi - \sin\theta\sin\phi)$

$x = X\cos\phi - Y\sin\phi$

$y = Y\cos\phi + X\sin\phi$

$$\boxed{\underbrace{\begin{bmatrix} \cos\phi & -\sin\phi \\ \sin\phi & \cos\phi \end{bmatrix}}_{A} \begin{bmatrix} X \\ Y \end{bmatrix} = \begin{bmatrix} x \\ y \end{bmatrix}}$$



formula to change

$P(x,y) \xrightarrow{\phi} P(X,Y)$

augmented matrix

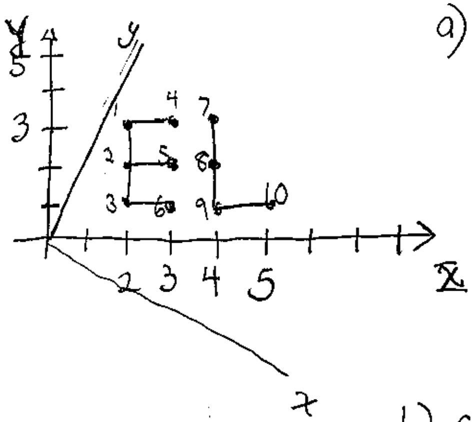
$$\left[\begin{array}{cc|c} \cos\phi & -\sin\phi & x \\ \sin\phi & \cos\phi & y \end{array}\right]$$

OR

$$A^{-1} = \frac{\begin{bmatrix} \cos\phi & \sin\phi \\ -\sin\phi & \cos\phi \end{bmatrix}}{\cos^2\phi + \sin^2\phi}$$

$$\boxed{\begin{bmatrix} X \\ Y \end{bmatrix} = \begin{bmatrix} \cos\phi & \sin\phi \\ -\sin\phi & \cos\phi \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix}}$$

HW



a) coordinates after rotate 30°?

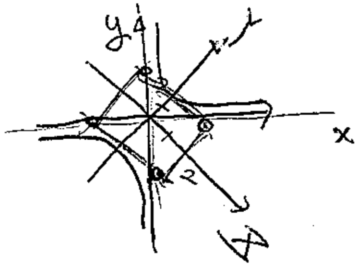
$$P\begin{bmatrix}X\\Y\end{bmatrix} \rightarrow \begin{bmatrix}x\\y\end{bmatrix}=\begin{bmatrix}\cos 30^\circ & -\sin 30^\circ\\ \sin 30^\circ & \cos 30^\circ\end{bmatrix}\begin{bmatrix}x\\y\end{bmatrix}$$

$$\begin{bmatrix}\frac{\sqrt3}{2} & -\frac12\\ \frac12 & \frac{\sqrt3}{2}\end{bmatrix}\begin{bmatrix}2 & 2 & 2 & 3 & 3 & 3 & 4 & 4 & 4 & 5\\ 1 & 2 & 3 & 3 & 2 & 1 & 3 & 2 & 1 & 1\end{bmatrix}$$

b) coordinates in XY system?
same as original

ex2 $xy = 2$ is a hyperbola? (Rotate axes thru 45°)

Rewrite in XY coord.



$$\begin{bmatrix}x\\y\end{bmatrix}=\begin{bmatrix}\cos 45^\circ & -\sin 45^\circ\\ +\sin 45^\circ & \cos 45^\circ\end{bmatrix}\begin{bmatrix}X\\Y\end{bmatrix}$$

$$=\begin{bmatrix}(X \mp Y)\\ X+Y\end{bmatrix}\frac{\sqrt2}{2}$$

$$xy = 2$$

$$\left(\frac{\sqrt2}{2}\right)^2 (X-Y)(X+Y) = 2$$

$$\frac{X^2-Y^2}{2} = 2 \rightarrow \frac{X^2}{4} - \frac{Y^2}{4} = 1$$

$$\left(\frac{X}{2}\right)^2 - \left(\frac{Y}{2}\right)^2 = 1$$

General Conics

$$Ax^2 + \boxed{Bxy} + Cy^2 + Dx + Ey + F = 0$$

Rewrite / Find ϕ that gets ~~rid~~ of $\underbrace{B}_{B'=0}$ in XY system

$$A'X^2 + C'Y^2 + D'X + E'Y + F' = 0$$

write in XY & get $B' = 0$

$$\begin{bmatrix}x\\y\end{bmatrix}=\begin{bmatrix}\cos\phi & -\sin\phi\\ +\sin\phi & \cos\phi\end{bmatrix}\begin{bmatrix}X\\Y\end{bmatrix}$$

$$A(X\cos\phi - \sin\phi\, Y)^2 + B(X\cos\phi - Y\sin\phi)(X\sin\phi + Y\cos\phi)$$

$$+C(X\sin\phi + Y\cos\phi)^2 + D\cancel{(X\cos\phi - Y\sin\phi)} + E\cancel{(X\sin\phi + Y\cos\phi)}$$

$$+F = 0$$

XY coeff: $-2A\cos\phi\sin\phi + B(\underbrace{\cos^2\phi - \sin^2\phi}_{\cos 2\phi})$
$+2C\sin\phi\cos\phi$

$$B' = (C-A)\sin 2\phi + B\cos 2\phi = 0$$

$$B\cos 2\phi = (A-c)\sin 2\phi$$

$$\boxed{\cot 2\phi = \frac{A-c}{B}}$$

$\boxed{\text{ex3}}$ HW : Show derivation

$$\overset{A}{6\sqrt{3}}\, x^2 + \underset{}{\overset{B}{6xy}} + \overset{C}{4\sqrt{3}}\, y^2 = 21\sqrt{3}$$

Rotate ϕ

Change to XY

① $$\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} \cos\overset{30°}{\phi} & -\sin\phi \\ \sin\phi & \cos\phi \end{bmatrix} \begin{bmatrix} X \\ Y \end{bmatrix}$$

$$= \begin{bmatrix} \frac{\sqrt{3}}{2}X - \frac{Y}{2} \\ \frac{X}{2} + \frac{\sqrt{3}}{2}Y \end{bmatrix}$$

② $$\cot 2\phi = \frac{A-C}{B} = \frac{\cancel{2}\sqrt{3}}{6}$$

$$\tan 2\phi = \sqrt{3}$$

$$2\phi = 60°$$

$$\phi = 30°$$

③ $$6\sqrt{3}\left(\frac{\sqrt{3}}{2}X - \frac{Y}{2}\right)^2 + \frac{6}{4}\left(\sqrt{3}X - Y\right)\left(X + \sqrt{3}Y\right) + 4\sqrt{3}\left(\frac{X}{2} + \frac{\sqrt{3}}{2}Y\right)^2 = 21\sqrt{3}$$

$$\frac{3\cancel{\sqrt{3}}}{2}\left(3X^2 - \underset{1}{\cancel{2\sqrt{3}XY}} + Y^2\right) + \frac{3}{2}\left(\cancel{\sqrt{3}}X^2 - \cancel{XY + 3XY} - \cancel{\sqrt{3}}Y^2\right) \leftarrow$$

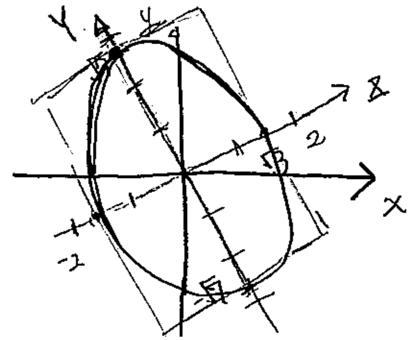
$$\underset{0}{\cancel{-9XY + 3XY + 2\cdot 3XY}} + \cancel{\sqrt{3}}\left(X^2 + \cancel{2\sqrt{3}XY} + 3Y^2\right) = 21\cancel{\sqrt{3}}$$

$$\Rightarrow X^2\left(\underbrace{\frac{9}{2} + \frac{3}{2} + 1}_{7}\right) + Y^2\left(\cancel{\frac{3}{2}}\cancel{-\frac{3}{2}} + 3\right) = 21$$

$$\frac{X^2}{3} + \frac{Y^2}{7} = 1$$

$$\left(\frac{X}{\sqrt{3}}\right)^2 + \left(\frac{Y}{\sqrt{7}}\right)^2 = 1$$

④

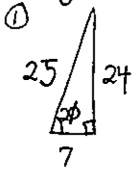


~~$B^2 - 4AC$~~ is invariant

- same as $B'=0$ $A'C'$ value
- $=0$ $(A'=C'=0)$ Parabola
- <0 $(A', C'$ same) ellipse
- >0 $(A', C'$ opp) hyperbola

$\boxed{\text{ex4}}$ $$64x^2 + 96xy + 36y^2 - 15x + 20y - 25 = 0$$

Triangle ①



$$\cot 2\phi = \frac{A-C}{B} = \frac{64-36}{96} = \frac{7}{24}$$

② $$\cos 2\phi = \frac{7}{25}$$

②' $\boxed{\text{HW}}$

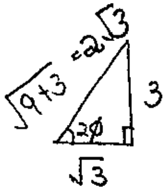
③ $$\cos\phi = \sqrt{\frac{1+\cos 2\phi}{2}} = \sqrt{\frac{1+\frac{7}{25}}{2}} = \frac{4}{5} \qquad \sqrt{\frac{\cancel{32}^{16}}{25\cdot\cancel{2}}}$$

$$\sin\phi = \sqrt{\frac{1-\cos 2\phi}{2}} = \sqrt{\frac{1-\frac{7}{25}}{2}} = \frac{3}{5} \qquad \sqrt{\frac{\cancel{18}^{9}}{25\cdot\cancel{2}}}$$

etc.

Similarly

$$\cot 2\phi = \frac{\sqrt{3}}{3}$$



$$\cos 2\phi = \frac{1}{2}$$

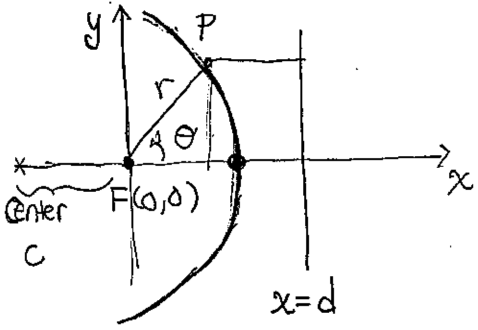
$$\cos\phi = \sqrt{\frac{1+\cos 2\phi}{2}} = \sqrt{\frac{1+\frac{1}{2}}{2}} = \frac{\sqrt{3}}{2} \checkmark$$

$$\sin\phi = \sqrt{\frac{1-\cos 2\phi}{2}} = \sqrt{\frac{1-\frac{1}{2}}{2}} = \frac{1}{2} \checkmark$$

HW 10.5 Pg(791) #19

follow $\begin{cases} a & ① \\ b \swarrow & ②\ ②' \\ c & ③ \end{cases}$

10.6 Polar Equations of Conics - all in terms of Focus & ℓ directrix



$$\frac{d(P,F)}{d(P,\ell)} = e \quad \text{eccentricity}$$

$e=1$ is parabola by definition

$0<e<1$ is ellipse?

$$\frac{r}{d-r\cos\theta} = e \Rightarrow r = \frac{ed}{1+e\cos\theta}$$

$$r = e(d-r\cos\theta)$$

$$r^2 = e^2(d-r\cos\theta)^2$$

$$x^2+y^2 = e^2(d-x)^2$$

$$= e^2(d^2-2dx+x^2)$$

$$(1-e^2)x^2 + 2de^2x + y^2 = e^2d^2$$

$$(1-e^2)\left[x^2 + \frac{2de^2}{1-e^2}x + \left(\frac{de^2}{1-e^2}\right)^2\right] + y^2 = e^2d^2 + \frac{d^2e^4}{1-e^2}$$

$$(1-e^2)\left(x+\frac{de^2}{1-e^2}\right)^2 + y^2 = \frac{e^2d^2 \cancel{-e^4d^2} \cancel{+d^2e^4}}{1-e^2}$$

$$\left(x+\frac{de^2}{1-e^2}\right)^2 + \frac{y^2}{1-e^2} = \frac{e^2d^2}{(1-e^2)^2}$$

$$h = -\frac{de^2}{1-e^2}, \quad a = \frac{ed}{1-e^2}, \quad b^2 = \frac{e^2d^2}{(1-e^2)}$$

Verify

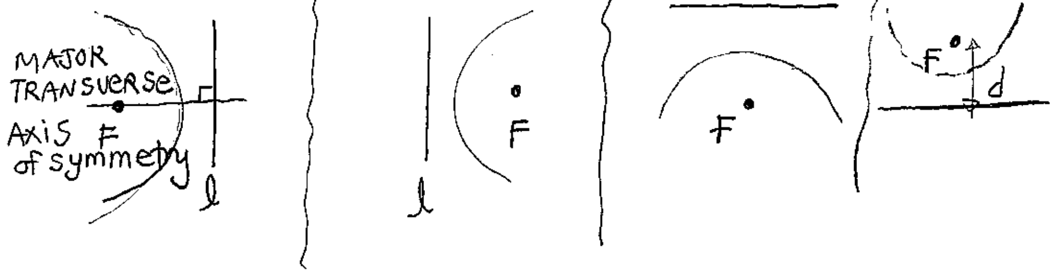
$$c^2 = a^2-b^2 = \frac{e^2d^2 - e^2d^2(1-e^2)}{(1-e^2)^2} = \frac{e^4d^2}{(1-e^2)^2}$$

$$c = -h = \boxed{\frac{e^2d}{1-e^2}}$$

$$e = \frac{c}{a} = \swarrow \quad \checkmark$$

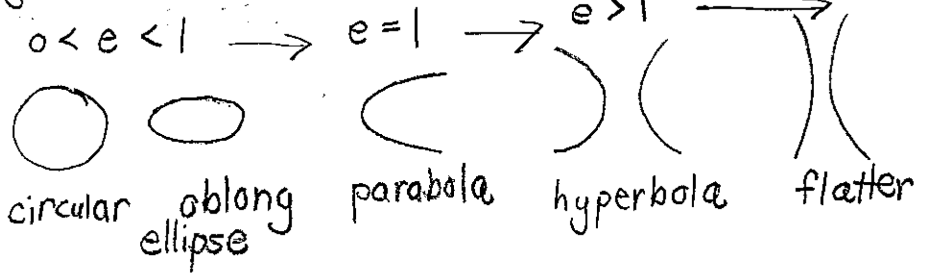
General:

$$r = \frac{ed}{1+e\cos\theta} \qquad r = \frac{ed}{1-e\cos\theta} \qquad r = \frac{ed}{1+e\sin\theta} \qquad r = \frac{ed}{1-e\sin\theta}$$



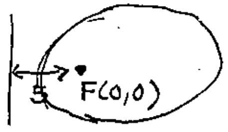
pg 799

$0<e<1 \longrightarrow e=1 \longrightarrow e>1 \longrightarrow$



$\boxed{ex2}$ $r = \frac{10}{3-2\cos\theta}$ Sketch. $\frac{ed}{1-e\cos\theta}$

$= \frac{\frac{10}{3}}{1-\frac{2}{3}\cos\theta} = \frac{\frac{2}{3}(5)}{1-\frac{2}{3}\cos\theta}$



$0 < e = \frac{2}{3} < 1$

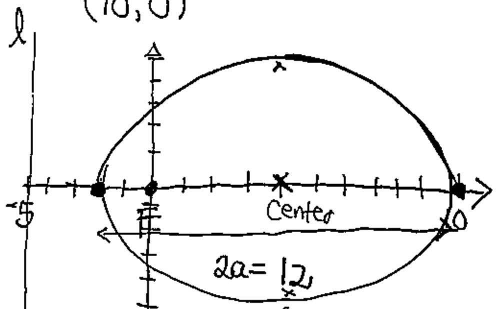
ellipse

Detailed plot

horizontal major axis

- r is minimum when $\cos\theta = -1$
  $(2, \pi)$
  $\theta = \pi$
  $r = \frac{10}{3+2} = 2$

- r is max when denominator smallest, $\cos\theta = 1$
  $(10, 0)$
  $\theta = 0$
  $r = \frac{10}{3-2} = 10$



$2a = 12$

$a = 6$

$e = \frac{c}{a} \Rightarrow \frac{2}{3} = \frac{c}{6}$

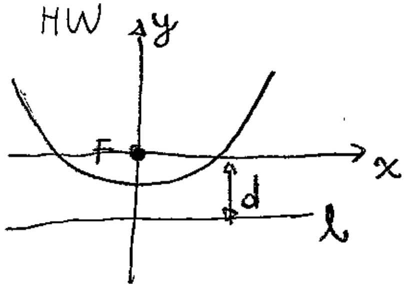
$c = 4 \Rightarrow 16 = a^2 - b^2$

$b = \sqrt{36-16} = \sqrt{20} = 2\sqrt{5} \approx 4.47$

$\boxed{ex3}$ $r = \frac{12}{2+4\sin\theta}$

$\boxed{ex4}$ $r = \frac{10}{3-2\cos(\theta - \pi/4)}$ It's ex2 but rotated by $\frac{\pi}{4}$ counterclockwise

HW



verify

$$\frac{d(P,F)}{d(P,\ell)} = e$$

is a hyperbola

$e > 1$