Ch10 6th edition Linear Algebra

6th
- 9.1 (5th) (Pg. 642) # 7, 15, 21, 33, 35, 43, 45. *55
  10.8 (P. 701) 8 14 17 30 31 37 40 *49
  eliminate let $u = \frac{1}{x}$ Graph

- 9.2 (Pg. 649) # 22, 23, 29, 33, 37, 40, 41, 47, 48, (50) 53, 54, 56, 57, 58. *75
  10.1 (P638) 37 43 47 52 53 56 63 64 (67) 68 69 72 73 74
  System of Eqn in 2 variables — ~~no soln~~ ∞, ∞, no soln

- 9.3 (Pg. 657) # 1, 3, 9, 11, (17, (19) 21, 23, (25, 29, 31,) 36, 37, 38. Bonus 39.
  10.2 (P. 646) 3 6 11 14 22) 24 25 27 30 34 35) 46 39 44 *47
  System in Several — 1, 1, ∞, ∅, ∅, ∞, 1

- Pg. 661 # 1b, 3.
  online
  Show $R_1 + R_2 \to R_2$ etc.

SPRING BREAK HW

- 9.4 (Pg. 684) # 9~13, 19, (29) 31, 32 39, (45) (ok to use calculator rref for these two: 51, 53).
  10.3 (P. 659) 5~10, 11~18 ab ~~c~~ 23 34 35 36 47 51 59 61
  rref * ex3~4 — 1, ∅, ∞ ∞, 1 Gauss, ∞ 2

- 9.5 (Pg. 684) # 1, 4, 5, 7, 8, 9, 11, 13, 15, (23, 27, 35) 40, 41, 45, (48) 49, 50,
  10.4 (P. 669) 1 6 8 9 11 12 14 15 18 20 (21b 28b 29a) 35 38 41 43 ~~x~~ 44 46 46
  matrix algebra

  51, 52. Bonus: # 48, 55, 56
  48 48 44 51 52 53

Quiz: 9.1~9.5.Gauss-Jordan elimination. rref.

- how many solutions matrix algebra bonus: sigma notation

- 9.6 (P. 697) # 9, 11, 16, 17, 19, 22, 25, 28, (CALC OK for below: 35, 36, 37, 38, 39, 40,
  10.5 (P.680) 3, formula 11 14 18 20 21 31 23 27 30 38 37 39 40 41 42
  A^{-1} — A^{-1}, A^{-1}, A^{-1} & Solve

  42, 43, (45, 47) Bonus # 50.
  43 46 44 50

  For what x, if any, does it have no inverse?

  CW $\begin{bmatrix} \cos x & \sin x \\ -\sin x & \cos x \end{bmatrix}$ $\begin{bmatrix} \sec x & \tan x \\ \tan x & \sec x \end{bmatrix}$ $\sec^2 x - \tan^2 x = 1$ same

  $\cos^2 x + \sin^2 x = 1 \neq 0$
  $\forall x$ ok inverse

det A
Cramer

- 9.7 (P. 713) # 4, 5, 9~14, 15, 18, 20, 22, 24, 25, 26, 27,
  10.6 (P690) 7, 10, 13, 17, 18 19 21 20 25 28 30 27 29 31

  any triangular has diagonal as determinant

  5 kids; Replace R_3 with $R_3 - R_2$; Replace C_1 with $C_1 + 2C_2$ ... get 2 0's

- Cramer 31, 39, 42, 44, 45, 46, 47, 49, 50, 58. Bonus # 55, 59, 62. 63
  36 43 45 48 49 50 52 53 54 60 62 59 63 66 67

  GJ faster
  Longer for 5x5 determinant

Bonus

- B1) Prove formula for inverse of 2x2 matrix (P. 690) 674
- B2) Use sigma notation to write the method for expanding a matrix in terms of cofactors and minors.
  - a) using ith row expansion
  - b) using jth column expansion

Partial fraction expansion

Short ← | Use shortcut

- 9.8 (P. 720) # 1, 4, 6, 7, 9, 10, | 21, (22,) 23, 24,
  10.7 (P.697) ~~1, 2~~ 4 5, 8 10 11, 12, 24 (23) 26 25

by hand, 2 students; factor

(CALC OK for the following: 26, 33, 34, (37,) 38, 42, 43, 44):
Set up augmented matrix 27 36 35 (40) 39 43 44 45 46
improper

- If you can use the shortcut method, please do.
- If you use a calculator, please set up the augmented matrix by equating coefficients.

see 6th 10.8

old → ("9.1" here 5th)

Don't need to find vertex but say bdd or not

- 9.9 (P. 726) # 6, 8, 11, 14, 15~18, 16 | 21, 22, 30, 36, 37, 38, 39, 40, 46.
  10.9 (P.708) 7 9 14 16 17~20 23 24 35 41 43 44 46 45 52

  not bdd; circle bdd; no soln; intro to linear prog.

Pg. 737 Linear Programming # 1, 3, 6, 13, 15, 16.
P. 716

Quiz: 9.6~9.9 and Linear Programming

9.1 (P.642) #7, 15, 21, 33, 35, 43, 45.

9.2 (P.649) 22, 23, 29, 33, 37, 40, 41, 47, 48,
0 ∞ ∞ 0 speed

50, 53, 54, 56, 57, 58
mixture

9.3 (P.657) #1, 3, 9, 11, 17, 19, 21, 23, 25, 29,
31, 36, 37, 38, *39

P661 # 1b, 3
rref method

GJ

9.4 (P.613) # 9, 10, 11, 12, 13, 19, 29, 31, 39, 45, 51, 53
rref
CALC OK
rref
Show augmented

9.5 (P.684) # 1, 4, 5, 7, 8, 9, 11, 13, 15, 23, 27, 35, 40, 41, 45, 48
49, 50, 51, 52, Bonus: 48, 55, 56

Look up SAT / sig processing
matrix word probs

9.6 (P.697) # 9, 11, 16, 17, 19, 22, 25, 28, 35, 36, 37, 38,
39, 40, 42, 43, 45, 47, *50
CALC OK to 47
3 5 7, 9 11, 16, 17, 19, 22, 23, 25, 27, 35, 37
2x2 x 3x3 *38, 42, 43, 47, 49

row/column to 0's

9.7 (P.713)
24 26
determinant: 4, 5, 9 ~ 14, 15, 18, 20, 22, 25, 27,
even in class

Cramer... 31, 39, 42, 44, 45, 46, 47, 49, 50, 58
BONUS 55, 59, 62

Best vs exact

9.8 (P.720) no CALC for shortcut & one by hand
1, 4, 6, 7, 9, 10, 21, 22, 23, 24,
CALC OK: 26, 33, 34, 37, 38, 42, 43, 44
rref only

21 23 24 26 33 34 37 38 42 43 44

9.9 6, 8, 11, 14, 15~18, Don't need to find vertex but say bdd or not
21, 22, 30, 36, 37, 38, 39, 40, 46

Linear Programming

P737 # 1, 3, 6, 13, 15, 16

1

Ch.9

9.1 My Summary/Map/Overview of Linear Algebra

Systems of Equations and Inequalities

Methods Overview

Aufmann
Baker, Nation

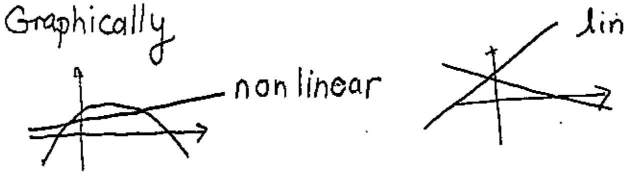
① Algebraic

(i) substitution $\boxed{y} = 2x \quad x + \boxed{y} = 3$

(ii) elimination
$$\begin{aligned} x+y&=3 \\ x-y&=2 \\ \hline 2x&=5 \end{aligned}$$

② Graphically

nonlinear — linear



③ Matrix (for linear systems)

$$\begin{cases} A_1x + B_1y + C_1z = D_1 \\ A_2x \quad B_2 \quad C_2 \quad D_2 \\ A_3 \quad B_3 \quad C_3 \quad D_3 \end{cases}$$

① Gaussian elimination

analogous

$$\begin{aligned} x + y + z &= 1 \\ 2x - y + z &= 2 \\ 3x + 2y - z &= 3 \end{aligned}$$

$\left[\begin{array}{ccc|c} 1 & 1 & 1 & 1 \\ 2 & -1 & 1 & 2 \\ 3 & 2 & -1 & 3 \end{array}\right]$ switch rows. Same; multiply × scalar. Same; adding 2 eqns (rows) $\rightarrow \left[\begin{array}{ccc|c} 1 & 0 & 0 & 2 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 3 \end{array}\right] \rightarrow$ $x=2$, $y=0$, $z=0$

coeff matrix

augmented matrix

$\boxed{\text{Always works}}$
Gaussian Elimination

reduced row echelon form

TI rref(A)

(ii) $A\vec{x} = \vec{b}$ ← * helps unique soln system only

$$\begin{bmatrix} 1 & 1 & 1 \\ 2 & -1 & 1 \\ 3 & 2 & -1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}$$

only if invertible
$n \times n$ (3 eqns 3 unknowns)
$|A| \neq 0$ iff A^{-1} exists

$\vec{x} = A^{-1}\vec{b}$
inverse matrix

(iii) Cramer's Rule
($n \times n$ system)
$D \neq 0$ for coeff matrix
det

(use: to find value of just 1 variable)

How many Solutions? 0, 1, infinitely many

independent
3 equations
2 unknowns

$$\begin{cases} Ax + By = C \\ Dx + Ey = F \\ Gx + Hy = L \end{cases}$$

could have 1 solution

more eqns than unknowns
Overdetermined
No solution or 1 solution

all ~~Dependent eqns~~
parallel
multiples of each other

$2x + 3y = 1$
$4x + 6y = 2$ //

no solution
"inconsistent"

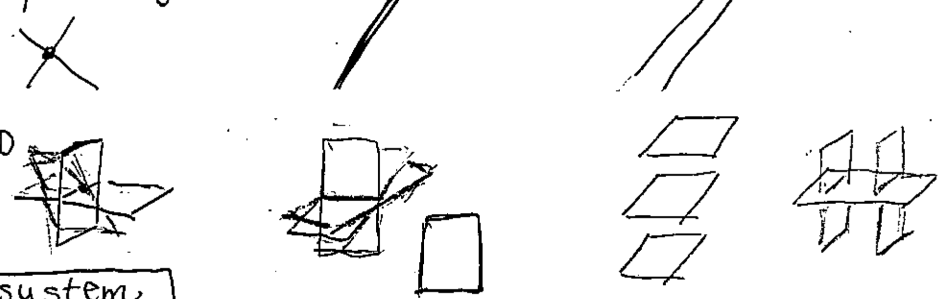
ID eqns < # unknowns

HOW MANY SOLUTIONS?

1	∞	0
independent system	dependent	inconsistent

$\begin{cases} Ax + By = C \\ \vdots \end{cases}$

$\begin{cases} Ax + By + Cz = D \\ \vdots \end{cases}$



★ $n \times n$ system unique soln
$\Leftrightarrow |A| \neq 0$
det of coeff matrix

$\Leftrightarrow$ A is invertible

ID eqns = # unknowns: 0, 1, ∞ many solns

ID eqns < # unknowns: 0 or ∞ many
(underdetermined)

ID eqns > # unknowns: 1, 0, or ∞ many
(overdetermined)

3

9.1 (10.8 6th) System of Nonlinear

Substitution Method

ex2 (1)

$$\begin{cases} x^2 + y^2 = 100 \\ 3x - y = 10 \end{cases}$$

(nonlinear)

① one in terms of another $\Rightarrow y = 3x - 10$

② Plug.

$x^2 + (3x-10)^2 = 100$

$x^2 + 9x^2 - 60x + 100 = 100$

$10x^2 - 60x = 0$

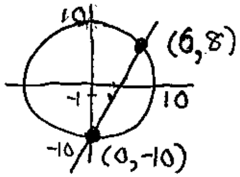
$10x(x-6) = 0$

$x = 0$ or 6

③ Back Sub.

$y = -10$ or 8

Note $y = 3x - 10$



ex4 ex2

Elimination method (SAT)

① $3x^2 + 2y = 26$

② $5x^2 + 7y = 3$

Adjust

$\rightarrow 15x^2 + 10y = 130$

$15x^2 + 21y = 9$

$-11y = 121$ eliminate

$y = -11$ Solve

$\rightarrow 3x^2 + 2(-11) = 26$ Back Sub

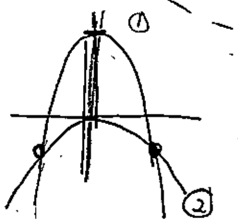
$3x^2 = 48$

$x^2 = 16$

$x = \pm 4$

answer:

$\boxed{(4,-11) \quad (-4,-11)}$



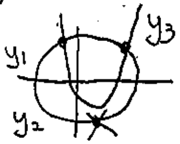
4

ex6 Graphically (calculus function know-how)

$$\begin{cases} x^2 + y^2 = 12 \\ y = 2x^2 - 5x \end{cases}$$

$\leftarrow y_1 = \sqrt{12 - x^2}$ $\quad y_2 = -\sqrt{12 - x^2}$

$\leftarrow y_3 = 2x^2 - 5x$



- ZOOM
- TRACE

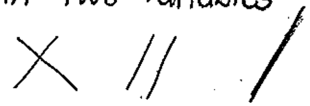
$\boxed{\text{Intersect}}$

TI-89: Solve

9.2 (6th 10.1)

System of Linear Equations in Two Variables

$$\begin{cases} ax + by = c \\ dx + ey = f \end{cases}$$



ex2 (ex5) no solution (inconsistent system)

$$\begin{cases} 8x - 2y = 5 \\ -12x + 3y = 7 \end{cases}$$

$24x - 6y = 15$

$-24x + 6y = 14$

$0 = 29$ false $\forall (x,y)$

no (x,y) would make this true

rref $\begin{bmatrix} 8 & -2 & 5 \\ -12 & 3 & 7 \end{bmatrix} \rightarrow \begin{bmatrix} 1 & -\frac{1}{4} & 0 \\ 0 & 0 & 1 \end{bmatrix}$

$x - \frac{1}{4}y = 0$

$0 = 1$

ex3 (ex6) Infinitely Many solns (dependent system)

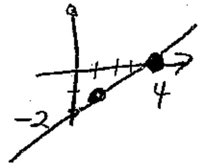
$$\begin{cases} 3x - 6y = 12 \\ 4x - 8y = 16 \end{cases}$$

$\xrightarrow{\times 4} 12x - 24y = 48$ $\quad x - 2y = 4$

$\xrightarrow{\times 3} 12x - 24y = 48$ $\quad x - 2y = 4$

Solutions: If $x = t$, then what's y? $y = \frac{3t-12}{6} = \frac{1}{2}t - 2$

$\boxed{(t, \frac{1}{2}t - 2)}$ eg $(1, -\frac{3}{2})$, $(4, 0)$



WORD PROBLEMS

- What are the variables?
- Eqns from Info
- Solve

ex4 (ex7) Row upstream from one point to another 4mi away in $1\frac{1}{2}$ hours

Return with current. 45min.

x Boat Speed relative to water? (mi/h)

y Current Speed?

$y \downarrow \quad x \uparrow \quad v = x - y$

$y \downarrow \quad x$

distance = speed × time

$$\begin{cases} 4 = (x-y)\frac{3}{2} \\ 4 = (x+y)\frac{3}{4} \end{cases}$$

$8 = 3x - 3y$

$16 = 3x + 3y$

$24 = 6x \Rightarrow x = \boxed{4}$ mph

$\rightarrow y = \frac{12-8}{3} = \boxed{\frac{4}{3}}$

$\boxed{ex5}$ WORD PROBLEM (mixture)

ex8

Liters	(x) wine	$\longrightarrow$	mixture
	0.1 alcohol	+0.7 soln (y)	0.16 alcohol
			1000L

$$\begin{cases} x+y = 1000 & \leftarrow \text{Total (Liters)} \\ 0.1x+0.7y = 0.16(1000) & \leftarrow \text{Amount Alcohol} \end{cases}$$

$$\begin{array}{r} x+y = 1000 \\ x+7y = 1600 \\ \hline 6y = 600 \end{array}$$

$$\boxed{y = 100L \Rightarrow x = 900L}$$

(40) $\begin{cases} ax+by=0 \\ x+y=1 \end{cases}$ $(a \neq b)$ $\rightarrow$ $\begin{array}{r} ax+by=0 \\ ax+ay=a \\ \hline (b-a)y=-a \end{array}$

$y = +\frac{a}{a-b}$

$x = \frac{-b}{a-b}$

9.3 / 10.2 Systems of Eqns in Several Variables

Linear	Nonlinear
$6x-3y+\sqrt{5}z=10$	$x^2+3y-\sqrt{z}=5$
	$xy+6x=-6$

How to Solve:

① Get an equivalent system using Gaussian elimination

- add eqns together
- multiply eqn by $c \neq 0$
- interchange eqns

② Triangular form $\begin{cases} x-2y-z=1 \\ y+2z=5 \\ z=3 \end{cases}$

$\boxed{ex1}$ $x=-2+3+1=2$

$y=-1$

③ Back-Substitution $\boxed{(2,-1,3)}$

Go to ex3

Replaced row must be involved in adding

$\boxed{ex2}$ Gaussian elimination always Like ①+② replace ② or ① (③+② replace ③ or ② not ①)

$$\begin{cases} x-2y+3z=1 \\ x+2y-z=13 \\ 3x+2y-5z=3 \end{cases} \longrightarrow \begin{cases} x-2y+3z=1 \\ 4y-4z=12 \\ 8y-14z=0 \end{cases} \longrightarrow \begin{cases} x-2y+3z=1 \\ 4y-4z=12 \\ 6z=24 \end{cases}$$

3 −6 9 3   8 −8 24

$$\left[\begin{array}{ccc|c} 1 & -2 & 3 & 1 \\ 1 & 2 & -1 & 13 \\ 3 & 2 & -5 & 3 \end{array}\right] \xrightarrow[R_3-3R_1]{R_2-R_1} \left[\begin{array}{ccc|c} 1 & -2 & 3 & 1 \\ 0 & 4 & -4 & 12 \\ 0 & 8 & -14 & 0 \end{array}\right]$$

$$\rightarrow \begin{cases} x-2y+3z=1 \\ y-z=3 \\ z=4 \end{cases}$$

$$\xrightarrow[2R_2-R_3]{\frac{1}{4}R_2} \left[\begin{array}{ccc|c} 1 & -2 & 3 & 1 \\ 0 & 1 & -1 & 3 \\ 0 & 0 & 6 & 24 \end{array}\right]$$

$\Rightarrow y=7$

$x=14-12+1=3$

$\boxed{(3,7,4)}$

will need show step in Quiz in ref & rref

row echelon form $\xrightarrow{-\frac{1}{6}R_3}$ $\left[\begin{array}{ccc|c} 1 & -2 & 3 & 1 \\ 0 & 1 & -1 & 3 \\ 0 & 0 & 1 & 4 \end{array}\right]$ $\leftarrow$ Show analogy

★ Check: Plug into all 3 eqns

$$\xrightarrow[R_2+R_3]{R_1-3R_3} \left[\begin{array}{ccc|c} 1 & -2 & 0 & -11 \\ 0 & 1 & 0 & 7 \\ 0 & 0 & 1 & 4 \end{array}\right]$$

reduced row echelon form $\xrightarrow{-R_1+2R_2}$ $\left[\begin{array}{ccc|c} 1 & 0 & 0 & 3 \\ 0 & 1 & 0 & 7 \\ 0 & 0 & 1 & 4 \end{array}\right] \rightarrow (3,7,4)$

Go to P6

$\boxed{\text{ex3}}$ Inconsistent System

$$\begin{cases} x+2y-2z=1 \\ 2x+2y-z=6 \\ 3x+4y-3z=5 \end{cases} \rightarrow \begin{cases} x+2y-2z=1 \\ 2y-3z=-4 \\ 2y-3z=-2 \end{cases} \rightarrow \begin{cases} x+2y-2z=1 \\ 2y-3z=-4 \\ \boxed{0=2} \end{cases}$$

always false

ref

$$\left[\begin{array}{ccc|c} 1 & 2 & -2 & 1 \\ 2 & 2 & -1 & 6 \\ 3 & 4 & -3 & 5 \end{array}\right] \rightarrow \left[\begin{array}{ccc|c} 1 & 2 & -2 & 1 \\ 0 & 2 & -3 & -4 \\ 0 & 2 & -3 & -2 \end{array}\right] \rightarrow \left[\begin{array}{ccc|c} 1 & 2 & -2 & 1 \\ 0 & 2 & -3 & -4 \\ 0 & 0 & 0 & 0 \end{array}\right]$$

Go to other page

$\boxed{\text{ex4}}$ Dependent System

$$\begin{cases} x-y+5z=-2 \\ 2x+y+4z=2 \\ 2x+4y-2z=8 \end{cases} \rightarrow \begin{cases} x-y+5z=-2 \\ 3y-6z=6 \\ 3y-6z=6 \end{cases} \rightarrow \begin{cases} x-y+5z=-2 \\ 3y-6z=6 \\ 0=0 \end{cases}$$

Infinitely many solns.

matrix Show result

$$\left[\begin{array}{ccc|c} 1 & 0 & 3 & 0 \\ 0 & 1 & -2 & 2 \\ 0 & 0 & 0 & 0 \end{array}\right]$$

If $\boxed{z=t}$,

$3y=6t+6$

$y=2t+2$

$x=2t+2-5t-2$

$=-3t$

3 ways

$b \quad t=\frac{b-2}{2}=\frac{b}{2}-1$

$\boxed{(-3t,\ 2t+2,\ t)}$

$t=-a/3$

$\boxed{\left(a,\ -\frac{2}{3}a+2,\ -\frac{a}{3}\right)} = (3\tilde{a},\ -2\tilde{a}+2,\ -\tilde{a})$

$\boxed{\left(-\frac{3b}{2}+3,\ b,\ \frac{b}{2}-1\right)} = (-3\tilde{b}+3,\ 2\tilde{b},\ \tilde{b}-1)$

ex. $t=2 \Rightarrow (-6,6,2)$

$\tilde{a}=-2 \quad 4+2$

$(-6,\ 4+2,\ 2) = (-6,\ 6,\ 2)$

$\tilde{b}=3$

$(-6,\ 6,\ 2) = (-6,\ 6,\ 2)$

Go to other p. GJ elim.

← show matrix 0, 1, ∞ forms

$\boxed{\text{P660}}$ Best vs. exact fit

CW Find $y=ax^2+bx+c$ Answer $y=x^2-2x+3$

(how many pts determine a,b,c?
3 eqns at least)

$(-1,6),\ (1,2),\ (2,3)$

$$\begin{cases} 1a-b+c=6 \\ a+b+c=2 \\ 4a+2b+c=3 \end{cases}$$

HW SHOW steps

$$\left[\begin{array}{ccc|c} 1 & -1 & 1 & 6 \\ 1 & 1 & 1 & 2 \\ 4 & 2 & 1 & 3 \end{array}\right] \xrightarrow[4R_1+R_3]{R_2-R_1} \left[\begin{array}{ccc|c} 1 & -1 & 1 & 6 \\ 0 & 2 & 0 & -4 \\ 0 & 2 & 3 & 5 \end{array}\right] \xrightarrow[R_3-R_2]{\frac{1}{2}R_2} \left[\begin{array}{ccc|c} 1 & -1 & 1 & 6 \\ 0 & 1 & 0 & -2 \\ 0 & 0 & 3 & 9 \end{array}\right] \xrightarrow{\frac{1}{3}R_3} \left[\begin{array}{ccc|c} 1 & -1 & 1 & 6 \\ 0 & 1 & 0 & -2 \\ 0 & 0 & 1 & 3 \end{array}\right] \xrightarrow{R_1-R_3} \left[\begin{array}{ccc|c} 1 & -1 & 0 & 3 \\ 0 & 1 & 0 & -2 \\ 0 & 0 & 1 & 3 \end{array}\right] \xrightarrow{R_1+R_2} \left[\begin{array}{ccc|c} 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & -2 \\ 0 & 0 & 1 & 3 \end{array}\right]$$

TRIANGULAR

Quiz ref

rref
QUADREG

10.3
9.4

System of Linear Eqns: Matrix

$$\text{matrix } A = \begin{array}{c} \text{column } 1 \quad\quad\quad\quad\quad\quad \text{column } n \\ \begin{bmatrix} a_{11} & a_{12} & a_{13} & \cdots & a_{1n} \\ a_{21} & a_{22} & a_{23} & \cdots & a_{2n} \\ \vdots & & & & \\ a_{mn} & a_{m2} & & \cdots & a_{mn} \end{bmatrix} \end{array} \begin{array}{l} \leftarrow \text{row } 1 \\ \\ \\ \leftarrow \text{row } n \end{array}$$

- Dimensions $m \times n$
- a_{ij}: i^{th} row, j^{th} colm

PROGRAM variables
MATLAB

$A = \begin{bmatrix} 1 & 3 & 0 \\ 2 & 4 & -1 \end{bmatrix}$ dimension 2×3

$a_{23} = \underline{-1}$

$B = \begin{bmatrix} 6 \\ -5 \\ 0 \\ 1 \end{bmatrix}$ dim 4×1

$-5 = \underline{b_{21}}$

$\boxed{\text{ex1}}$
$$\begin{cases} 6x - 2y - z = 4 \\ x + \quad 3z = 1 \\ \quad 7y + z = 5 \end{cases}$$

Augmented matrix
$$\left[\begin{array}{ccc|c} 1 & -2 & -1 & 4 \\ 1 & 0 & 3 & 1 \\ 0 & 7 & 1 & 5 \end{array}\right]$$

Matrix Eqn
$$A\vec{x} = \vec{b}$$
$$\underbrace{\begin{bmatrix} 6 & -2 & -1 \\ 1 & 0 & 3 \\ 0 & 7 & 1 \end{bmatrix}}_{\text{coeff matrix}} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 4 \\ 1 \\ 5 \end{bmatrix}$$

---

elementary row operations: system is equivalent

1. add multiple of one row to another
2. multiply row by nonzero constant
3. switch rows

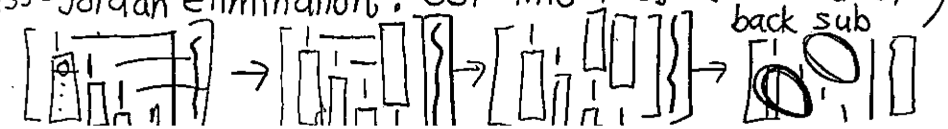
Gaussian Elimination: ✓

-use elementary row ops to get ref $\begin{bmatrix} 1 & 2 & 3 & 4 \\ 0 & 1 & 2 & 3 \\ 0 & 0 & 1 & 2 \end{bmatrix}$ (i.e. triangular, start w/ 1)

Leading variable corresponds to leading entry in ref

Back substitute

Gauss-Jordan Elimination: Get into rref (no need for back sub)



---

ref
$$\begin{bmatrix} 1 & 3 & -6 & 10 & 0 \\ 0 & 0 & 1 & 4 & -3 \\ 0 & 0 & 0 & 1 & 1/2 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

- Row Start with 1
- Below 1 all 0's

Not ref
$$\begin{bmatrix} 0 & 1 & -\frac{1}{2} & 0 & 7 \\ 1 & 0 & 3 & 4 & -5 \\ 0 & 0 & 0 & 1 & 0.4 \\ 0 & 1 & 1 & 0 & 0 \end{bmatrix} \begin{array}{c} (3) \\ (1) \\ (4) \\ (2) \end{array}$$
switch order

Go back to ex2

CW ✓ Show

$\boxed{\text{ex3} \sim 4}$
$$\begin{cases} 4x + 8y - 4z = 4 \\ 3x + 8y + 5z = -11 \\ -2x + y + 12z = -17 \end{cases}$$

$$\left[\begin{array}{ccc|c} 4 & 8 & -4 & 4 \\ 3 & 8 & 5 & -11 \\ -2 & 1 & 12 & -17 \end{array}\right] \xrightarrow{\frac{1}{4}R_1} \left[\begin{array}{ccc|c} 1 & 2 & -1 & 1 \\ 3 & 8 & 5 & -11 \\ -2 & 1 & 12 & -17 \end{array}\right] \xrightarrow[R_3+2R_1]{R_2-3R_1} \left[\begin{array}{ccc|c} 1 & 2 & -1 & 1 \\ 0 & 2 & 8 & -14 \\ 0 & 5 & 10 & -15 \end{array}\right]$$

$$\xrightarrow[\frac{1}{5}R_3]{\frac{1}{2}R_2} \left[\begin{array}{ccc|c} 1 & 2 & -1 & 1 \\ 0 & 1 & 4 & -7 \\ 0 & 1 & 2 & -3 \end{array}\right] \xrightarrow[R_2-R_3]{} \left[\begin{array}{ccc|c} 1 & 2 & -1 & 1 \\ 0 & 1 & 4 & -7 \\ 0 & 0 & 2 & -4 \end{array}\right] \xrightarrow[\frac{1}{2}R_3]{} \left[\begin{array}{ccc|c} 1 & 2 & -1 & 1 \\ 0 & 1 & 4 & -7 \\ 0 & 0 & 1 & -2 \end{array}\right]$$

$$\xrightarrow[R_2-R_3 \times 4]{R_1+R_3'} \left[\begin{array}{ccc|c} 1 & 2 & 0 & -1 \\ 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & -2 \end{array}\right] \xrightarrow{R_1-2R_2} \left[\begin{array}{ccc|c} 1 & 0 & 0 & -3 \\ 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & -2 \end{array}\right]$$

$$\boxed{\begin{aligned} x &= -3 \\ y &= 1 \\ z &= -2 \end{aligned}}$$

Check rref

no soln "inconsistent"
$$\left[\begin{array}{ccc|c} 1 & 0 & 0 & 2 \\ 0 & 1 & 0 & 3 \\ 0 & 0 & 0 & 1 \end{array}\right]$$
$0 \neq 1$

$$\left[\begin{array}{ccc|c} 1 & 0 & 0 & 2 \\ 0 & 1 & 0 & 3 \\ 0 & 0 & 1 & 0 \end{array}\right]$$
OK one soLN

(dependent)
∞ many soln
$$\left[\begin{array}{ccc|c} 1 & 2 & -3 & 1 \\ 0 & 1 & 5 & -2 \\ 0 & 0 & 0 & 0 \end{array}\right]$$
z anything $= t$
Then
$y + 5z = -2$
$y = -2 - 5t$
$x = 1 + 3t - 2(-2-5t)$
$= 5 + 13t$

Soln Set $\{(5+13t, -2-5t, t) : t \in \mathbb{R}\}$

$\left(a, -2-5\left(\frac{a-5}{13}\right), \frac{a-5}{13}\right)$

$t = \frac{a-5}{13}$

$\boxed{\text{ex5}}$
$$\begin{cases} x-3y+2z=12 \\ 2x-5y+5z=14 \\ x-2y+3z=20 \end{cases}$$

Gauss elim, to ref (before rref)

ref CALC $\Rightarrow \begin{bmatrix} 1 & -3 & 2 & 12 \\ 0 & 1 & 1 & -10 \\ 0 & 0 & 0 & 1 \end{bmatrix} \leftarrow$ NO SOLN $0 \neq 1$

$\boxed{\text{ex6}}$
$$\begin{cases} -3x-5y+36z=10 \\ -x \quad +7z=5 \\ x+y-10z=-4 \end{cases}$$
CALC ref $\Rightarrow \left[\begin{array}{ccc|c} 1 & \boxed{1} & -10 & -4 \\ 0 & 1 & -3 & 1 \\ 0 & 0 & 0 & 0 \end{array}\right] \leftarrow$ ∞ many SOLNS

Get into rref by hand $\xrightarrow[R_1-R_2]{\text{Get 0's above 1st 1 on Right}} \left[\begin{array}{ccc|c} 1 & 0 & -7 & -5 \\ 0 & 1 & -3 & 1 \\ 0 & 0 & 0 & 0 \end{array}\right]$

$x=-5+7t$
$y=1+3t$
$z=t$

$2<3$
$\Rightarrow 3-2=1$ (t)

Soln Set: $\{(-5+7t, 3t+1, t) \mid t \in \mathbb{R}\}$

eg $(-12,-2,-1)$, $(-5,1,0)$

COMMON SENSE

ref
$m < n$
ID eqns unknowns
$\Rightarrow n-m$ rows nonleading variables
are $[0\,0\,0\,0 \mid b]$
IF $[0\,0\,0\,0\,0]$
∞ many solns
& you'll have that many s, t's

show ✓
$\boxed{\text{ex7}}$ by hand
$$\begin{cases} x+2y-3z-4w=10 \\ x+3y-3z-4w=15 \\ 2x+2y-6z-8w=10 \end{cases}$$

ID
2 eqns $<$ 4 unknowns
$4-2=2$ nonleading variables $(s \,\&\, t)$

$\left[\begin{array}{cccc|c} 1 & 2 & -3 & -4 & 10 \\ 1 & 3 & -3 & -4 & 15 \\ 2 & 2 & -6 & -8 & 10 \end{array}\right] \xrightarrow[R_3+R_1(-2)]{R_2-R_1} \left[\begin{array}{cccc|c} 1 & 2 & -3 & -4 & 10 \\ \boxed{0} & 1 & 0 & 0 & 5 \\ \boxed{0} & 2 & 0 & 0 & 10 \end{array}\right] \xrightarrow{\frac{1}{2}R_3} \left[\begin{array}{cccc|c} 1 & 2 & -3 & -4 & 10 \\ 0 & 1 & 0 & 0 & 5 \\ 0 & \boxed{1} & 0 & 0 & 5 \end{array}\right]$

$2 \quad 4 \quad -6 \quad -8 \quad 20$

$\xrightarrow{R_3-R_2} \left[\begin{array}{cccc|c} 1 & 2 & -3 & -4 & 10 \\ \boxed{0} & 1 & 0 & 0 & 5 \\ \boxed{0} & \boxed{0} & 0 & 0 & 0 \end{array}\right] \xrightarrow{R_1-R_2 2} \begin{array}{c} \left[\begin{array}{cccc|c} 1 & \boxed{0} & -3 & -4 & 0 \\ \boxed{0} & 1 & 0 & 0 & 5 \\ \boxed{0} & \boxed{0} & 0 & 0 & 0 \end{array}\right] \\ x \quad y \quad z \quad w \end{array}$

$x=4t+3s$
$y=5$
$z=s$
$w=t$

in Quiz ref
① show steps
② show when ref
③ Get to rref

$$\boxed{\left\{(4t+3s, 5, s, t) \;\middle|\; \begin{array}{l} s \in \mathbb{R} \\ t \in \mathbb{R} \end{array}\right\}}$$

Check: $(11,5,1,2)$

After ex 3

- what happens if by elimination, we get... Triangular form

$$\begin{cases} x - y + 3z = 1 \\ \quad y + z = 2 \\ \quad\quad 0z = \cancel{-6} \ 1 \ \cancel{=0} \end{cases}$$

no soln.

$$\begin{cases} x - y + z = 1 \\ \quad y + z = 2 \\ \quad\quad 0 = 0 \end{cases}$$

∞ many

$\underline{z=0} \Rightarrow y=2 \Rightarrow x=3$

$\underline{z=-1} \Rightarrow y=3 \Rightarrow x=5$

$z=t \Rightarrow y=2-t \Rightarrow x=1-t+2-t$
$= 3-2t$

$(3-2t, 2-t, t)$

- See example 2... augmented matrix
Gaussian elimination. Triangular, row echelon form

$$\left[\begin{array}{ccc|c} 1 & -1 & 3 & 1 \\ 0 & 1 & 1 & 2 \\ 0 & 0 & \cancel{*} & 0 \end{array}\right]$$
$0 \quad -1$

$$\left[\begin{array}{ccc|c} 1 & -1 & 1 & 1 \\ 0 & 1 & 1 & 2 \\ 0 & 0 & 0 & 0 \end{array}\right]$$

Go to ex4

show matrix forms
0, 1, ∞ soln.

Gauss-Jordan elimination rref

- Can keep eliminating until:

$$\begin{cases} x + 0y + 0z = 1 \\ \quad y + 0z = 2 \\ \quad\quad z = 3 \end{cases}$$

$$\left[\begin{array}{ccc|c} 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & 2 \\ 0 & 0 & 1 & 3 \end{array}\right]$$

$$\begin{cases} x + 0y + 0z = 1 \\ \quad y = 2 \\ \quad\quad z = 0 \end{cases}$$

$$\left[\begin{array}{ccc|c} 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & 2 \\ 0 & 0 & \boxed{1} & \boxed{0} \end{array}\right]$$

OK!

$$\begin{cases} x + 0y + 0z = 1 \\ \quad y = 2 \\ \quad\quad 0z = 1 \end{cases}$$

no soln.

$$\left[\begin{array}{ccc|c} 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & 2 \\ 0 & 0 & \boxed{0} & \boxed{1} \end{array}\right]$$

NOT OK!

$$\begin{cases} x + 0y + 0z = 1 \\ \quad y = 2 \\ \quad\quad 0z = 0 \end{cases}$$

∞ many soln.

$(1, y, t)$

$$\left[\begin{array}{ccc|c} 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & 2 \\ 0 & 0 & \boxed{0} & \boxed{0} \end{array}\right]$$

Go to P. 7

So how rref?

~~9.5~~ 10.4 Matrix Algebra

m rows $\times$ n columns

PROGRAMMING, SIGNAL PROCESSING, Relativity

- Matrix $A = B$ means $a_{ij} = b_{ij}$ $\forall i = 1, 2, \dots m$, $j = 1, 2, \dots n$
- Element-wise

$A \pm B = [a_{ij} \pm b_{ij}]$ ← commutative? ✓ associative? ✓

distributive? ✓

- Scalar Product

$cA = [ca_{ij}]$ ← associative? ✓ comm? ✓

ex2

$$A = \begin{bmatrix} 2 & -3 \\ 0 & 5 \\ 7 & -\frac{1}{2} \end{bmatrix} \quad B = \begin{bmatrix} 1 & 0 \\ -3 & 1 \\ 2 & 2 \end{bmatrix} \quad C = \begin{bmatrix} 7 & -3 & 0 \\ 0 & 1 & 5 \end{bmatrix}$$

a) $A + B = \begin{bmatrix} 3 & -3 \\ -3 & 6 \\ 9 & 1.5 \end{bmatrix}$

b) $B + C$ cannot

c) $5A = \begin{bmatrix} 10 & -15 \\ 0 & 25 \\ 7 & -5/2 \end{bmatrix}$

---

ex3 Solve Matrix Eqn

$$2\underline{X} - A = B$$

$$2\underline{X} - \begin{bmatrix} 2 & 3 \\ -5 & 1 \end{bmatrix} = \begin{bmatrix} 4 & -1 \\ 1 & 3 \end{bmatrix}$$

$$\underline{X} = \tfrac{1}{2}(B + A) = \frac{1}{2}\begin{bmatrix} 6 & 2 \\ -4 & 4 \end{bmatrix} = \boxed{\begin{bmatrix} 3 & 1 \\ -2 & 2 \end{bmatrix}}$$

★ MULTIPLY Matrices

③ $\underset{m \times k}{C} = \underset{(m \times n)\,(n \times k)}{AB} = \begin{bmatrix} a_{11} & a_{12} & \cdots & a_{1n} \\ a_{21} & a_{22} & \cdots & a_{2n} \\ a_{i1} & a_{i2} & a_{ij} & a_{in} \\ a_{m1} & a_{m2} & \cdots & a_{mn} \end{bmatrix} \begin{bmatrix} b_{11} & \cdots & b_{1j} & b_{1k} \\ b_{21} & \cdots & \vdots & b_{2k} \\ \vdots & & & \\ b_{n1} & \cdots & b_{nj} & b_{nk} \end{bmatrix}$

i^{th} Row, j^{th} column

$$[c_{ij}] = [A_i \times B_j]$$

① defined in terms of row $\times$ column "Inner produce" DOT PRODUCT

$$[a_1 \ a_2 \dots a_n] \begin{bmatrix} b_1 \\ b_2 \\ \vdots \\ b_n \end{bmatrix} = a_1 b_1 + a_2 b_2 + \dots + a_n b_n$$

i.e. $\vec{a} = \begin{bmatrix} a_1 \\ \vdots \\ a_n \end{bmatrix}$, $\vec{b} = \begin{bmatrix} b_1 \\ \vdots \\ b_n \end{bmatrix}$ $\quad \vec{a} \cdot \vec{b} = \underbrace{\vec{a}^T}_{\text{TRANSPOSE}} \vec{b}$

$$A = \begin{bmatrix} 1 & 2 & 3 & 4 \\ 5 & 6 & 7 & 8 \\ 9 & 10 & 11 & 12 \end{bmatrix} \quad A^T = \begin{bmatrix} 1 & 5 & 9 \\ 2 & 6 & 10 \\ 3 & 7 & 11 \\ 4 & 8 & 12 \end{bmatrix}$$

i^{th} Row → i^{th} column

MULTIPLY

ex4 ①

$$\begin{bmatrix} 1 & 3 & 1 \\ -1 & 0 & 2 \end{bmatrix} \begin{bmatrix} 1 & 2 & 1 \\ 1 & 3 & 0 \\ 2 & 4 & 1 \end{bmatrix} = \begin{bmatrix} 6 & 15 & 2 \\ 3 & 6 & 1 \end{bmatrix}$$

$1+3+2$

$2 \times \underline{3}$ $\quad$ $\underline{3} \times 3$ $\quad$ 2×3

ex4 AB

$$\begin{bmatrix} 1 & 3 \\ -1 & 0 \end{bmatrix} \begin{bmatrix} -1 & 5 & 2 \\ 0 & 4 & 7 \end{bmatrix} = \begin{bmatrix} -1 & 17 & 23 \\ 1 & -5 & -2 \end{bmatrix}$$

BA ? cannot

$$\begin{bmatrix} -1 & 5 & 2 \\ 0 & 4 & 7 \end{bmatrix} \begin{bmatrix} 1 & 3 \\ -1 & 0 \end{bmatrix}$$

NOT COMMUTATIVE!

PROPERTIES

$A(BC) = (AB)C$

$A(B+C) = AB + AC$

$(B+C)A = BA + CA$

ex5 $A = \begin{bmatrix} 5 & 7 \\ -3 & 0 \end{bmatrix}$ $B = \begin{bmatrix} 1 & 2 \\ 9 & -1 \end{bmatrix}$

$AB = \begin{bmatrix} 68 & 3 \\ -3 & -6 \end{bmatrix}$

① $A_1 B_1 = C_{11}$

② $C_{22} = A_2 B_2$

$C = BA$ $C_{21} = B_2 A_1 = [9 \ -1]\begin{bmatrix} 5 \\ -3 \end{bmatrix} = 45+3 = \boxed{48}$

ex6 Linear System as Matrix

$\begin{bmatrix} 1 & -1 & 3 \\ 1 & 2 & -2 \\ 3 & -1 & 5 \end{bmatrix}\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 4 \\ 10 \\ 14 \end{bmatrix}$ multiply

$\begin{bmatrix} x - y + 3z \\ x + 2y - 2z \\ 3x - y + 5z \end{bmatrix} = \begin{bmatrix} 4 \\ 10 \\ 14 \end{bmatrix}$

ex7 matrix word problems

$$\begin{array}{c|ccc} & \text{① } 18\sim30 & \text{Age ② } 31\sim50 & \text{③ } >50 \\ \hline D & 0.3 & 0.6 & 0.5 \\ R & 0.5 & 0.35 & 0.25 \\ I & 0.2 & 0.05 & 0.25 \end{array} = A$$

$$\begin{array}{c|cc} & M & F \\ \hline 18\sim30 & 5000 & 6000 \\ 31\sim50 & 10000 & 12000 \\ >50 & 12000 & 15000 \end{array} = B$$

b) How many males are democrats?

30% ① + 60% ② + 50% ③

inner/DOT PRODUCT of vectors = scalar

$[0.3 \ \ 0.6 \ \ 0.5]\begin{bmatrix} 5000 \\ 1000 \\ 12000 \end{bmatrix} = \boxed{13500}$

c) How many female Republicans?

$[0.5 \ \ 0.35 \ \ 0.25]\begin{bmatrix} 6000 \\ 12000 \\ 15000 \end{bmatrix} = \boxed{10950}$

a) what does $AB = C$ represent?

C 3×2

C_{IF} = # of female independents

C_{RM} = # of male Repub.

C_{12} = # Female Dem

C_{31} I, M

$C_{32} = I, F$

$C_{23} =$? dne

* show CALC how-to

9.6 / 10.5 Inverse Matrix

can multiply?

$\begin{bmatrix} 1 & 1 & 2 \\ 3 & 1 & 0 \end{bmatrix}\begin{bmatrix} 1 \\ 2 \\ 1 \end{bmatrix}$ ✓

$\begin{bmatrix} \cdot & \cdot \\ \cdot & \cdot \\ \cdot & \cdot \end{bmatrix}\begin{bmatrix} \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot \end{bmatrix}$ ×

$\begin{bmatrix} \cdot \\ \cdot \\ \cdot \end{bmatrix}[\cdot]$ ✓

$\begin{bmatrix} \cdot \\ \cdot \\ \cdot \end{bmatrix}[\cdot\ \cdot\ \cdot] = \begin{bmatrix} \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot \end{bmatrix}$ ✓

$[\cdot\ \cdot\ \cdot]\begin{bmatrix} \cdot \\ \cdot \\ \cdot \end{bmatrix} = [\cdot]$ ✓

$\boxed{\text{Identity}}$

$1 \cdot a = a \cdot 1 = a$

$\boxed{\text{ID matrix}}$ $I_n = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$ $n \times n$ square

$A \cdot I_n = A$ $\quad I_n \cdot B = B$

when product is defined

$\boxed{\text{ex1}}$ Keeps row

$\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}\begin{bmatrix} 3 & 5 & 6 \\ -1 & 2 & 7 \end{bmatrix} = \begin{bmatrix} 3 & 5 & 6 \\ -1 & 2 & 7 \end{bmatrix}$

$\begin{bmatrix} 3 & 5 & 6 \\ -1 & 2 & 7 \end{bmatrix}\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 3 & 5 & 6 \\ -1 & 2 & 7 \end{bmatrix}$

B is the $\boxed{\text{inverse}}$ of A $\quad$ ★SQUARE matrix

(A^{-1}) $\quad n \times n$

$\boxed{BA = AB = I_n}$

not reciprocal

($\frac{1}{2}$ is the inverse of 2 $\quad \frac{1}{2} \times 2 = 2 \times \frac{1}{2} = 1$)

Inverse of a 2×2 Matrix

$A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$ $\quad \boxed{A^{-1} = \dfrac{\begin{bmatrix} d & -b \\ -c & a \end{bmatrix}}{|A|}}$ $\quad$ exists iff $ad - bc \neq 0$

(HW BONUS)

Proof: elementary row ops to get ID on left

$\left[\begin{array}{cc|cc} a & b & 1 & 0 \\ c & d & 0 & 1 \end{array}\right] \xrightarrow{aR_2 - cR_1} \left[\begin{array}{cc|cc} a & b & 1 & 0 \\ 0 & \underbrace{ad-bc}_{D} & -c & a \end{array}\right] \xrightarrow{\frac{R_2}{D}} \left[\begin{array}{cc|cc} a & b & 1 & 0 \\ 0 & 1 & \frac{-c}{D} & \frac{a}{D} \end{array}\right]$

$\xrightarrow{R_1 - bR_2} \left[\begin{array}{cc|cc} a & 0 & 1+\frac{bc}{D} & 0-\frac{ba}{D} \\ 0 & 1 & \frac{-c}{D} & \frac{a}{D} \end{array}\right] \xrightarrow{\frac{R_1}{a}} \left[\begin{array}{cc|cc} 1 & 0 & \frac{1}{D}\left(\frac{\cancel{a}d \cancel{-bc} \cancel{+bc}}{\cancel{a}}\right) & \frac{1}{D}(-b) \\ 0 & 1 & \frac{1}{D}(-c) & \frac{1}{D}(a) \end{array}\right]$

Why does this work?!

Proof omitted

See Linear Algebra

Linear transformation,

$\boxed{\text{ex3}}$ $A = \begin{bmatrix} 4 & 5 \\ 2 & 3 \end{bmatrix}$

$A^{-1} = \dfrac{\begin{bmatrix} 3 & -5 \\ -2 & 4 \end{bmatrix}}{12 - 10} = \begin{bmatrix} 3/2 & -5/2 \\ -1 & 2 \end{bmatrix}$

Check: $AA^{-1} = \begin{bmatrix} 6-5 & -10+10 \\ 3-3 & -5+6 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = I_{2\times2}$

$A^{-1}A = I_{2\times2}$

$\boxed{\text{General method}}$

$\boxed{\text{ex4}}$ $A = \begin{bmatrix} 1 & -2 & -4 \\ 2 & -3 & -6 \\ -3 & 6 & 15 \end{bmatrix}$ $\quad$ Like Gauss-Jordan rref

$\left[\begin{array}{ccc|ccc} 1 & -2 & -4 & 1 & 0 & 0 \\ 2 & -3 & -6 & 0 & 1 & 0 \\ -3 & 6 & 15 & 0 & 0 & 1 \end{array}\right] \xrightarrow[R_3+3R_1]{R_2-2R_1} \left[\begin{array}{ccc|ccc} 1 & -2 & -4 & 1 & 0 & 0 \\ 0 & 1 & 2 & -2 & 1 & 0 \\ 0 & 0 & 3 & 3 & 0 & 1 \end{array}\right] \xrightarrow{\frac{1}{3}R_3} \left[\begin{array}{ccc|ccc} 1 & -2 & -4 & -3 & 2 & 0 \\ 0 & 1 & 2 & -2 & 1 & 0 \\ 0 & 0 & 1 & 1 & 0 & \frac{1}{3} \end{array}\right]$

$\xrightarrow[R_2-2R_3]{R_1+2R_2} \left[\begin{array}{ccc|ccc} 1 & 0 & 0 & -3 & 2 & 0 \\ 0 & 1 & 0 & -4 & 1 & -2/3 \\ 0 & 0 & 1 & 1 & 0 & \frac{1}{3} \end{array}\right]$

A^{-1}

check $\quad A^{-1}A = \begin{bmatrix} -3 & 2 & 0 \\ -4 & 1 & -2/3 \\ 1 & 0 & 1/3 \end{bmatrix}\begin{bmatrix} 1 & -2 & -4 \\ 2 & -3 & -6 \\ -3 & 6 & 15 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

AA^{-1}

$\boxed{\text{CALC}}$ $\boxed{x^{-1}}$ enter

$\boxed{\text{ex5}}$ $A = \begin{bmatrix} 2 & -3 & -7 \\ 1 & 2 & 7 \\ 1 & 1 & 4 \end{bmatrix} \rightarrow \left[\begin{array}{ccc|ccc} 1 & 0 & 1 & 3/7 & 3/7 & 0 \\ 0 & 1 & 3 & -1/7 & 2/7 & 0 \\ 0 & 0 & 0 & -1/7 & -5/7 & 1 \end{array}\right]$

ROW OF ZEROS cannot form ID on left $\Rightarrow$ $\boxed{\text{not invertible}}$

matrix equations

$$\begin{cases} x-2y-4z=7 \\ 2x-3y-6z=5 \\ -3x+6y+15z=0 \end{cases}$$

① $\begin{bmatrix} 1 & -2 & -4 \\ 2 & -3 & -6 \\ -3 & 6 & 15 \end{bmatrix}\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 7 \\ 5 \\ 0 \end{bmatrix}$

$A \quad \vec{x} = \vec{b}$

$\underbrace{A^{-1} A} \quad \vec{x} = A^{-1}\vec{b}$

$I_{3\times3} \quad \vec{x} = A^{-1}\vec{b}$

② $\boxed{\vec{x} = A^{-1}\vec{b}}$ ← only works if SQUARE invertible

③ from ex4 $A^{-1} = \begin{bmatrix} -3 & 2 & 0 \\ -4 & 1 & -2/3 \\ 1 & 0 & 1/3 \end{bmatrix}$

④ $\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} -3 & 2 & 0 \\ -4 & 1 & -2/3 \\ 1 & 0 & 1/3 \end{bmatrix}\begin{bmatrix} 7 \\ 5 \\ 0 \end{bmatrix} = \boxed{\begin{bmatrix} -11 \\ -23 \\ 7 \end{bmatrix}}$

ex6 $\begin{cases} 2x-5y=15 \\ 3x-6y=36 \end{cases}$ Solve.

$\underbrace{\begin{bmatrix} 2 & -5 \\ 3 & -6 \end{bmatrix}}_{A}\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 15 \\ 36 \end{bmatrix}$

$\begin{bmatrix} x \\ y \end{bmatrix} = A^{-1}\begin{bmatrix} 15 \\ 36 \end{bmatrix} = \begin{bmatrix} -2 & \frac{5}{3} \\ -1 & 2/3 \end{bmatrix}\begin{bmatrix} 15 \\ 36 \end{bmatrix} = \begin{bmatrix} -30+60 \\ -15+24 \end{bmatrix}$

$A^{-1} = \dfrac{\begin{bmatrix} -6 & 5 \\ -3 & 2 \end{bmatrix}}{-12+15} = \begin{bmatrix} -2 & 5/3 \\ -1 & 2/3 \end{bmatrix} \qquad = \boxed{\begin{bmatrix} 30 \\ 9 \end{bmatrix}}$

ex7

	hamster	Daily	gerbil
protein	340mg		480
fat	280		360
carb	440		680

Pet food (1g)

KD	Pellet	Chow
10	0	20
10	20	10
5	10	30

How many grams of each food to feed hamster? gerbil?

Soln: Hamster: how much protein   CALC $\boxed{A^{-1}\vec{b}}$

$\begin{bmatrix} 10 & 0 & 20 \\ 10 & 20 & 10 \\ 5 & 10 & 30 \end{bmatrix}\begin{bmatrix} K \\ P \\ C \end{bmatrix} = \begin{bmatrix} 340 \\ 280 \\ 440 \end{bmatrix} \Rightarrow \begin{bmatrix} K \\ P \\ C \end{bmatrix} = \begin{bmatrix} 10 \\ 3 \\ 12 \end{bmatrix}$

Gerbil $\begin{bmatrix} & A & \end{bmatrix}\begin{bmatrix} K \\ P \\ C \end{bmatrix} = \begin{bmatrix} 480 \\ 360 \\ 680 \end{bmatrix} \Rightarrow \begin{bmatrix} 8 \\ 4 \\ 20 \end{bmatrix}$

~~10.6~~ 9.7 Determinant (scalar) (only for SQUARE matrix)

$A = \begin{bmatrix} a & b \\ c & d \end{bmatrix} \quad \det A = |A| \triangleq \underbrace{ad-bc}_{\text{scalar}}$

$\begin{vmatrix} a & b \\ c & d \end{vmatrix}$

ex1 $A = \begin{bmatrix} 6 & -3 \\ 2 & 3 \end{bmatrix} \quad |A| = 18+6 = 24$

det of $n\times n$ matrix

expand along any convenient row or column

A: $\begin{vmatrix} 2 & 3 & -1 \\ 0 & 2 & 4 \\ -2 & 5 & 6 \end{vmatrix}$

minor M_{21}, M_{22}, M_{23}

$= -0\begin{vmatrix} 2 & 4 \\ 5 & 6 \end{vmatrix} + 2\begin{vmatrix} 2 & -1 \\ -2 & 6 \end{vmatrix} - (4)\begin{vmatrix} 2 & 3 \\ -2 & 5 \end{vmatrix}$

cofactor a_{21}

$= 0 + 2(12-2) - 4(10+6) =$

$= 20 - 64 = -44$

Checkerboard $\begin{vmatrix} + & - & + \\ - & + & - \\ + & - & + \end{vmatrix}$

$= +2\begin{vmatrix} 2 & 4 \\ 5 & 6 \end{vmatrix} - 0\begin{vmatrix} 3 & -1 \\ 5 & 6 \end{vmatrix} + (-2)\begin{vmatrix} 3 & -1 \\ 2 & 4 \end{vmatrix}$

$= 2(12-20) - 0 - 2(12+2)$

$= -16 - 28 = -44$

matrix A $\begin{cases} \text{Minor of } a_{32} & M_{ij} = \text{det from deleting } i^{th} \text{ row } j^{th} \text{ colm} \\ & M_{32} = \begin{vmatrix} 2 & -1 \\ 0 & 6 \end{vmatrix} \\ \text{cofactor of } a_{32} & A_{ij} = \text{minor with checkerboard sign } (-1)^{i+j} M_{ij} \quad (\text{even+odd} = -) \\ & A_{32} = -\begin{vmatrix} 2 & -1 \\ 0 & 6 \end{vmatrix} \end{cases}$

Determinant of matrix A $(n\times n)$

$$\det A = \begin{vmatrix} a_{11} & \cdots & a_{1n} \\ a_{i1} & \cdots & a_{in} \\ a_{n1} & \cdots & a_{nn} \end{vmatrix} = \sum_{j=1}^{n} a_{ij} A_{ij} = \sum_{j=1}^{n} a_{ij} (-1)^{i+j} M_{ij}$$

i^{th} row expansion

cofactor — checker board — minor

Bonus:

$$\det A = \sum_{i=1}^{n} a_{ij} (-1)^{i+j} M_{ij}$$

j^{th} colm expansion

ex $A = \begin{bmatrix} 1 & 2 & 0 & 4 \\ 0 & 0 & 0 & 3 \\ 5 & 6 & 2 & 6 \\ 2 & 4 & 0 & 9 \end{bmatrix}$

note [ ] matrix

| | determinant

which expansion? by 2nd row

$$|A| = 3\begin{vmatrix} 1 & 2 & 0 \\ 5 & 6 & 2 \\ 2 & 4 & 0 \end{vmatrix} = 3\left(-2\begin{vmatrix} 1 & 2 \\ 2 & 4 \end{vmatrix}\right) = 3(-2)(4-4) = \boxed{0}$$

OR by 3rd colm

$$|A| = 2\begin{vmatrix} 1 & 2 & 4 \\ 0 & 0 & 3 \\ 2 & 4 & 9 \end{vmatrix} = 2(-3)\begin{vmatrix} 1 & 2 \\ 2 & 4 \end{vmatrix} = -6(4-4) = \boxed{0}$$

Thm A is invertible $\iff \det A \neq 0$

invertible? NO! Shortcut in QUIZ

Diagonal matrix

$$D = \begin{bmatrix} \lambda_1 & 0 & 0 \\ 0 & \lambda_2 & 0 \\ 0 & 0 & \lambda_3 \end{bmatrix} \qquad \boxed{|D| = \lambda_1\lambda_2\lambda_3}$$

OR TRIANGULAR

cw

(37)

$$\begin{array}{c|ccc} & \text{hamster} & \text{gerbil} & \text{dormouse} \\ \hline \end{array}$$

$$\underbrace{\begin{bmatrix} 3 & -2 \\ -4 & 3 \end{bmatrix}}_{A} \begin{bmatrix} x & y & z \\ u & v & w \end{bmatrix} = \begin{bmatrix} 1 & 0 & -1 \\ 2 & 1 & 3 \end{bmatrix}$$

$$\begin{bmatrix} x & y & z \\ u & v & w \end{bmatrix} = A^{-1}\begin{bmatrix} 1 & 0 & -1 \\ 2 & 1 & 3 \end{bmatrix}$$

(43) $A = \begin{bmatrix} 1 & e^x & 0 \\ e^x & -e^{2x} & 0 \\ 0 & 0 & 2 \end{bmatrix}$ DIAGONALIZE FIRST THEN $\div \to I$

$$\left[\begin{array}{ccc|ccc} 1 & e^x & 0 & 1 & 0 & 0 \\ e^x & -e^{2x} & 0 & 0 & 1 & 0 \\ 0 & 0 & 2 & 0 & 0 & 1 \end{array}\right] \xrightarrow{e^{-x}R_2} \left[\begin{array}{ccc|ccc} 1 & e^x & 0 & 1 & 0 & 0 \\ 1 & -e^x & 0 & 0 & e^{-x} & 0 \\ 0 & 0 & 2 & 0 & 0 & 1 \end{array}\right]$$

$$\xrightarrow{R_1-R_2} \left[\begin{array}{ccc|ccc} 1 & e^x & 0 & 1 & 0 & 0 \\ 0 & 2e^x & 0 & 1 & -e^{-x} & 0 \\ 0 & 0 & 2 & 0 & 0 & 1 \end{array}\right] \xrightarrow{2R_1-R_2} \left[\begin{array}{ccc|ccc} 2 & 0 & 0 & 1 & e^{-x} & 0 \\ 0 & 2e^x & 0 & 1 & -e^{-x} & 0 \\ 0 & 0 & 2 & 0 & 0 & 1 \end{array}\right]$$

$$\xrightarrow{e^{-x}R_2} \left[\begin{array}{ccc|ccc} 2 & 0 & 0 & 1 & e^x & 0 \\ 0 & 2 & 0 & 1 & -e^{-2x} & 0 \\ 0 & 0 & 2 & 0 & 0 & 1 \end{array}\right]$$

invertible $\forall x$

$$A^{-1} = \frac{1}{2}\begin{bmatrix} 1 & e^{-x} & 0 \\ 1 & -e^{-2x} & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$2\begin{vmatrix} 1 & 2 & 4 \\ 0 & 0 & 3 \\ 2 & 4 & 9 \end{vmatrix} = 2\left(-3\begin{vmatrix} 1 & 2 \\ 2 & 4 \end{vmatrix}\right) = -6(4-4) = 0$$

A SQUARE ONLY

★Determinant facts:

- A^{-1} exists $\iff |A| \neq 0$
  $\iff A\vec{x} = \vec{b}$ has only one solution for $\vec{x}$

$|A| = \lambda_1\lambda_2\cdots\lambda_n$ Triangular

- (Other) ways to solve a system:
  1. rref Gauss-Jordan
  2. $A\vec{x} = \vec{b}$, $\vec{x} = A^{-1}\vec{b}$ } $|A| \neq 0$ one soln.
  3. Cramer's Rule

Row & Column → Same determinant!

R_1+2R_2 $\quad C_1+3C_2$ $\quad$ (adding mult of a row to another row) gives same determinant

why??

*But multiply 1 row×2 → $2|A|$. * $\underset{n\times n}{2A} \to 2^n|A|$

ex5 $A = \begin{bmatrix} 8 & 2 & -1 & -4 \\ 3 & 5 & -3 & 11 \\ 24 & 6 & 1 & -12 \\ 2 & 2 & 7 & -1 \end{bmatrix}$ · Use a determinant to see if A has an inverse. Hint: Get more 0's first

Soln:

$$R_3-3R_1 \to \quad \det A = \begin{vmatrix} 8 & 2 & -1 & -4 \\ 3 & 5 & -3 & 11 \\ 0 & 0 & 4 & -0 \\ 2 & 2 & 7 & -1 \end{vmatrix}$$

$$\underbrace{\begin{vmatrix} 1 & 2 \\ 3 & 2 \end{vmatrix}}_{-4} \xrightarrow{2R_1-R_2} \underbrace{\begin{vmatrix} 1 & 2 \\ -1 & 2 \end{vmatrix}}_{4}$$

$$+0-0+4\underbrace{\begin{vmatrix} 8 & 2 & -4 \\ 3 & 5 & 11 \\ 2 & 2 & -1 \end{vmatrix}}_{|B|}-0 = 4(-150) = \boxed{-600}$$

$$C_1+2C_3 \qquad \det B = \begin{vmatrix} 0 & 2 & -4 \\ 25 & 5 & 11 \\ 0 & 2 & -1 \end{vmatrix} = +0-25\begin{vmatrix} 2 & -4 \\ 2 & -1 \end{vmatrix}+0$$

$$= -25(-2+8) = -150$$

Cramer's Rule

$$\begin{cases} ax+by=r \\ cx+dy=s \end{cases} \to \overbrace{\begin{bmatrix} a & b \\ c & d \end{bmatrix}}^{D}\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} r \\ s \end{bmatrix} \qquad \det D = |D|$$

condition: A is (SQUARE) INVERTIBLE ⟷ $\det A \neq 0$

$$x = \frac{\begin{vmatrix} r & b \\ s & d \end{vmatrix}}{|D|} = \frac{|D_x|}{|D|} \qquad y = \frac{\begin{vmatrix} a & r \\ c & s \end{vmatrix}}{|D|} = \frac{|D_y|}{|D|}$$

GENERAL n eqns in n unknowns $x_1, \dots x_n$ $\quad D\underline{X} = B$

$$x_1 = \frac{|D_{x_1}|}{|D|},\quad x_2 = \frac{|D_{x_2}|}{|D|}, \dots$$

ex6 $\begin{cases} 2x+6y=-1 \\ x+8y=2 \end{cases}$ $\quad \begin{bmatrix} 2 & 6 \\ 1 & 8 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} -1 \\ 2 \end{bmatrix}$

$$|D| = \begin{vmatrix} 2 & 6 \\ 1 & 8 \end{vmatrix} = 16-6 = 10$$

$$|D_x| = \begin{vmatrix} -1 & 6 \\ 2 & 8 \end{vmatrix} = -8-12 = -20$$

$$|D_y| = \begin{vmatrix} 2 & -1 \\ 1 & 2 \end{vmatrix} = 4+1 = 5$$

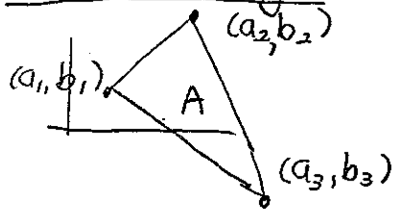
$$x = \frac{-20}{10} = -2 \qquad y = \frac{5}{10} = \frac{1}{2} \quad \Rightarrow \boxed{\left(-2, \tfrac{1}{2}\right)}$$

ex7 $\begin{cases} 2x-3y+4z=1 \\ x+6z=0 \\ 3x-2y=5 \end{cases}$ $\quad \left(\frac{39}{19}, \frac{11}{19}, -\frac{13}{38}\right)$

$$|D| = \begin{vmatrix} 2 & -3 & 4 \\ 1 & 0 & 6 \\ 3 & -2 & 0 \end{vmatrix} = -38 \qquad |D_x| = \begin{vmatrix} 1 & -3 & 4 \\ 0 & 0 & 6 \\ 5 & -2 & 0 \end{vmatrix} = -78$$

$$|D_y| = \begin{vmatrix} 2 & 1 & 4 \\ 1 & 0 & 6 \\ 3 & 5 & 0 \end{vmatrix} = -22 \qquad |D_z| = \begin{vmatrix} 2 & -3 & 1 \\ 1 & 0 & 0 \\ 3 & -2 & 5 \end{vmatrix} = 13$$

Area of Triangle



$$A = \pm\frac{1}{2}\begin{vmatrix} a_1 & b_1 & 1 \\ a_2 & b_2 & 1 \\ a_3 & b_3 & 1 \end{vmatrix}$$

9.7

HW #59 Answers

(10.6 63)

9.8 / 10.7 Partial Fraction Expansion

degree num < deg den

PROPER

$$r(x) = \frac{P(x)}{Q(x)} = \frac{P(x)}{\boxed{(x-r_1)}\boxed{(x-r_2)^3}(ax^2+bx+c)(cx^2+dx+e)^4} = \boxed{\frac{A}{x-r_1}} + \boxed{\frac{B}{x-r_2} + \frac{C}{(x-r_2)^2} + \frac{D}{(x-r_2)^3}} + \frac{Ex+F}{ax^2+bx+c}$$

$$+ \frac{Gx+H}{cx^2+dx+e} + \frac{Ix+J}{(cx^2+dx+e)^2} + \frac{Kx+L}{(cx^2+dx+e)^3} + \frac{Mx+N}{(cx^2+dx+e)^4}$$

Rational function

$$Q(x) = \overset{(x+(i+3))}{(x-r_1)}\overset{(x-(i+3))}{(x-r_2)}\overset{(x-3)}{(x-r_3)}\cdots(x-r_n) = (x^2-10)(x-3)$$

- polynomial degree n
- can always factor into n roots (complex #'s)
- Linear (real) factors & irreducible QUADRATIC.

Why? imaginary in conjugate pairs. Multiply is Quadratic

$\boxed{\text{ex1}}$ DISTINCT LINEAR FACTORS (is in BC CALCULUS)

Partial fraction Decomposition

$$\frac{5x+7}{x^3+2x^2-x-2} = \frac{5x+7}{(x-1)(x+1)(x+2)} = \frac{\overset{2}{A}}{x-1} + \frac{\overset{-1}{B}}{x+1} + \frac{\overset{-1}{C}}{x+2}$$

- Guess a factor: -2
- $Q(x) = (x+2)(x^2+0x-1)$

OR factor $x^2(x+2)-1(x+2) = (x+2)(x^2-1)$
$= (x+2)(x-1)(x+1)$

(shortcut $(x-1)(2x^2+3x+1) = 2x^3+x^2-2x-1$)

$$(5x+7) = A(x+1)(x+2) + B(x-1)(x+2) + C(x-1)(x+1)$$

Not in book

SHORTCUT:

$x=1 \quad 12 = A(6)+0+0 \Rightarrow A=2$

$x=-2 \quad -3 = 0+0+C(+3) \Rightarrow C=-1$

$x=-1 \quad 2 = 0+B(-2)+0 \Rightarrow B=-1$

Slow way see Pg 717
new: 694

ex2

$$\frac{x^2+1}{x(x-1)^3} = \frac{\overset{-1}{A}}{x} + \frac{\overset{1}{B}}{x-1} + \frac{\overset{0}{C}}{(x-1)^2} + \frac{\overset{2}{D}}{(x-1)^3}$$

$$x^2+1 = A(x-1)^3 + Bx(x-1)^2 + Cx(x-1) + Dx$$

(No shortcut?)

$$= A(x^3-3x^2+3x-1) + B(x^3-2x^2+x) + C(x^2-x) + Dx$$

$$= (A+B)x^3 + (-3A-2B+C)x^2 + (3A+B-C+D)x - A$$

equate coeffs

$$\begin{cases} A+B = 0 \\ -3A-2B+C = 1 \\ 3A+B-C+D = 0 \\ -A = 1 \end{cases} \qquad \left[\begin{array}{cccc|c} 1 & 1 & 0 & 0 & 0 \\ -3 & -2 & 1 & 0 & 1 \\ 3 & 1 & -1 & 1 & 0 \\ -1 & 0 & 0 & 0 & 1 \end{array}\right] \text{ rref}$$

OR BACK SUB

ex3

$$\frac{2x^2-x+4}{\underbrace{x^3+4x}_{x(x^2+4)}} = \frac{\overset{4}{A}}{x} + \frac{\overset{1}{B}x+\overset{-1}{C}}{x^2+4}$$

$$2x^2-x+4 = A(x^2+4) + (Bx+C)x$$

$$= (A+B)x^2 + Cx + (4A)$$

$$\begin{cases} A+B = 2 \quad \rightarrow B = 1 \\ C = -1 \\ 4A = 4 \end{cases} \qquad \text{rref}\begin{bmatrix} 1 & 1 & 0 & 2 \\ 0 & 0 & 1 & -1 \\ 4 & 0 & 0 & 4 \end{bmatrix}$$

ex4

$$\frac{x^5-3x^2+12x-1}{x^3(x^2+x+1)(x^2+2)^3} = \frac{A}{x} + \frac{B}{x^2} + \frac{C}{x^3} + \frac{Dx+E}{x^2+x+1} + \frac{Fx+G}{x^2+2} + \frac{Hx+I}{(x^2+2)^2} + \frac{Jx+K}{(x^2+2)^3}$$

11 eqns, 11 unknowns. Where from? Because $\underset{\text{11th power}}{P(x)} = \underbrace{(x-r_1)(x-r_2)\cdots(x-r_{11})}_{\text{irreducible}} \rightarrow \left(\underbrace{\frac{Bx+C}{x^2-\Delta} + \frac{}{x^2-\square} + \cdots}_{5}\right. + \left.\frac{A}{x}\right)$ $2\times 5 + 1 = 11$ unknowns.

11 linear factors. Even if ...

11 eqns: coeff. of x^{11}

Highest Power $x^{10} = x^{11} \div x$, & constant

A, B, C ... K

$P(x) = x^5-3x^2+12x-1$

$\frac{x^{11}}{x} \rightarrow x^{10}$ & constant matchup gives the 11 eqns.

ex5 When starting with improper rational

$$\frac{2x^4+4x^3-2x^2+x+7}{x^3+2x^2-x-2} = 2x + \underbrace{\frac{5x+7}{x^3+2x^2-x-2}}_{\text{ex 1}}$$

$$\begin{array}{r} 2x \\ x^3+2x^2-x-2 \overline{)\, 2x^4+4x^3-2x^2+x+7} \\ \underline{2x^4+4x^3-2x^2-4x} \\ 5x+7 \end{array}$$

$$= \boxed{2x + \frac{\overset{A}{2}}{x-1} - \frac{\overset{B}{1}}{x+1} - \frac{\overset{C}{1}}{x+2}}$$

$$5x+7 = A(x+1)(x+2) + B(x-1)(x+2) + C(x-1)(x+1)$$

• $x=-1 \quad 2 = B(-2) \Rightarrow B = -1$ etc.

Use shortcut if possible

$\frac{10.9}{9.9}$ Systems of Inequalities

Graph $\quad y > x^2 \quad \{(x,y) : y > x^2\}$

① Graph the Equation(s)

② Test one point in each region. Shade

ex1 a) $x^2 + y^2 < 25$

common sense

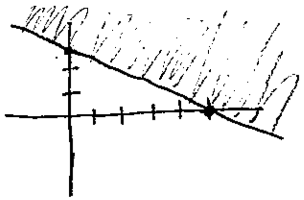
$0 < 25$

distance < 5

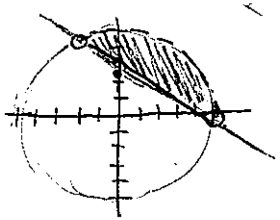
$100 + 100 \not< 25$

b) $x + 2y \geq 5$

$y \geq \frac{-x+5}{2} = \frac{-(x-5)}{2}$



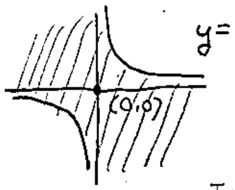
ex2 $\begin{cases} x^2 + y^2 < 25 \\ x + 2y \geq 5 \end{cases}$



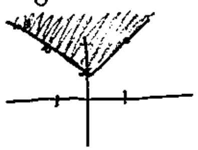
Aufmann & Baker P633

ex $\quad xy \leq 1$

$y = \frac{1}{x}$



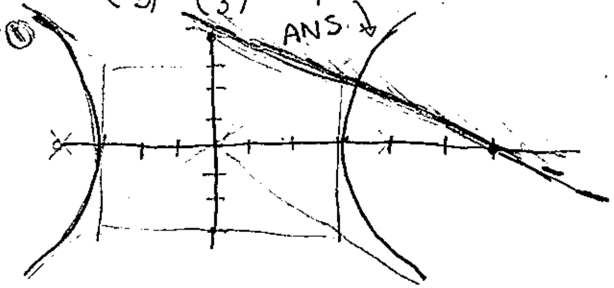
ex $\quad y \geq |x| + 1$



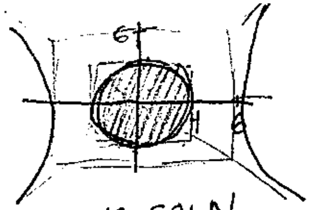
ex $\quad x^2 - y^2 \leq 9 \leftarrow \left(\frac{x}{3}\right)^2 - \left(\frac{y}{3}\right)^2 = 1$

$2x + 3y > 12$

$y > \frac{-2(x-6)}{3}$



ex $\begin{cases} x^2 + y^2 \leq 16 \\ x^2 - y^2 \geq 36 \end{cases}$

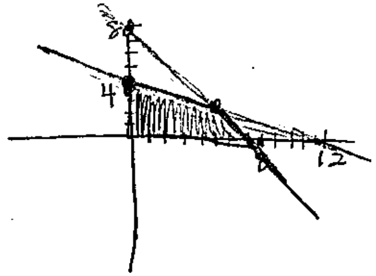


NO SOLN

ex3 $\begin{cases} x + 3y \leq 12 \\ x + y \leq 8 \\ x \geq 0 \\ y \geq 0 \end{cases}$

$y \leq \frac{-(x-12)}{3}$

$y \leq -x + 8$



Application: Feasible Regions

Linear Programming Pg 735 P716

		Cut	Sew	Profit
ex1	Oxfords = x	15 min	10 min	\$15
	Loafers = y	15	20 min	\$20
		8 hours	8 hours	

how many of each to maximize profit?

constraints $\begin{cases} 15(x+y) \leq 480 \\ 10x + 20y \leq 480 \end{cases}$

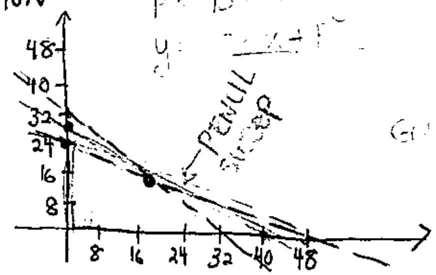
HOURS

$\begin{cases} \frac{1}{4}(x+y) \leq 8 \\ \frac{1}{6}x + \frac{1}{3}y \leq 8 \\ x \geq 0 \\ y \geq 0 \end{cases}$

FEASIBLE REGION

MINUTES

OBJECTIVE FUNCTION $P(x,y) = 15x + 20y$



③ TEST VERTICES

(x,y)	P
(0,24)	480
(16,16)	$\boxed{560}$
(32,0)	480
(0,0)	0

VERTICES: $x + y = 32$

$x + 2y = 48$

$y = 16 \rightarrow x = 16$