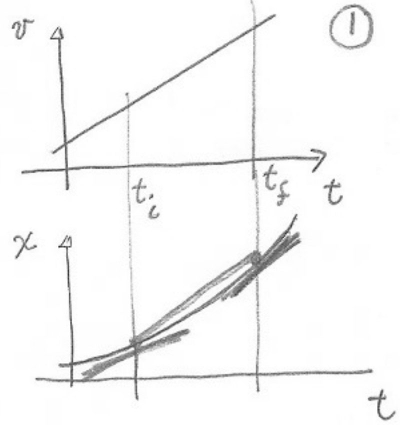
# Chapter 2




① Constant acceleration

Average $\begin{matrix}\text{speed}\\\text{velocity}\end{matrix}$ = average of initial & final velocities

secant slope of $x(t)$ = avg of tangent slopes

$$\frac{\Delta x}{\Delta t} = \frac{v_i + v_f}{2}$$

Proof: $x(t) = x_0 + v_0 t + \frac{1}{2}at^2 \qquad v(t) = v_0 + at$

$$\frac{\Delta x}{\Delta t} = \frac{x_f - x_i}{t_f - t_i} = \frac{\cancel{x_0} + v_0 t_f + \frac{1}{2}at_f^2 - (\cancel{x_0} + v_0 t_i + \frac{1}{2}at_i^2)}{\Delta t}$$

$$= \frac{v_0\cancel{(t_f - t_i)} + \frac{1}{2}a\cancel{(t_f - t_i)}(t_f + t_i)}{\cancel{\Delta t}} = \frac{\underbrace{(v_0 + at_i)} + \underbrace{(at_f + v_0)}}{2}$$

$$= \frac{v_i + v_f}{2}$$

② How to explain the kinematic equations

Constant acceleration $a$

✓ $v(t) = v_0 + at$ (reason. $a$ is the slope)

? $x(t) = x_0 + v_0 t + \frac{1}{2}at^2$

$$\underbrace{x(t) - x_0}_{\text{displacement}} = \underbrace{v_0 t}_{\text{gain if constant speed}} + \underbrace{\frac{1}{2}at^2}_{\text{more gain due to speed up/down}}$$

Reasonable: x-t curve. Secant slope = avg of tangents

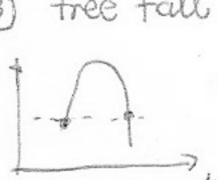
$$\bar{v} = \frac{v_0 + v_f}{2} \qquad \text{Let } \begin{matrix} v_i = v_0 \\ t = 0 \text{ as start}\end{matrix}$$

$$\frac{\Delta x}{\Delta t \, (t-0)} = \frac{v_0 + (v_0 + a\Delta t)}{2} = v_0 + \frac{1}{2}at$$

$$\Delta x = v_0 t + \frac{1}{2}at^2 \quad \#$$

HS Physics idea: Galileo's inclined planes ...

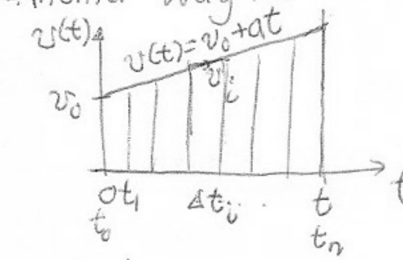
③ free fall $g = -9.80 \ m/s^2$




At same height, speed same
1) parabola symmetry. Tangent same
2) amount velocity lost going up = gained back in same amount of time going down

★ ex 2.12 estimates are great!

④ Another way to look at ②




$\bar{v} \equiv \frac{\int_{t_0}^{t} v(\tau)\,d\tau}{t - t_0}$

$\bar{v} \equiv \frac{\Delta x}{\Delta t}$

$\Delta x = \bar{v}\,\Delta t, \quad \bar{v} \in [v_0, v(t)]$

$\lim_{n\to\infty} \sum_{i=1}^{n} \bar{v}_i \, \Delta t_i$ — $\bar{v}_i$ doesn't need to be the mean (MVT for integrals) (but for Linear $v$ it is)

$= \int_{t_0}^{t} v(\tau)\,d\tau$

TRAPEZOID area under curve

$= \frac{1}{2}(v_0 + v(t)) \times t$

$\therefore \bar{v} = \frac{v_0 + v(t)}{2}$

Deriving the kinematic Equations

Method 1: $\Delta x = \bar{v}\,\Delta t$

$x(t) - x_0 = \frac{1}{2}(v_0 + v(t))\,t = \frac{1}{2}(v_0 + v_0 + at)\,t$

$\therefore x(t) = x_0 + v_0 t + \frac{1}{2}at^2$ by the "avg" ②

Method 2: Pictorially by definite integral, area of trapezoid

Method 3: AP Calculus AB $f(t) = f(a) + \int_a^t f'(\tau)\,d\tau$ accumulating Fundamental Thm

$a(t) = a$

$\frac{dv}{dt} = a \Rightarrow v(t) = v_0 + \int_0^t a\,d\tau = v_0 + at$

$v(t) = \frac{dx}{dt} \Rightarrow x(t) = x_0 + \int_0^t v(\tau)\,d\tau = x_0 + \int_0^t (v_0 + a\tau)\,d\tau$

$= x_0 + v_0 t + \frac{1}{2}at^2$

Method 4: separable differential eqn with initial condition

$\frac{dv}{dt} = a, \quad v(0) = v_0$

$v(t) = \int dv = \int a\,dt = at + C \rightarrow C = v_0$

fn whose deriv. is ?

$v(t) = \frac{dx}{dt} = at + v_0, \quad x_0 = x(0)$

$x(t) = \int (at + v_0)\,dt$

$= \frac{1}{2}at^2 + v_0 t + C_2$

# Chapter 2 : 1D Motion

## Kinematic Eqns for constant acceleration

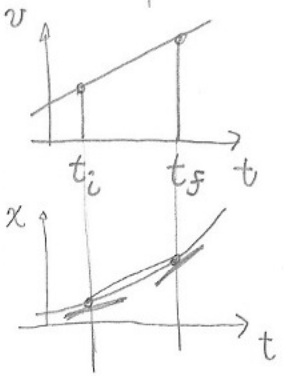
① $\bar{v} = \dfrac{v_0 + v_f}{2}$

$\dfrac{\Delta x}{\Delta t}$

Reasonable, OR proof

avg speed = avg of initial & final velocities

secant slope = avg of tangent slopes
of parabola



$$\bar{v} \equiv \frac{\int_{t_i}^{t_f} v(t)\,dt}{\Delta t} = \frac{\frac{1}{2}(v_i + v_f)\,\Delta t}{\Delta t}$$

②

$$v(t) = v_0 + \overset{\text{gained}}{\widetilde{at}}$$

$$x(t) = \underbrace{x_0}_{\text{displace}} + \underbrace{v_0 t}_{\text{gain}} + \underbrace{\tfrac{1}{2}at^2}_{\text{more gain}}$$

Derive it

$\dfrac{dv}{dt} = a = \text{constant}$

$f(t) = f(a) + \int_a^t f'(\tau)\,d\tau$

accumulation method

$$v(t) = v_0 + \int_0^t v'(\tau)\,d\tau$$

$$= v_0 + at$$

DE $\dfrac{dx}{dt} = v_0 + at$

$$\int dx = \int (v_0 + at)\,dt$$

$x = v_0 t + \frac{1}{2}at^2 + C$ General Soln

At $t = 0$ ← I.C.

$x_0 = C$

Plug

← Particular Soln

③ $\boxed{v_f^2 = v_0^2 + 2a(x_f - x_i)}$

Assume didn't know,
Use ① & ② to write relationship
of $\Delta v$ & $\Delta x$ w/o time

$v(t) - v_0 = at$

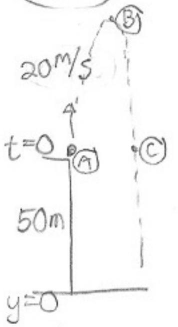
Verify

$v_f^2 - v_0^2 = 2a(x_f - x_i)$

$(v_f - v_0)\dfrac{(v_f + v_0)}{2} = a\,(x_f - x_i)$ ↑ derived

$\Delta v \cdot \dfrac{\Delta x}{\Delta t} \overset{\checkmark}{=} a\,\Delta x$

(ex 2.12) Serway & Beichner $20.0\,m/s$




a) time of max height

b) max height

$t_c$ c) return to start (time)

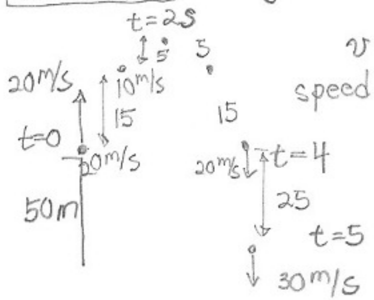
d) velocity $v_c$

e) $t=5.00s$, $v$? $y$?

9.80

$g \approx -10\,m/s^2$

Estimate




$v(t) = v_0 + gt$

speed lost = speed gained
up & down

$\Delta y = \bar{v}\,\Delta t$

a) $t = 2s$

b) $50 + 20 = 70m$

c) $t_c = 4s$

d) $v_c = -20\,m/s$

e) $t = 5.00s \Rightarrow v \approx -30\,m/s$

$y(t) \approx 25\,m$

$t=0 \rightarrow t=2s$

$v=20 \rightarrow 0$

Avg. $10 \times 2 = 20m$

accurately

a) $v(t) = v_0 - gt = 0 \quad \Rightarrow \quad t_B = \underbrace{\boxed{2.04\,s}}_{}$ 3 sig figs

$20.0 - 9.80t = 0$

b) $y(t) = y_0 + v_0 t + \frac{1}{2}at^2 \Rightarrow y(t_B) = \boxed{70.4\,m}$

c) $y(t_c) = \cancel{50.0} = \cancel{50.0} + 20.0t + \frac{1}{2}(-9.80)t^2$

$t(20.0 - 4.90t) = 0$

$t = 0$ or $\boxed{4.08s}$

d) $v(t_c) = v_0 + at_c = 20.0 - 9.80(4.08) = \boxed{-20.0\,m/s}$

e) $v(5) = 20.0 - 9.80 \times 5.00 = \boxed{-29.0\,m/s}$

$y(5) = 50.0 + 20.0 \times 5.00 + \frac{1}{2}(-9.80)\,5.00^2 = \boxed{-22.5\,m}$