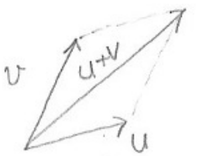
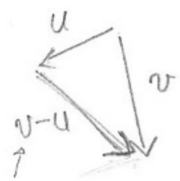
AP Physics C – Vector Review

(Precalculus 6th ed by James Stewart 9.1 ~ 9.5)

- Addition

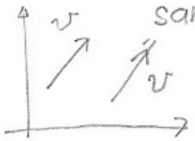


Parallelogram or triangle

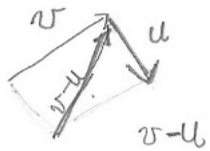


$(\vec{v}-\vec{u})+\vec{u} = \vec{v}$

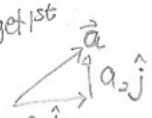
the arrow added to 2nd to get 1st



same dir & mag



- $\vec{a} = \langle a_1, a_2 \rangle = a_1\hat{i} + a_2\hat{j}$



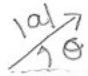
$\vec{a} \pm \vec{b} = \langle a_1 \pm b_1, a_2 \pm b_2 \rangle$

$c\vec{a} = \langle ca_1, ca_2 \rangle$

- magnitude, norm, absolute value, length: $|\vec{a}| = \sqrt{a_1^2 + a_2^2} = \sqrt{\vec{a}\cdot\vec{a}}$

dir. $\tan\theta = \frac{a_y}{a_x}$

θ Quadrant – be careful



$\vec{a} = |a|\cos\theta\,\hat{i} + |a|\sin\theta\,\hat{j}$

$\boxed{\text{Dot Product}}$ scalar $W = \vec{F}\cdot\vec{d}$

$$\boxed{\begin{aligned}\vec{u}\cdot\vec{v} &\equiv u_1v_1 + u_2v_2 \\ &= |\vec{u}||\vec{v}|\cos\theta\end{aligned}}$$

+ more for each dimension

by defn, $\vec{u}\cdot\vec{v}$ commutative, distributive over +, scalar associative

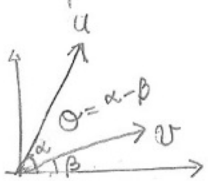
Proof of Dot Product Thm,

Geometric $\langle |\vec{u}|\cos\alpha, |\vec{u}|\sin\alpha \rangle \cdot \langle v\cos\beta, v\sin\beta \rangle$

$= uv\cos\alpha\cos\beta + uv\sin\alpha\sin\beta$

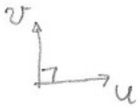
$= uv(\cos(\alpha-\beta))$

$= uv\cos\theta$



orthogonal, normal, perpendicular

$\boxed{\vec{u}\cdot\vec{v} = 0}$ $|u||v|\cos\theta = 0$ iff $\theta = 90°$



- Projections (vector) & component (magnitude) along a direction

$u\cos\theta = \frac{\vec{u}\cdot\vec{v}}{|\vec{v}|}$

$|u\cos\theta|$ $\hat{v}$ OK!

$u\cos\theta$ is negative

Projection of $\vec{u}$ onto $\vec{v}$

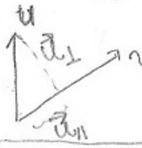
- $\text{proj}_{\vec{v}}\vec{u} = u\cos\theta\,\hat{v} = \boxed{\frac{\vec{u}\cdot\vec{v}}{|\vec{v}|}\left(\frac{\vec{v}}{|\vec{v}|}\right)}$

normalizing to a unit direction vector

- $\vec{u} =$

as orthogonal vectors

$\vec{u} = \vec{u}_{\parallel} + \vec{u}_{\perp}$



$\vec{u}_{\parallel} = \text{proj}_{\vec{v}}\vec{u}$

$\vec{u}_{\perp} = \vec{u} - \text{proj}_{\vec{v}}\vec{u}$

Properties

$\vec{u}\cdot\vec{v} = \vec{v}\cdot\vec{u}$

$(a\vec{u})\cdot\vec{v} = a(\vec{u}\cdot\vec{v}) = \vec{u}\cdot(a\vec{v})$

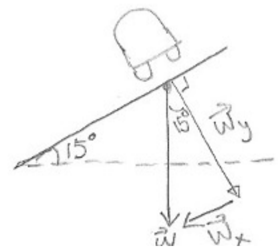
$(\vec{u}+\vec{v})\cdot\vec{w} = \vec{u}\cdot\vec{w} + \vec{v}\cdot\vec{w}$

$|\vec{u}|^2 = \vec{u}\cdot\vec{u}$

9.2 (ex4) 3000 lb car, inclined 15°

a) magnitude of friction to keep from rolling

$w_x = 3000\sin 15° \approx 776$ lb.



b) force pressing on driveway surface

$w_y = w\cos 15° = 2898$ lb

ex6 9.2 $\vec{u} = \langle -2, 9 \rangle$

$\vec{v} = \langle -1, 2 \rangle$

a) $\text{proj}_{\vec{v}}\vec{u} = \frac{\vec{u}\cdot\vec{v}}{|\vec{v}|}\frac{\vec{v}}{|\vec{v}|} = \frac{2+18}{1+4}\langle -1, 2\rangle = \langle -4, 8 \rangle$

orthogonal vectors

b) $\underset{\substack{\uparrow \\ \text{parallel} \\ \text{to } \vec{v}}}{\vec{u} = \vec{u}_{\parallel}} + \vec{u}_{\perp} \Rightarrow$ $\vec{u}_{\parallel} = \langle -4, 8 \rangle$

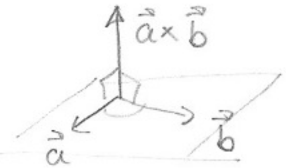
$\vec{u}_{\perp} = \langle -2, 9 \rangle - \langle -4, 8 \rangle = \langle 2, 1 \rangle$

9.5 Cross Product

- $\vec{a}\times\vec{b} \equiv \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \end{vmatrix} = \langle a_2b_3 - a_3b_2, a_3b_1 - a_1b_3, a_1b_2 - a_2b_1 \rangle$

  vector

- $\vec{a}\times\vec{b} \begin{cases} \perp \vec{a} \\ \perp \vec{b} \end{cases}$



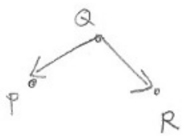
Right Hand Rule

ex3 Vector perpendicular to plane thru $P(1,4,6)$, $Q(-2,5,-1)$, $R(1,-1,1)$

$\vec{a}$ $\vec{b}$

$\vec{a} = \langle 3, -1, 7 \rangle$

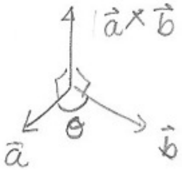
$\vec{b} = \langle 3, -6, 2 \rangle$



$\vec{a}\times\vec{b} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 3 & -1 & 7 \\ 3 & -6 & 2 \end{vmatrix} = \langle 40, 15, -15 \rangle$

Length

$\boxed{|\vec{a}\times\vec{b}| = |\vec{a}||\vec{b}|\sin\theta}$



dir: right hand

mag: $ab\sin\theta$

Proof

$|\vec{a}\times\vec{b}|^2 = (a_2b_3 - a_3b_2)^2 + (a_3b_1 - a_1b_3)^2 + (a_1b_2 - a_2b_1)^2$

$= a_2^2b_3^2 - 2a_2a_3b_2b_3 + a_3^2b_2^2$

$+ a_3^2b_1^2 - 2a_3a_1b_1b_3 + a_1^2b_3^2$

$+ a_1^2b_2^2 - 2a_1a_2b_1b_2 + a_2^2b_1^2$

$\vdots$ Lots of work

$|\vec{a}\times\vec{b}|^2 \cancel{= (\vec{a}\times\vec{b})}\cdot(\vec{a}\times\vec{b})$

$$\boxed{\begin{vmatrix} u_1 & u_2 & u_3 \\ a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \end{vmatrix} \quad \vec{u}\cdot(\vec{a}\times\vec{b}) = \cancel{\vec{u}\cdot\vec{a}\times\vec{u}\cdot\vec{b}}}$$

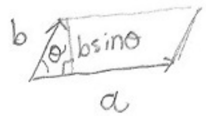
$(a_1^2 + a_2^2 + a_3^2)(b_1^2 + b_2^2 + b_3^2) - (a_1b_1 + a_2b_2 + a_3b_3)^2$

$= |\vec{a}|^2|\vec{b}|^2 - (\vec{a}\cdot\vec{b})^2$

$= |\vec{a}|^2|\vec{b}|^2 - |\vec{a}|^2|\vec{b}|^2\cos^2\theta$

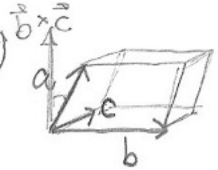
$= |\vec{a}|^2|\vec{b}|^2(\underbrace{1-\cos^2\theta}_{\sin^2\theta})$

Area of Parallelogram



Area $= |\vec{a}\times\vec{b}|$

Volume of Paralleliped



$V = Ah$ — component of projection

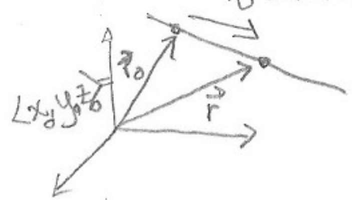
$= \cancel{|\vec{b}\times\vec{c}|}\ \frac{\vec{a}\cdot(\vec{b}\times\vec{c})}{\cancel{|\vec{b}\times\vec{c}|}}$

$\boxed{V = \vec{a}\cdot(\vec{b}\times\vec{c})}$

9.6

Eqn of Line

$\vec{v}=\langle a,b,c\rangle$



$$\boxed{\vec{r}=\vec{r}_0+t\vec{v}}$$

$$=\langle x_0+ta,\ y_0+tb,\ z_0+tc\rangle$$

parametric eqns of Line

ex2 Line thru $(-1,2,6)$ & $(2,-3,-7)$

Parametric eqns

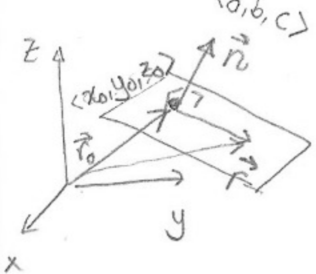
$\vec{v}=\langle 3,-5,-13\rangle$

$\vec{r}_0$ P $=\langle -1,2,6\rangle$

$\vec{r}=\vec{r}_0+t\vec{v}=\langle -1+3t,\ 2-5t,\ 6-13t\rangle$

Eqn of a plane

Point & normal



$\{\vec{r}:\boxed{\vec{n}\cdot(\vec{r}-\vec{r}_0)=0}\}$

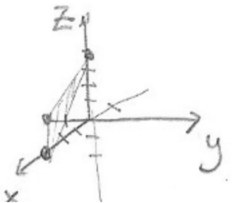
$\langle a,b,c\rangle\cdot\langle x-x_0,\ y-y_0,\ z-z_0\rangle=0$

$a(x-x_0)+b(y-y_0)+c(z-z_0)=0$

ex3 $\vec{n}=\langle 4,-6,3\rangle$, $P(3,-1,-2)$

a) eqn of plane $\cancel{\vec{n}}\ 4(x-3)-6(y+1)+3(z+2)=0$

b) sketch



$z=4$

$x,y=0\Rightarrow -12-\cancel{6}+3z+\cancel{6}=0$

$y,z=0\Rightarrow x=3$

$x,\cancel{y}z=0\Rightarrow y=-2$

ex4 eqn of plane

$P(1,4,6)$, $Q(-2,5,-1)$, $R(1,-1,1)$

$\vec{a}=\langle 3,-1,7\rangle$

$\vec{b}=\langle 3,-6,+2\rangle$

$$\vec{n}=\begin{vmatrix}\hat{i}&\hat{j}&\hat{k}\\3&-1&7\\3&-6&2\end{vmatrix}=\langle 40,15,-15\rangle$$

$$\rightarrow\langle 8,3,-3\rangle$$

$\vec{n}\cdot(\vec{r}-\langle 1,4,6\rangle)=0$

$8(x-1)+3(x-4)-3(z-6)=0$ ← not unique

$\boxed{8x+3x-3z=2}$ unique

HW