Vector Worksheet

Reference: Precalculus 6th Edition by Stewart, Redlin, Watson

9.2 Find the component of u along v

27. $\mathbf{u} = 7\mathbf{i}$, $\mathbf{v} = 8\mathbf{i} + 6\mathbf{j}$

29–34 ■ (a) Calculate $\text{proj}_{\mathbf{v}}\,\mathbf{u}$. (b) Resolve $\mathbf{u}$ into $\mathbf{u}_1$ and $\mathbf{u}_2$, where $\mathbf{u}_1$ is parallel to $\mathbf{v}$ and $\mathbf{u}_2$ is orthogonal to $\mathbf{v}$.

29. $\mathbf{u} = \langle -2, 4 \rangle$, $\mathbf{v} = \langle 1, 1 \rangle$

DISCOVERY ■ DISCUSSION ■ WRITING

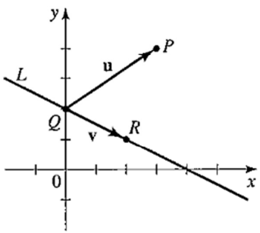
53. Distance from a Point to a Line Let L be the line $2x + 4y = 8$ and let P be the point $(3, 4)$.

(a) Show that the points $Q(0, 2)$ and $R(2, 1)$ lie on L.

(b) Let $\mathbf{u} = \overrightarrow{QP}$ and $\mathbf{v} = \overrightarrow{QR}$, as shown in the figure. Find $\mathbf{w} = \text{proj}_{\mathbf{v}}\,\mathbf{u}$.

(c) Sketch a graph that explains why $|\mathbf{u} - \mathbf{w}|$ is the distance from P to L. Find this distance.

(d) Write a short paragraph describing the steps you would take to find the distance from a given point to a given line.



9.4

29–32 ■ Determine whether or not the given vectors are perpendicular.

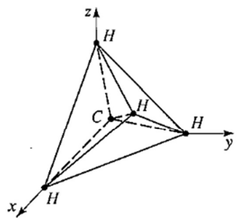
29. $\langle 4, -2, -4 \rangle$, $\langle 1, -2, 2 \rangle$ **30.** $4\mathbf{j} - \mathbf{k}$, $\mathbf{i} + 2\mathbf{j} + 9\mathbf{k}$

48. Central Angle of a Tetrahedron A *tetrahedron* is a solid with four triangular faces, four vertices, and six edges, as shown in the figure. In a *regular* tetrahedron, the edges are all of the same length. Consider the tetrahedron with vertices $A(1, 0, 0)$, $B(0, 1, 0)$, $C(0, 0, 1)$, and $D(1, 1, 1)$.

(a) Show that the tetrahedron is regular.

(b) The center of the tetrahedron is the point $E(\frac{1}{2}, \frac{1}{2}, \frac{1}{2})$ (the "average" of the vertices). Find the angle between the vectors that join the center to any two of the vertices (for instance, $\angle AEB$). This angle is called the *central angle* of the tetrahedron.

NOTE: In a molecule of methane (CH_4) the four hydrogen atoms form the vertices of a regular tetrahedron with the carbon atom at the center. In this case chemists refer to the central angle as the *bond angle*. In the figure, the tetrahedron in the exercise is shown, with the vertices labeled H for hydrogen, and the center labeled C for carbon.



9.5

17–20 ■ Find a vector that is perpendicular to the plane passing through the three given points.

(17.) $P(0, 1, 0)$, $Q(1, 2, -1)$, $R(-2, 1, 0)$

25–28 ■ Find the area of $\triangle PQR$.

(25.) $P(1, 0, 1)$, $Q(0, 1, 0)$, $R(2, 3, 4)$

29–34 ■ Three vectors **a**, **b**, and **c** are given. **(a)** Find their scalar triple product $\mathbf{a} \cdot (\mathbf{b} \times \mathbf{c})$. **(b)** Are the vectors coplanar? If not, find the volume of the parallelepiped that they determine.

(33.) $\mathbf{a} = 2\mathbf{i} - 2\mathbf{j} - 3\mathbf{k}$, $\mathbf{b} = 3\mathbf{i} - \mathbf{j} - \mathbf{k}$, $\mathbf{c} = 6\mathbf{i}$

DISCOVERY ■ DISCUSSION ■ WRITING

37. Order of Operations in the Triple Product Given three vectors **u**, **v**, and **w**, their scalar triple product can be performed in six different orders:

$$\mathbf{u} \cdot (\mathbf{v} \times \mathbf{w}), \quad \mathbf{u} \cdot (\mathbf{w} \times \mathbf{v}), \quad \mathbf{v} \cdot (\mathbf{u} \times \mathbf{w}),$$
$$\mathbf{v} \cdot (\mathbf{w} \times \mathbf{u}), \quad \mathbf{w} \cdot (\mathbf{u} \times \mathbf{v}), \quad \mathbf{w} \cdot (\mathbf{v} \times \mathbf{u})$$

(a) Calculate each of these six triple products for the vectors:

$$\mathbf{u} = \langle 0, 1, 1 \rangle, \quad \mathbf{v} = \langle 1, 0, 1 \rangle, \quad \mathbf{w} = \langle 1, 1, 0 \rangle$$

(b) On the basis of your observations in part (a), make a conjecture about the relationships between these six triple products.

(c) Prove the conjecture you made in part (b).

9.6

CONCEPTS

1. A line in space is described algebraically by using __________ equations. The line that passes through the point $P(x_0, y_0, z_0)$ and is parallel to the vector $\mathbf{v} = \langle a, b, c \rangle$ is described by the equations $x =$ __________, $y =$ __________, $z =$ __________.

2. The plane containing the point $P(x_0, y_0, z_0)$ and having the normal vector $\mathbf{n} = \langle a, b, c \rangle$ is described algebraically by the equation __________.

8 ■ Find parametric equations for the line that passes through the point P and is parallel to the vector $\mathbf{v}$.

7. $P(1, 1, 1)$, $\mathbf{v} = \mathbf{i} - \mathbf{j} + \mathbf{k}$

15–20 ■ A plane has normal vector $\mathbf{n}$ and passes through the point P. **(a)** Find an equation for the plane. **(b)** Find the intercepts and sketch a graph of the plane.

15. $\mathbf{n} = \langle 1, 1, -1 \rangle$, $P(0, 2, -3)$

27–30 ■ A description of a line is given. Find parametric equations for the line.

27. The line crosses the x-axis where $x = -2$ and crosses the z-axis where $z = 10$.

29. The line perpendicular to the xz-plane that contains the point $(2, -1, 5)$.

31–34 ■ A description of a plane is given. Find an equation for the plane.

32. The plane that crosses the x-axis where $x = 1$, the y-axis where $y = 3$, and the z-axis where $z = 4$.

33. The plane that is parallel to the plane $x - 2y + 4z = 6$ and contains the origin.

34. The plane that contains the line $x = 1 - t$, $y = 2 + t$, $z = -3t$ and the point $P(2, 0, -6)$. [*Hint:* A vector from any point on the line to P will lie in the plane.]

DISCOVERY ■ DISCUSSION ■ WRITING

35. Intersection of a Line and a Plane A line has parametric equations

$$x = 2 + t, \qquad y = 3t, \qquad z = 5 - t$$

and a plane has equation $5x - 2y - 2z = 1$.

(a) For what value of t does the corresponding point on the line intersect the plane?

(b) At what point do the line and the plane intersect?

36. Lines and Planes A line is parallel to the vector **v**, and a plane has normal vector **n**.

(a) If the line is perpendicular to the plane, what is the relationship between **v** and **n** (parallel or perpendicular)?

(b) If the line is parallel to the plane (that is, the line and the plane do not intersect), what is the relationship between **v** and **n** (parallel or perpendicular)?

(c) Parametric equations for two lines are given. Which line is parallel to the plane $x - y + 4z = 6$? Which line is perpendicular to this plane?

Line 1: $x = 2t, \qquad y = 3 - 2t, \qquad z = 4 + 8t$

Line 2: $x = -2t, \qquad y = 5 + 2t, \qquad z = 3 + t$