NAME:

AP Physics C Summer Homework

Questions labeled in [brackets] are required only for students who have completed AP Calculus AB

1.

Sketch each function and fill in the blanks

Function	Sketch	Domain	Range	One Point on Curve	Asymptotes (equation or "none")
A) $y = e^{-x} - 1$					
B) $y = \sqrt{x}$					
c) $y = \frac{1}{x}$					
d) $y = \log_5(-x)$					
e) $y = \log_{\frac{2}{5}} x$					
f) $y = x^4$					
g) $y = (\frac{1}{3})^x$					
h) $y = \lfloor x \rfloor$					
i) $y = \sqrt[3]{x}$					

2. Fill in the radian conversion of each angle and the trigonometric value at each angle on the chart.

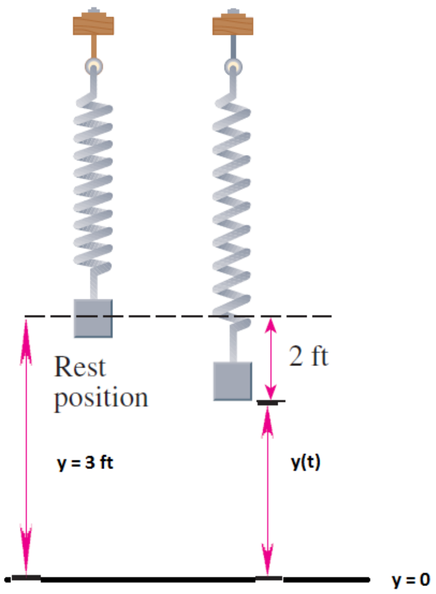
Degree θ	0°	30°	45°	60°	90°	180°	270°	360°
Radian θ								
sinθ								
cosθ								
tanθ								
cscθ								
secθ								
cotθ								

3. $y=\frac{1}{2}-\frac{1}{2}\cos(-2x-\frac{\pi}{3})$

a) Amplitude = ______ b) Period = _______ c) Phase Shift = _________

d) Graph one period with the 5 important points below. The x and y axes should be clearly labeled:

4. As the tides change, the water level in a bay varies sinusoidally. At high tide today at 8 A.M., the water level was 15 feet; at low tide, 6 hours later at 2 P.M., it was 3 feet. Model the water level as a function of time using a sinusoid.



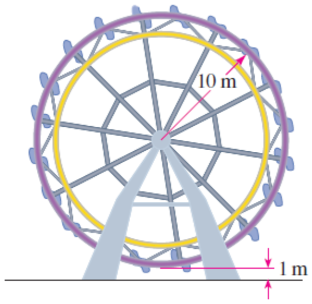
5. Spring–Mass System A mass suspended from a spring is pulled down a distance of 2 ft from its rest position, as shown in the figure. The mass is released at time *t* = 2 and allowed to oscillate. If the mass returns to this position after 3 s, find an equation that describes its motion.

y = position above the ground

t = time in seconds

y(t) = ______________________________

6. Ferris Wheel A ferris wheel has a radius of 10 m, and the bottom of the wheel passes 1 m above the ground. If the ferris wheel makes one complete revolution every 20 s, find an equation that gives the height above the ground of a person on the ferris wheel as a function of time.



MVT and Average

(A review of the Extreme Value, Intermediate Value, and Mean Value Theorems for Derivatives and Integrals is in the Calculus C Ch. 4 Notes, Page 14)

[1.] Let $f(x) = x^{2/3}$ on [- 8, 27]. Show that the conclusion to the Mean Value Theorem fails and figure out why. Hint: Sketch the graph of f(x).

[2.] Find all values of x^* in the interval [0,2] that satisfy the Mean-Value Theorem for Integrals, where $f(x) = x^3$.

2. Find the average value of $f(x) = (x + 1)^2$ over [−1, 2].

- Find a point x^* in [−1, 2] such that $f(x^*) = f_{\text{ave}}$.
- Sketch the graph of $f(x) = (x + 1)^2$ over [−1, 2] and construct a rectangle over the interval whose area is the same as the area under the graph of f over the interval.

[3.] Derive the Mean Value Theorem for Integrals by applying the Mean Value Theorem for Derivatives to the antiderivative F(x).

Differentials

1. Let $y = \frac{1}{2}x^2 + 1$.

- Find Δy if $\Delta x = 1$ and the initial value of x is $x = 1$.
- Find dy if $dx = 1$ and the initial value of x is $x = 1$.
- Make a sketch of $y = \frac{1}{2}x^2 + 1$ and show Δy and dy in the picture.

2. A cylindrical rod (circular right cylinder) of height 1 cm is supposed to have a radius of 2.5 cm. However, the manufacturing process introduces an uncertainty of 0.001 in the radius. Use differentials to approximate the uncertainty in the volume of the cylindrical rod.

Vector Calculus

Read the Vector Calculus Notes P. 5~7. Complete the following 2 questions. (See p. 7)

1. A projectile is shot from the origin at angle θ from the positive x-axis with initial speed v_0 ft/sec. Neglect friction.

a. What is the velocity $\mathbf{v}(t)$=? Position $\mathbf{r}(t)$ = ?

b. Show the path is a parabola.

2. $x = 3\cos(t)$, $y = 2\sin(t)$

are the parametric equations for point P moving in a plane. t = time.

a. Graph the path of point P on the x-y axes.

b. Find $\mathbf{v}(t)$=?, $|\mathbf{v}(t)|$=? , $\mathbf{a}(t)$ = ? Sketch $\mathbf{r}(t)$, $\mathbf{v}(t)$, and $\mathbf{a}(t)$ on the graph drawn in part (a)

c. Find the maximum and minimum values of speed and where they occur (in x-y coordinates)

d. Show the acceleration vector of point P always points to the origin

e. When is $|\mathbf{a}(t)|$ largest? smallest?

Unit Conversion Factors

1. To convert km/h to m/s, multiply by unit conversion factors so that the units you don't want cancel out and the units you want remain. Show your work as in the following example:

$$\left(\frac{25\text{ km}}{\text{h}}\right)\left(\frac{1000\text{ m}}{1\text{ km}}\right)\left(\frac{1\text{ h}}{3600\text{ s}}\right) =$$

2. How many seconds are in a year?

3. Convert the speed of light, 3×10^8 m/s, to km/day.

4. The SI unit for liquid volume is 1 milliliter, approximately the volume of a large drop of water. The corresponding SI unit for solid volume is 1 cm^3.   1 drop $\approx$ 1 mL = 1 cm^3.

How many liters can a 1 m^3 tank hold?

How many drops of water is this?

Taylor Series

Taylor and Maclaurin series are an important topic in AP Calculus C and AP Physics C. The idea is foundational in engineering and science because it allows for polynomial approximations to non-polynomial functions, thereby greatly simplifying many analyses. Please read this Taylor reference.

[#1] What is the Taylor series approximation for sin(x) based at $\pi/2$?

[a.] Sketch sin(x) from 0 to 2π radians.

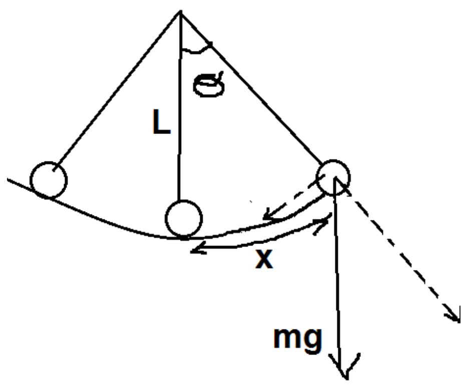
[b.] Use derivatives and then sketch the first-order Taylor approximation for sin(x) based at $\pi/2$ on the graph in part (a).

[c.] Use derivatives and then sketch the second-order Taylor approximation for sin(x) based at $\pi/2$ on the graph in part (a).

[d.] What is the second-order Taylor approximation for $\sin(3\pi/4)$ based at $\pi/2$? What is the error between your quadratic approximation and the actual answer? Sketch a distance on the graph in part (a) to visualize the error.

[e.] Now find the Maclaurin series approximation for sin(x) (i.e. based at 0).

[#2]



Here is an example of a common application of series in physics. Why is the oscillation of a pendulum considered simple harmonic motion? It is actually considered SHM only for the case of small-amplitude oscillations. The definition of "simple harmonic" is that the magnitude of the restoring force is directly proportional to displacement from equilibrium. That is, $F = -k x$, like Hooke's Law. In the diagram of the pendulum, the restoring force is directed tangent to the pendulum's path. Show that $F \approx -kx$, where $k=mg/L$. Hint: Rewrite $\sin\theta$ using its Maclaurin series. When θ is "small", say $\theta \approx 0.01$, the higher-order terms with θ^2, θ^3,... are negligible and can be ignored.

The Fun Part of Calculus is Being Able to Derive/Prove Formulas in Physics

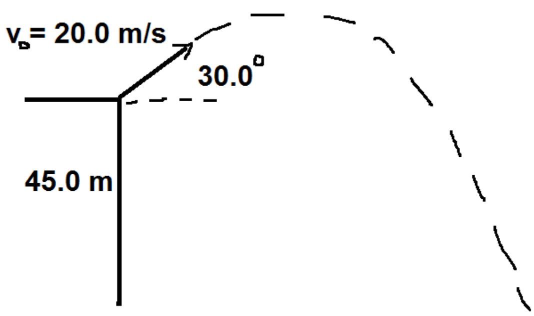
Kinematics

Hint for the next 3 problems: See [Physics Chapter 2 Kinematics Notes]

[#1] Derive the kinematic equations for constant acceleration from definition and calculus.

$$v_f = v_i + a\Delta t \qquad x_f = x_i + v_i\Delta t + \frac{1}{2}a\Delta t^2 \qquad v_f^{\ 2} = v_i^{\ 2} + 2a(\Delta x)$$

[#2] Prove that the slope of the secant line connecting two points of a parabola is equal to the arithmetic average of the slopes of the tangent lines at those two points.



#3 A stone is thrown from the top of a building upward at an angle of 30.0° to the horizontal and with an initial speed of 20.0 m/s as shown. If the height of the building is 45.0 m,

a) How long is it before the stone hits the ground?

b) What is the speed of the stone just before it strikes the ground?

c) Where does the stone strike the ground?

d) Try part a) without a calculator and using only the idea of $\Delta y = v_{avg}\Delta t$. You may use $g \approx 10\ m/s^2$ (Hint: See the "estimation" technique in Example 2.12 of the "Chapter 2 Notes")

Where the Laws Come From

[#4] Derive the work-kinetic energy theorem (W = 1/2 mv_f^2 – 1/2 mv_i^2). Hint: Start with the definition of work, $W = \int_{x_i}^{x_f} F(x)dx$. Replace F (net force) with Newton's Second Law. Replace acceleration with dv/dt. Notice dx/dt is velocity. Change the limits of integration from x_i and x_f , to v_i and v_f. Then evaluate the definite integral.

[#5] Rewrite Newton's Second Law ($\Sigma F = ma$) as the impulse-momentum theorem ($\Sigma F = dp/dt$).

[#6] The Law of Conservation of Momentum states that for an isolated system, the total momentum before and after a collision is the same. Why is this a natural consequence of Newton's Laws? Hint: $\Sigma F = dp/dt$.

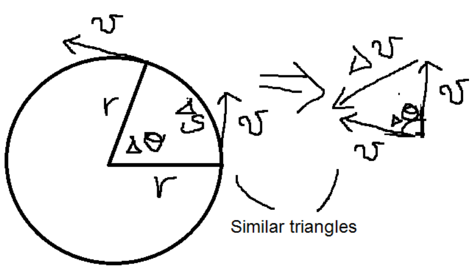
[#7] Starting with the definition that $\omega = d\theta/dt$, and the formula for arc length $s = r\theta$, prove

[a.] $v_t = r\omega$

[b.] $a_t = r\alpha$

[#8] For uniform circular motion, prove $a_c = v^2/r = r\,\omega^2$. Please use 2 methods:

[a.] Geometrically.



[b.] Vectors and Calculus.

$\vec{s}(t) = < r\cos\omega t, r\sin\omega t >$

Take the derivative with respect to time twice

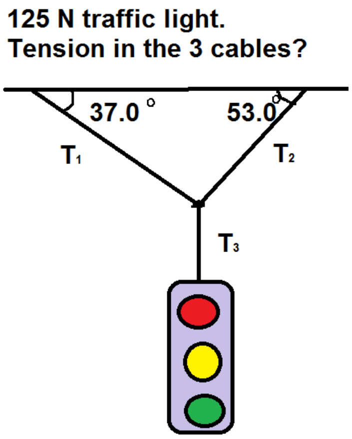
(just take the derivative of each component) to show that

$\vec{a}(t) = -\omega^2\vec{s}(t)$. That is, acceleration is directed toward the center of the circle.

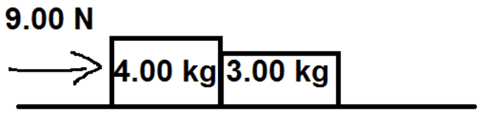
The magnitude can be shown from this to be $r\omega^2$.

Dynamics

#1 The traffic light is in static equilibrium.



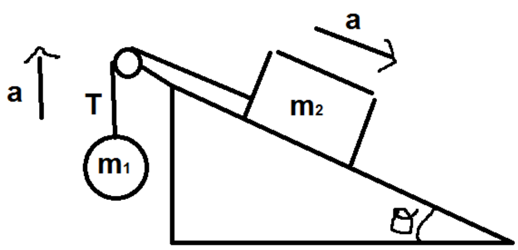
#2



a. Find the system's acceleration

b. Find the magnitude of the contact force.

#3

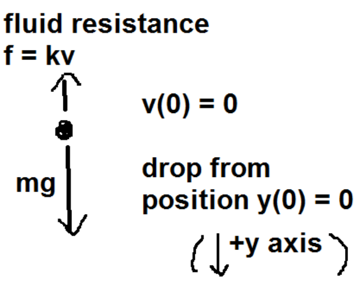


a. Draw a free body diagram for m_2.

b. Find the acceleration.

c. Find the tension in the string.

#4 The force of air resistance on a falling object is approximately proportional to the speed v of the object. As the object gains speed, air resistance increases until the object reaches dynamic equilibrium. The free body diagram of a falling object is shown.



a) Show that the terminal speed of the object is $v_T = mg/k$.

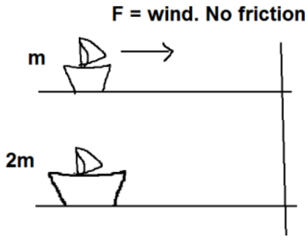
b) Show $v(t) = v_T[1 - e^{-(k/m)t}]$

Hint: This is a calculus question. Start with $\Sigma F = ma$, and replace a with dv/dt. You will have a first-order separable differential equation. Solve for v.

c) Find a(t) =

d) Find y(t) =

e) Sketch a(t) vs. t, v(t) vs. t, y(t) vs. t.



#1 Two iceboats hold a race on a frictionless horizontal lake. The two iceboats have masses *m* and 2*m*. Each iceboat has an identical sail, so the wind exerts the same constant force **F** on each iceboat. The two iceboats start from rest and cross the finish line a distance *s* away. Describe your reasoning in the following questions. The answers could be: 1m iceboat, 2m iceboat, or both are the same.

a) Which iceboat crosses the finish line with greater kinetic energy?

b) Which iceboat wins the race?

c) Which iceboat spends more time along the surface?

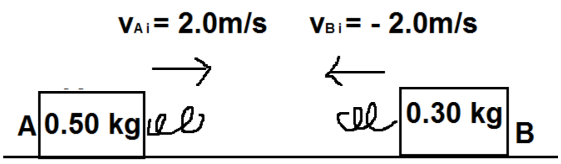
d) Which iceboat has the greater change in momentum?

e) If the iceboat of mass *m* has momentum of 350 kg.m/s at the finish line, how much momentum does the 2*m* iceboat have?

f) If the iceboats go off a ledge at the finish line and the 2*m* iceboat reaches the ground 10 m from the ledge, at what distance does the 1*m* iceboat reach the ground?

g) Which iceboat spends more time in the air, from cliff to ground?

*h) BONUS: By how many seconds does the winning iceboat win? Write the answer in terms of the mass *m*, distance *s*, and force *F*.



#7 Two gliders move toward each other on a frictionless linear air track. They have ideal spring bumpers so the collision is elastic. What are the velocities of A and B after the collision? Hint: Set up 2 equations: $p_i = p_f$ and $K_i = K_f$. Actually, instead of using the fact that kinetic energy stays the same, it is faster to use the consequential result that relative velocities after the collision have the same magnitude but opposite sign. That is, $v_{Af} - v_{Bf} = -(v_{Ai} - v_{Bi})$. ← Try to prove that too.

7 ** BONUS: For an elastic collision, prove that $v_{Af} - v_{Bf} = -(v_{Ai} - v_{Bi})$.

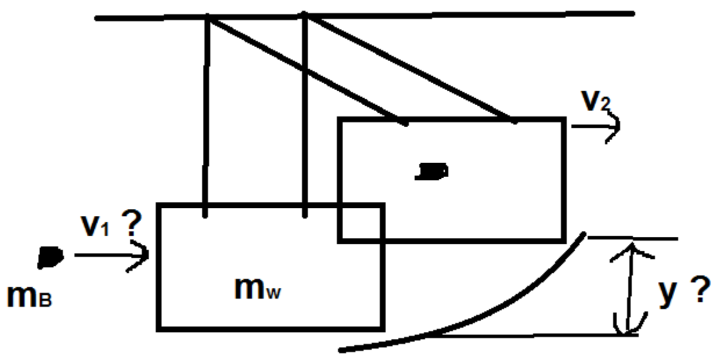
Hint: Use the 2 equations from conservation of momentum and conservation of kinetic energy.

#8 A ballistic pendulum is a system for measuring the speed of a bullet. The bullet, with mass m_B, is fired into a block of wood with mass m_W, suspended like a pendulum, and makes a completely inelastic collision with it. After the impact of the bullet, the block swings up to a maximum height y. Given the values of y, m_B, and m_W, show that the initial speed v_i of the bullet is

$$v_1 = \frac{m_B + m_w}{m_B}\sqrt{2gy}$$

Hint:

- During the inelastic collision, momentum is conserved. The only forces involved in the collision are horizontal.
- After the collision, mechanical energy is conserved and kinetic energy is transformed into gravitational potential energy.



Topic #7: Physics Formulas with Calculus. (See the Calculus Tutorial for AP Physics C)

(Source: Young and Freedman 13th Edition)

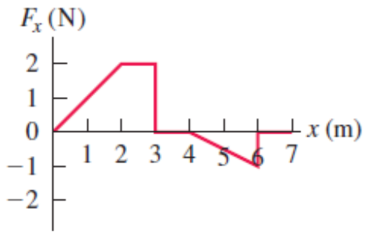
2.8 • **CALC** A bird is flying due east. Its distance from a tall building is given by $x(t) = 28.0 \text{ m} + (12.4 \text{ m/s})t - (0.0450 \text{ m/s}^3)t^3$. What is the instantaneous velocity of the bird when $t = 8.00$ s?

2.17 • **CALC** A car's velocity as a function of time is given by $v_x(t) = \alpha + \beta t^2$, where $\alpha = 3.00$ m/s and $\beta = 0.100 \text{ m/s}^3$. (a) Calculate the average acceleration for the time interval $t = 0$ to $t = 5.00$ s. (b) Calculate the instantaneous acceleration for $t = 0$

4.59 • **CALC** An object with mass m moves along the x-axis. Its position as a function of time is given by $x(t) = At - Bt^3$, where A and B are constants. Calculate the net force on the object as a function of time.

6.41 • A force $\vec{F}$ is applied to a 2.0-kg radio-controlled model car parallel to the x-axis as it moves along a straight track. The x-component of the force varies with the x-coordinate of the car as shown in Fig. E6.41. Calculate the work done by the force $\vec{F}$ when the car moves from (a) $x = 0$ to $x = 3.0$ m; (b) $x = 3.0$ m to $x = 4.0$ m; (c) $x = 4.0$ m to $x = 7.0$ m; (d) $x = 0$ to $x = 7.0$ m; (e) $x = 7.0$ m to $x = 2.0$ m.

Figure **E6.41**

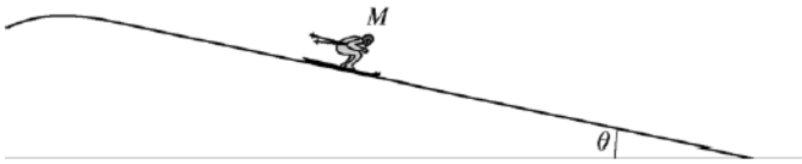


6.71 • **CALC** An object is attracted toward the origin with a force given by $F_x = -k/x^2$. (Gravitational and electrical forces have this distance dependence.) (a) Calculate the work done by the force F_x when the object moves in the x-direction from x_1 to x_2. If

Remember, solutions to FRQs are available at AP Central.

PHYSICS C: MECHANICS
SECTION II
Time—45 minutes
3 Questions

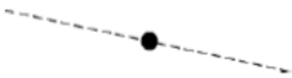
Directions: Answer all three questions. The suggested time is about 15 minutes for answering each of the questions, which are worth 15 points each. The parts within a question may not have equal weight. Show all your work in the pink booklet in the spaces provided after each part, NOT in this green insert.



Mech. 1.

A skier of mass M is skiing down a frictionless hill that makes an angle θ with the horizontal, as shown in the diagram. The skier starts from rest at time $t = 0$ and is subject to a velocity-dependent drag force due to air resistance of the form $F = -bv$, where v is the velocity of the skier and b is a positive constant. Express all algebraic answers in terms of M, b, θ, and fundamental constants.

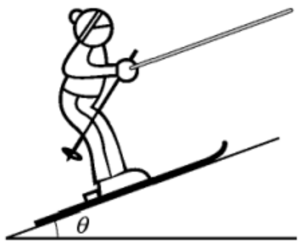
(a) On the dot below that represents the skier, draw a free-body diagram indicating and labeling all of the forces that act on the skier while the skier descends the hill.



(b) Write a differential equation that can be used to solve for the velocity of the skier as a function of time.

(c) Determine an expression for the terminal velocity v_T of the skier.

(d) Solve the differential equation in part (b) to determine the velocity of the skier as a function of time, showing all your steps.



Mech. 3.

A skier of mass m will be pulled up a hill by a rope, as shown above. The magnitude of the acceleration of the skier as a function of time t can be modeled by the equations

$$a = a_{max} \sin\frac{\pi t}{T} \qquad (0 < t < T)$$
$$= 0 \qquad (t \geq T),$$

where a_{max} and T are constants. The hill is inclined at an angle θ above the horizontal, and friction between the skis and the snow is negligible. Express your answers in terms of given quantities and fundamental constants.

(a) Derive an expression for the velocity of the skier as a function of time during the acceleration. Assume the skier starts from rest.

(b) Derive an expression for the work done by the net force on the skier from rest until terminal speed is reached.

(c) Determine the magnitude of the force exerted by the rope on the skier at terminal speed.

(d) Derive an expression for the total impulse imparted to the skier during the acceleration.

(e) Suppose that the magnitude of the acceleration is instead modeled as $a = a_{max}e^{-\pi t/2T}$ for all $t > 0$, where a_{max} and T are the same as in the original model. On the axes below, sketch the graphs of the force exerted by the rope on the skier for the two models, from $t = 0$ to a time $t > T$. Label the original model F_1 and the new model F_2.