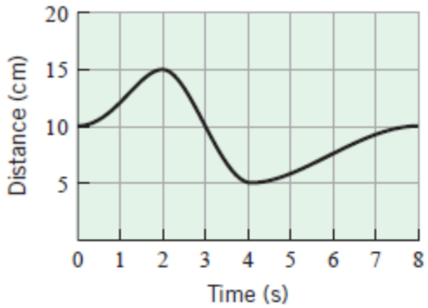
Calculus Worksheet for AP Physics C

Questions are from Anton, Howard, Irl Bivens, and Stephen Davis. *Calculus: Early Transcendentals. 10th ed*. John Wiley & Sons, 2012.

Topic #1: Rates of Change (Anton 2.1)

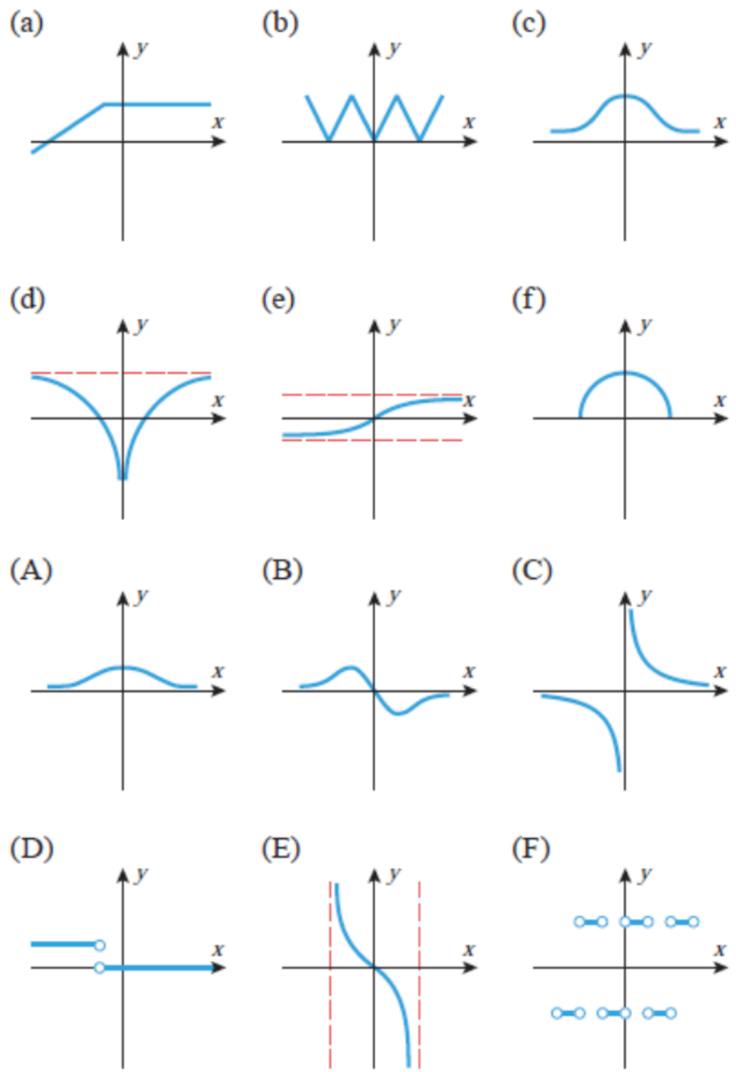
3. The accompanying figure shows the position versus time curve for a certain particle moving along a straight line. Estimate each of the following from the graph:
(a) the average velocity over the interval $0 \le t \le 3$
(b) the values of t at which the instantaneous velocity is zero
(c) the values of t at which the instantaneous velocity is either a maximum or a minimum
(d) the instantaneous velocity when $t = 3$ s.



◀ **Figure Ex-3**

(Anton 2.2)

23. Match the graphs of the functions shown in (a)–(f) with the graphs of their derivatives in (A)–(F).



Topic #2, 5: Memorize the Derivative Formulas

(Anton 2.4) Product Rule

5–20 Find $f'(x)$.

5. $f(x) = (3x^2 + 6)\left(2x - \frac{1}{4}\right)$

6. $f(x) = (2 - x - 3x^3)(7 + x^5)$

7. $f(x) = (x^3 + 7x^2 - 8)(2x^{-3} + x^{-4})$

8. $f(x) = \left(\dfrac{1}{x} + \dfrac{1}{x^2}\right)(3x^3 + 27)$

Quotient Rule

21–24 Find $dy/dx|_{x=1}$.

21. $y = \dfrac{2x - 1}{x + 3}$

22. $y = \dfrac{4x + 1}{x^2 - 5}$

(Anton Ch. 2 Review) Product, Quotient, Chain Rules

29–32 Find $f'(x)$.

29. (a) $f(x) = x^8 - 3\sqrt{x} + 5x^{-3}$

(b) $f(x) = (2x + 1)^{101}(5x^2 - 7)$

30. (a) $f(x) = \sin x + 2\cos^3 x$

31. (a) $f(x) = \sqrt{3x + 1}(x - 1)^2$

(b) $f(x) = \left(\dfrac{3x + 1}{x^2}\right)^3$

(b) $f(x) = \dfrac{1}{2x + \sin^3 x}$

(Anton 3.2)

1–26 Find dy/dx. ■

1. $y = \ln 5x$

7. $y = \ln\left(\dfrac{x}{1+x^2}\right)$

19. $y = \ln(\ln x)$

25. $y = \log(\sin^2 x)$

(Anton 3.3)

15–26 Find dy/dx. ■

15. $y = e^{7x}$

17. $y = x^3 e^x$

Topic # 3: Memorize the antiderivative formulas (Anton 5.2)

2. In each part, confirm that the stated formula is correct by differentiating.

(a) $\displaystyle\int x \sin x \, dx = \sin x - x \cos x + C$

5–8 Find the derivative and state a corresponding integration formula. ■

7. $\dfrac{d}{dx}[\sin(2\sqrt{x})]$

11–14 Evaluate each integral by applying Theorem 5.2.3 and Formula 2 in Table 5.2.1 appropriately. ■

11. $\displaystyle\int \left[5x + \frac{2}{3x^5}\right] dx$ 1

13. $\displaystyle\int [x^{-3} - 3x^{1/4} + 8x^2] \, dx$

14. $\displaystyle\int \left[\frac{10}{y^{3/4}} - \sqrt[3]{y} + \frac{4}{\sqrt{y}}\right] dy$

21. $\displaystyle\int \left[\frac{2}{x} + 3e^x\right] dx$

Topic #4: Definite Integral as Area and the Fundamental Theorem

(Anton 5.5)

Draw a picture and use geometric shape formulas.

18. In each part, evaluate the integral, given that

$$f(x) = \begin{cases} 2x, & x \le 1 \\ 2, & x > 1 \end{cases}$$

(a) $\int_0^1 f(x)\,dx$ (b) $\int_{-1}^1 f(x)\,dx$

(c) $\int_1^{10} f(x)\,dx$ (d) $\int_{1/2}^5 f(x)\,dx$

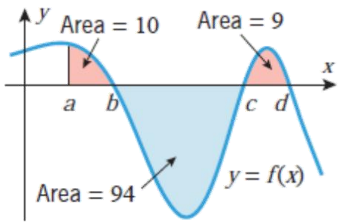
FOCUS ON CONCEPTS

19–20 Use the areas shown in the figure to find

(a) $\int_a^b f(x)\,dx$ (b) $\int_b^c f(x)\,dx$

(c) $\int_a^c f(x)\,dx$ (d) $\int_a^d f(x)\,dx$. ■

20.



21. Find $\int_{-1}^2 [f(x) + 2g(x)]\,dx$ if

$$\int_{-1}^2 f(x)\,dx = 5 \quad \text{and} \quad \int_{-1}^2 g(x)\,dx = -3$$

(Anton 5.6)

Use the Fundamental Theorem of Calculus to find the following areas:

17. $\int_4^9 2x\sqrt{x}\,dx$

19. $\int_{-\pi/2}^{\pi/2} \sin\theta\,d\theta$

21. $\int_{-\pi/4}^{\pi/4} \cos x\,dx$

23. $\int_{\ln 2}^3 5e^x\,dx$

24. $\int_{1/2}^1 \frac{1}{2x}\,dx$

Topic #6: u-substitution for Integrals.

(Anton 5.3)

15–56 Evaluate the integrals using appropriate substitutions.

Show u = , du= for at least 2 problems. If you become comfortable, you can skip explicitly showing u and just balance out constants as shown in the Tutorial on page 4.

15. $\int (4x-3)^9\,dx$

17. $\int \sin 7x\,dx$

29. $\int \frac{x^3}{(5x^4+2)^3}\,dx$

31. $\int e^{\sin x}\cos x\,dx$

33. $\int x^2 e^{-2x^3}\,dx$

~~**35.**~~ $\cancel{\int \frac{e^x}{1+e^{2x}}\,dx}$ *bonus

37. $\int \frac{\sin(5/x)}{x^2}\,dx$

~~**39.**~~ $\cancel{\int \cos^4 3t \sin 3t\,dt}$ *bonus

(Anton 5.9)

13. $\int_{-2}^{-1} \frac{x}{(x^2+2)^3}\,dx$

15. $\int_{-\ln 3}^{\ln 3} \frac{e^x}{e^x+4}\,dx$

Topic #10: Differential Equations

(Anton 5.2)

▶ **Example 6** Solve the initial-value problem

$$\frac{dy}{dx} = \cos x, \quad y(0) = 1$$

(Anton 8.2)

▶ **Example 2** Solve the initial-value problem

$$(4y - \cos y)\frac{dy}{dx} - 3x^2 = 0, \quad y(1) = 2\pi$$

Obtain an equation with x, y and constants. You do not need to isolate y.

a) Use the indefinite integral method.

b) Use the definite integral method.

Topic #11: Slice, Approximate, Integrate.

A right circular cylinder of radius R and height H is completely filled with water. Find the work needed to pump all the water out of the cylinder. Use density ρ and gravitational constant g.

Hint: slice = circular disc of thickness Δy. $\Delta W = \Delta mgh$. where h depends on y.

(Barron's #3.45)

After a volcano erupts, pieces can be found scattered around the center of the blast. The density of volcano fragments lying x meters from the point of eruption is given by

$N(x) = \dfrac{2x}{1+x^{3/2}}$ fragments per square meter. How many fragments will be found within 20 meters of the point where the volcano exploded? (Set up the definite integral. You can't evaluate it easily by hand, so use a graphing calculator.)

(A) 13 (B) 278 (C) 556 (D) 712 (E) 4383