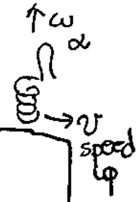
APB Giancoli   Ch8 Rotational Motion

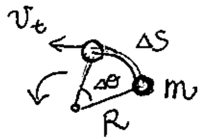
Course Description 4%
- ✓ Uniform Circular
- ✓ Torque & rotational statics
- × Rotational dynamics / kinematics
- × angular momentum / conservation

τ ≙ ?
τ from gravity
conditions of transl & rotational equil.



8.1 (optional) Rotational Kinematics

Rigid Body



$s = R\theta$

$A = \frac{1}{2}R^2\theta$

$$\omega = \frac{d\theta}{dt} \quad rad/s$$

$$\alpha = \frac{d\omega}{dt} \quad rad/s^2 = \frac{d^2\theta}{dt^2}$$

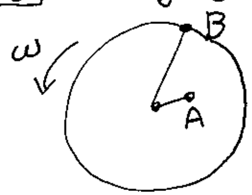
$$v_t = \lim_{\Delta t \to 0} \frac{\Delta s}{\Delta t} = \boxed{R\omega} \quad m/s$$

$$a_t = \lim_{\Delta t \to 0} \frac{\Delta v_t}{\Delta t} = \frac{d^2 s}{dt^2} = \frac{r\,d^2\theta}{dt^2} = \boxed{R\alpha} \quad m/s^2$$

$$a_c = \boxed{\frac{v_t^2}{R} = r\omega^2} \quad m/s^2$$

$$\vec{a}_{Total} = \vec{a}_t + \vec{a}_r$$

ex3 Merry-go-round / carousel

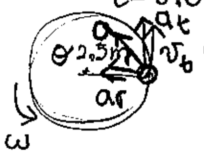


- Which has bigger v_t? B $v_t = R\omega$
- Which has bigger ω? same

ex4~5 Carousel initially at rest $t=0$

$\alpha = 0.060\ rad/s^2$ constant

$t = 8.0\,s$ a) $\omega = ?$ b) $v_t = ?$ c) a_t? d) a_r? e) a_{total}? f) Sketch & θ all



a) $\omega = \alpha t = 0.48\ rad/s$

b) linear velocity $v_t = r\omega = 1.2\ m/s$

c) $r\alpha = 0.15\ m/s^2$

d) $r\omega^2$ $0.58\ m/s^2$

e) $\sqrt{a_t^2 + a_r^2}$ $0.60\ m/s^2$

$\theta = \tan^{-1}\left(\frac{a_t}{a_r}\right) = 0.25\ rad$

8.2 (optional)

Kinematic eqns.

linear	rotational
a = constant	α = constant
x, v	θ, ω
$v_f = v_0 + at$	$\omega_f = \omega_0 + \alpha t$
$\Delta x = v_0 t + \frac{1}{2}at^2$	$\Delta\theta = \omega_0 t + \frac{1}{2}\alpha t^2$
$v_f^2 = v_0^2 + 2a\Delta x$	$\omega_f^2 = \omega_0^2 + 2\alpha\Delta\theta$
$v_{avg} = \frac{\Delta x}{\Delta t} = \frac{v_0 + v_f}{2}$	$\omega_{avg} = \frac{\Delta\theta}{\Delta t} = \frac{\omega_0 + \omega_f}{2}$

ex6 Centrifuge

rest to 20000 rpm in 30s

a) $\bar{\alpha}$? b) how many revolutions has it turned assuming α = constant

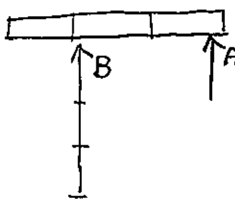
a) $$\bar{\alpha} = \frac{\Delta\omega}{\Delta t} = \frac{2\times10^4}{30\,s} \times \left(\frac{2\pi\ rad}{60\ s}\right) \approx \frac{2\times6\times10^3}{3\times6\ 10} = \approx 70\ rad/s^2$$

b) $$\Delta\theta = \cancel{\omega_0 t} + \frac{1}{2}\alpha t^2 = \frac{70}{2}\times30^2\ rad \times \frac{1\ rev}{2\pi\ rad} \approx \boxed{5.0\times10^3\ rev}$$

8-3 Skip. Bonus ex7 HW

8-4 Torque ✪ Mandatory

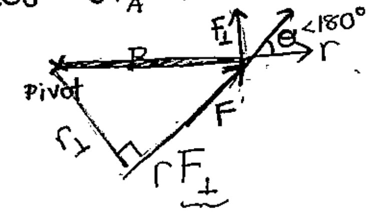
Open a door



← won't rotate

- F_A is better at rotating
- F_B would need × $3F_A$ to have same α

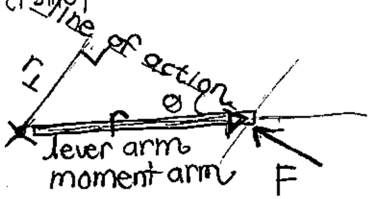
* rotates better if r big, F direction right · $\alpha \sim \tau$

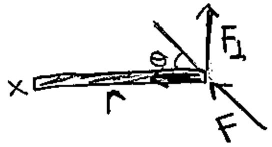


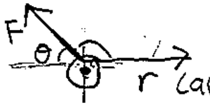
$$\boxed{\vec{\tau} = \vec{r} \times \vec{F} \quad \tau = rF\sin\theta}$$

3 ways $= r_\perp F$ ($r\sin\theta$) $= rF_\perp$ ($F\sin\theta$) ✶

N·m
NOT Joule
(Farther, F small, R big, Δs big) work same





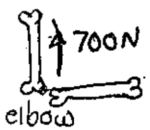


r (away from pivot)

$\tau_{out\ of\ page}$, tends to turn counterclockwise

ex8 Biceps Torque

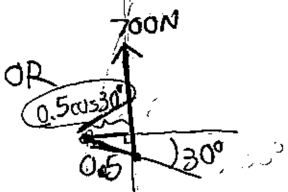
a)



$\tau = 700 \times 0.05$
$= 35$ Nm

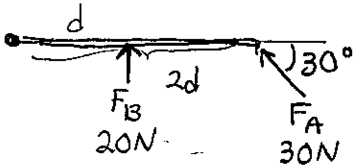
b)

OR



$\tau = 700\cos 30° \times 0.05 \approx 30$ Nm ($\cos 30° = \frac{\sqrt{3}}{2}$)

exB



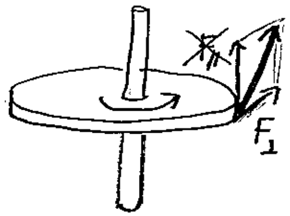
Which has greater torque?

$\tau_B = F_B d$

$\tau_A = F_A \sin 30° \times 2d$

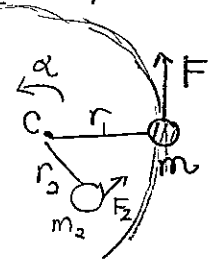
same

* Force won't tilt axis.
We assume rigid body, fixed axis



(we only consider in the plane of rotation)

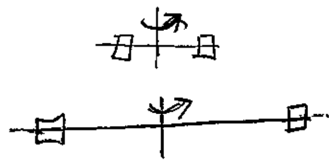
8.5 (Optional) Rotational Dynamics Intro.



tangentially
$\Sigma F = ma$
$F = m_1 r_1 \alpha$
$rF = mr^2\alpha$
$\tau = I\alpha$

system: apply to each
$\tau_1 = m_1 r_1^2 \alpha$
$+\tau_2 \quad +m_2 r_2^2 \alpha$
⋮
$$\boxed{\Sigma \vec{\tau} = I\vec{\alpha}}$$

- I = moment of inertia $\equiv \boxed{\Sigma m_i r_i^2}$
  rotational inertia
  kg·m²



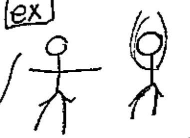
higher rotational inertia
more difficult to spin

- Figure 8-20 : Moments for uniform shapes
- HW : read & do ex 9, 10, 11
- Bonus : 8-7 on test
  8-8 or
  8-9 wherefrom

Linear (m)	Rotational (I)
$K = \frac{1}{2}mv^2$	$K = \frac{1}{2}I\omega^2$
$\vec{p} = m\vec{v}$	$\vec{L} = I\vec{\omega}$
$F = \frac{\Delta p}{\Delta t}$	$\tau = \frac{\Delta L}{\Delta t}$

See P. 219

conservation
ex



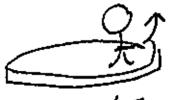
$I\omega = I\omega$
slow fast

ex 16 Youtube - try it?
bike wheel

- 

at rest

$L_i = 0$



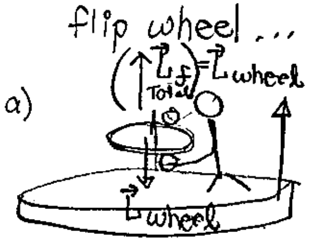
(Push back
Torque equal & opposite)
Fr

$I_{person}\vec{\omega}_p = -I_{table}\vec{\omega}_{table}$
cancel

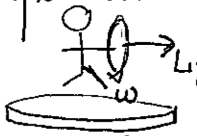
- 

flip wheel ...

a) $\vec{L}_f{}_{Total} = \vec{L}_{wheel} + \vec{L}_{table}$ ($2 L_{wheel}$)



b) L_{total} move so ...

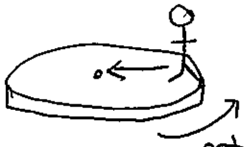


then

(← friction cancels it)

(b) rotate slower than before

- exF



Walk to center, What happens?

- rotates faster

$I_{before}\omega = I_{after}\,\omega$

(APC too 10.7)

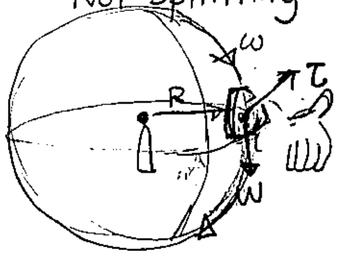
- Gyroscope & Precession

$$\vec{\tau} = \frac{\Delta \vec{L}}{\Delta t}$$

$$\vec{\tau}\,\Delta t = \Delta \vec{L}$$

- see Video:
Torque the wheel, angular momentum catches up with it!

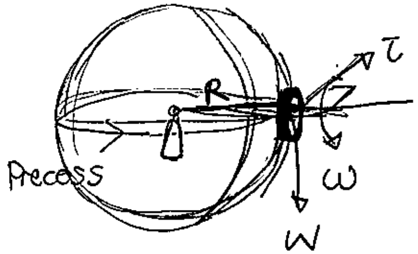
Not Spinning



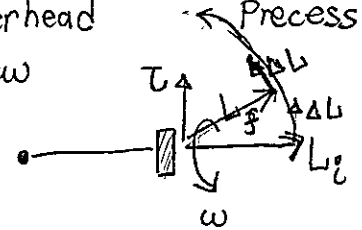
τ tangent to sphere ⇒ $\vec{\Delta L} = \vec{L}_f - \vec{L}_i$
↑ same dir as $\vec{\tau}$ 0

∴ drops

Spinning



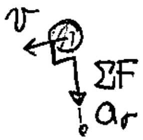
overhead View



★ catch up with $\vec{\tau}$
$\Delta\vec{L}$ in dir of $\vec{\tau}$
catch up to torque
$\vec{\tau} \perp \vec{\omega}$
axis of rotation

$\Delta\vec{L} \perp \vec{L}$
axis turns

Linear:
$\Sigma F \perp \vec{v}$
$\Delta\vec{p} \perp \vec{p}$
uniform circular motion



- Video

$\Sigma\tau = 0$

Ch8 HW Rotational Motion

C

8.1 } kinematics
8.2 } α=const. ⇒ Probs

Questions 5, 7
Misconc 2, 3, 4, 5, 6, 10, 11, 12, 13
Probs 3, 10, 29, 81,

8.3 skip. Bonus ex7

8.4 Torque ⇒ Probs.

8.5 Rotational Dynamics ex9 (10, 11)

Bonus: Sections 8.7 ~ 8.9 read & do examples ex 12, 13, 14, 15, 16
(Bonus on test or where from)

Ch9 Static Equilibrium

- 9.1, 9.2, 9.4 Probs 6, 8, 16, 17, 18, 37. Search/Learn 5
  Nice too 19, 27, 28, 30, 36, 60, 62, 66, 69; search 3
- elastigity & fractue Bonus * 9.5 ~ 9.7 ex10 ~ 13

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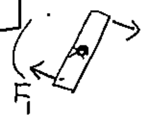
Ch 9 Static Equilibrium 9.1, 9.2, 9.4

(elasticity & fracture * 9.5 ~ 9.7) Bonus

- Architecture, balance, bone health
- AP tests Static equilib w/ torque

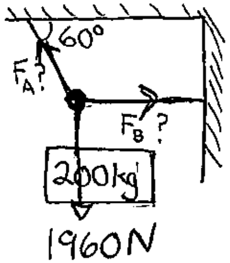
- Conditions for Equilibrium

① $\Sigma \vec{F} = \vec{0}$	not moving in 1st place, will stay still
② $\Sigma \vec{\tau} = \vec{0}$	



($\Sigma \vec{\tau} \neq 0$, $\Sigma \vec{F} = 0$)

ex2 (Skip. Like summer HW) Review in HW



★ Draw free-body diagram!

$\Sigma F_x = 0$

$\Sigma F_y = 0$

etc.

Solving Tips

① ★ free-body diagram at point of action
- W at CM
- Choose a dir (if value < 0, just means opp)

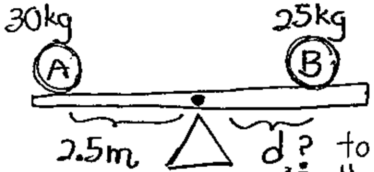
② Coordinates $\Sigma F_x = 0$, $\Sigma F_y = 0$

- our probs: all forces in same plane

③ - choose an axis ⊥ plane (arbitrary!) PUt torques consistent $\Sigma \tau = 0$ (+ counterclock, − other way)

④ 3 eqns

ex4

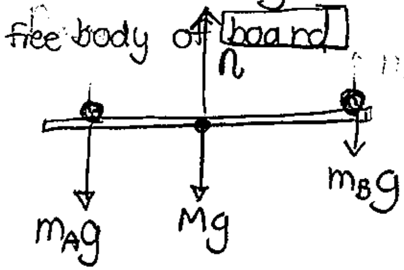


ⓑ d? to balance the seesaw? (further)

Board $M = 4.0$ kg

ⓒ normal of pivot on board?

a) free body of board



ⓑ $\Sigma F_x = 0$ OK

- ($\Sigma F_y = 0$ not used yet)
- $\Sigma \tau = 0$

$m_A g d_A = m_B g d_B$

$d_B = \frac{30}{25} \times 2.5 = \boxed{3\,m}$

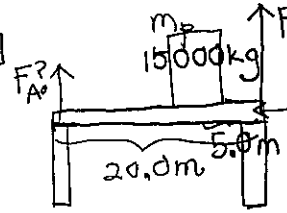
ⓒ $\Sigma F_y = 0$

$n = (m_A + M + m_B) g$

$\approx (30 + 4 + 25) \times 10$

≈ 590 N

ex5



$m_{beam} = 1500$ kg

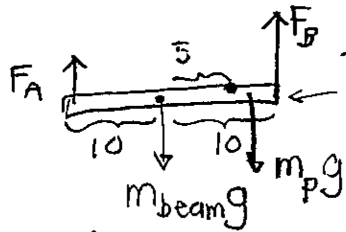
NO CALC ⤳ in terms of g

$\Sigma F_x = 0$ of course all up/down

$\Sigma F_y = 0$ (~~don't~~ need)

- $\Sigma \tau = 0$

$F_B 10 = F_A 10 + m_p g 5$



free-body of object NOT moving

- $\Sigma F_y = 0$

$F_A + F_B = (m_{beam} + m_p) g$

$-10 F_A + 10 F_B = m_p g 5$

$20 F_B = (10 m_{Beam} + 15 m_p) g$

$F_B = \frac{1}{4}(2 \times 1500 + 3 \times 15000) \times 10$

$= 18 \times 1500\, g = \boxed{12000\, g}$

$F_A = (m_B + m_p) g - F_B$

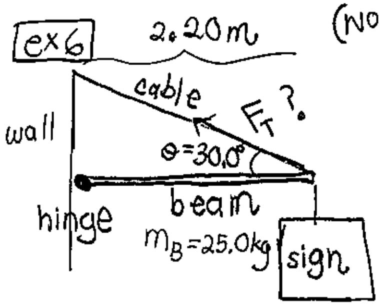
$= 1500(1 + 10) g - 12000\, g$

$= 100 g (11 \times 15 - 120)$

$= 1500 g (11 \times 1 - 8) = \boxed{4500 g}$

- cable supports force along its length only (or will bend)
- hinge / rigid device: force in any direction

ex6 (NO CALC in g)



M=28.0kg

$\vec{F}_{Hinge\ on\ beam}$?

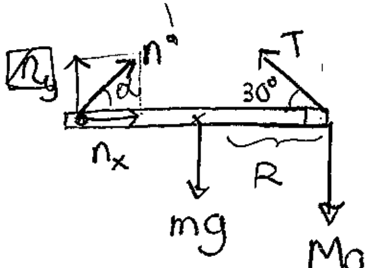
(NO CALC. write in g)

$\star\ \Sigma F_x = 0$ ① $n_x = T\cos 30°$

$\Sigma F_y = 0$ ② $n_y + T\sin 30° = (m+M)g$

$\Sigma \tau = 0$ ③ $T\sin 30°\, R = \sout{n\sin\alpha}\, R + MgR$ (n_y $+mg\frac{R}{2}$)

Beam free-body



$\Downarrow$ $[(m+M)g - n_y]R$

$T = 2(M + \frac{m}{2})g$

$= (2M+m)g$

$= (56+25)g$

$= \boxed{81g}$

$n_y = \frac{(m+M)g - Mg}{2} = \frac{mg}{2}$

$\approx \frac{250}{2} = \boxed{12.5g} \approx 123\,N$

$\Downarrow$

② $\frac{T}{2} = (m+M)g - n_y$

$= (m+M)g - \frac{mg}{2}$

$= (\frac{m}{2} + M)g$

$T = (m+2M)g \approx \boxed{81kg} \approx 794\,N$

25+56

① $n_x = T\frac{\sqrt{3}}{2}$

$= \boxed{40.5\sqrt{3}\,g}$

$\approx 687\,N$

$n_y = (m+M)g - \frac{T}{2}$

$= (53 - \frac{81}{2})g$

$= \boxed{12.5g}$

ex7 Ladder (more difficult. was in APC book & exam sample)

NO CALC (in terms of g)

5.0m ladder

at rest 4.0m above floor

m = 12.0kg ladder

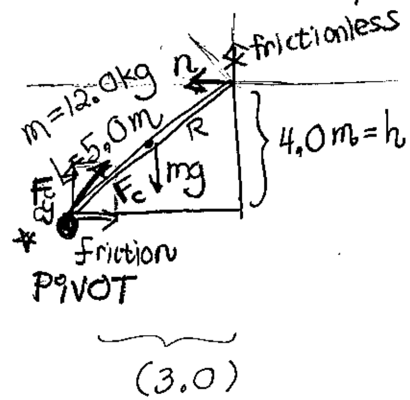
wall frictionless

floor not

forces on ladder by floor & wall?

3 unknowns

3 eqns



① $\Sigma F_x = 0$ $F_{cx} = n$

② $\Sigma F_y = 0$ $F_{cy} = mg$

③ $\Sigma \tau = 0$ $n \cdot h + F_{cx}\,0 = F_{cy} \times 0 + mg\,1.5$

3 eqns

3 unknowns

② $F_{cy} = \boxed{12g}$

③ $nh = mg1.5$

$n = \frac{12 \times \frac{3}{2}}{4} g = \frac{9}{2} g = \boxed{4.5g} = 44N$

① $F_{cx} = \boxed{4.5g}$

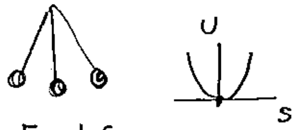
$\Rightarrow F_c = \boxed{\sqrt{(\frac{9}{2})^2 + 12^2}\ g} = 126N \approx 130N$

$\angle\theta$ $\theta = \tan^{-1}\left(\frac{12}{9/2}\right) = \tan^{-1}\left(\frac{24}{9}\right) \approx \boxed{69°}$

Bonus : examples

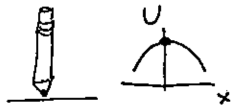
9-4 Stability & Balance

• Stable equilibrium



$F = -ks$

Unstable

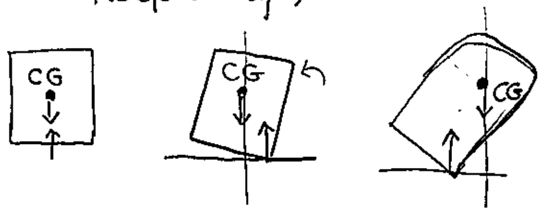


Torque

Neutral

tiny base

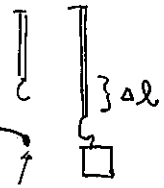
• Stable equilibrium or balance
- when CG below point of support (pendulum)
- when CG above base of support & vertical line in base (torque from normal won't topple over but keep it up)

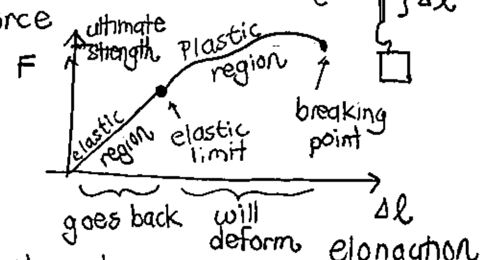


9-5

★ Hooke's Law $F = k\Delta l$

for small force linear





★ Practical, not Hooke's Law but

$$\Delta l = \frac{1}{E}\frac{F}{A} l_0$$

l_0: more force, long, stretch more

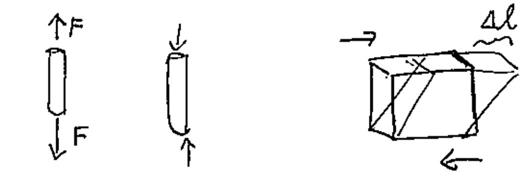
A: more area, less stretch

$$\Rightarrow F = E \frac{A}{l_0} \Delta l$$

elastic modulus
Young's modulus

$$E = \frac{F/A}{\Delta l / l_0} = \frac{\text{stress}}{\text{strain}}$$ } proportional in elastic linear region

[N/m^2]



3 types of stress

Shear modulus G $\quad \Delta l = \frac{1}{G}\frac{F}{A} l_0$

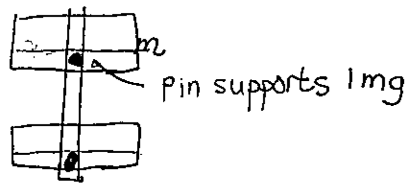
Bulk modulus B $\quad \Delta V = \frac{-1}{B} \Delta P \, V_0$

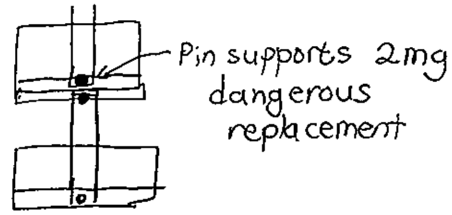
under water

decreases with increased pressure

9-6 Fracture : too much Δl, breaks

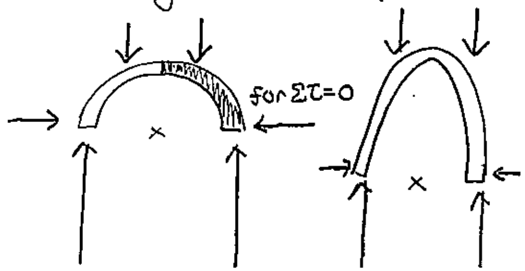
max F/A or stress



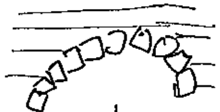


9-7 A pointed arch, dome needs less buttressing

Longer lever arm, more torque to hold it up



columns cannot span much space
stones compress well but easily break from tension/shear



arch: mostly
• compression
• Pointed more vertical less horizontal supp needed