APB Giancoli

Ch7 Linear Momentum

7.1 ~ 7.3

① momentum $\boxed{\vec{p} = m\vec{v}}$ (m) $\rightarrow v$

inertia of motion

$\boxed{1\ kg\cdot m/s = 1\ N\cdot s}$

② 7-1 ~ 7-3

$\Sigma\vec{F} = m\vec{a}$

$= m\dfrac{d\vec{v}}{dt} = \dfrac{d}{dt}(m\vec{v})$

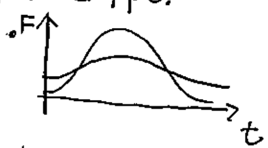
$\boxed{\Sigma\vec{F} = \dfrac{d\vec{p}}{dt}}$ object & system

$\left(\Sigma\vec{F}_1 + \Sigma\vec{F}_2 + \ldots = \dfrac{d\vec{p}_1}{dt} + \dfrac{d\vec{p}_2}{dt} + \ldots = \dfrac{d}{dt}\vec{P}_{total}\right)$

③ Impulse-Momentum Thm

$\boxed{\underbrace{\Sigma\vec{F}\,\Delta t}_{\vec{J}} = \Delta\vec{p}}$

- CISE ppt.

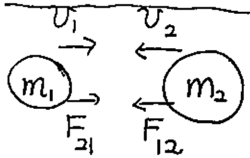


$\boxed{\Delta\vec{p} = \text{area under } F\text{-}t \text{ curve}}$

tennis vs golf, same $0 \sim v$ (area same)
which is which?

- See ex1~2
- See ex6
- CISE PB 26 Buick, Mini-Cooper/ice-boat

④



$\left(\Sigma\vec{F}_{system}\,\Delta t = \Delta\vec{p}_{tot}\right)$

$\Sigma\vec{F}_1 + \Sigma\vec{F}_2 = \dfrac{\Delta\vec{p}}{\Delta t}$

3rd Law

$0 = \dfrac{\Delta\vec{p}}{\Delta t}$

$\Delta\vec{p} = 0$

$\vec{p}_i = \vec{p}_f$
- Conservation of momentum
- condition: $\Sigma\vec{F}_{external} = 0$

(See ex3~5)

contact forces in collision make others negligible (gravity/friction)

7.1 ex1 Tennis champ

$m = 0.060$ kg

$v = 55$ m/s

$\Delta t = 4$ millisec $= 0.004$ s

a) $\Delta\vec{p}$

b) $F_{avg} = ?$

c) can lift 60kg person?

d) compare to other forces on ball

a) $\Delta p = mv - 0 = \boxed{3.3\ N\cdot s}$

b) $F_{avg}\,\Delta t = \Delta p \Rightarrow F_{avg} = \boxed{830\ N}$

c) yes $m_{pers}g \approx 600N$

d) $m_{ball}\,g \approx 0.6N$

★ Contact forces make others negligible! ★

ex2

$F_{water\ on\ car}$?

$F\Delta t = \Delta p$

$F = \dfrac{\Delta p}{\Delta t} = \dfrac{m\Delta v}{\Delta t} = \dfrac{1.5kg}{1s}\cdot 20\,m/s$ ($1.5kg \times 20\,m/s$)

a) $= \boxed{30N}$ left

b) If water splashes back $\Rightarrow$ (F bigger)

ex6

Karate blow breaks board & bounce back

$v = 10$ m/s ($m_{hand} \approx 1kg$, $\Delta t \approx 2ms$, $\Delta x \approx 1cm$)

Estimate a) Impulse?

b) Force?

a) Impulse $\Delta p = m \times 20 = \boxed{20\ N\cdot s}$

b) $F\Delta t = \Delta p$

$F = \dfrac{20}{0.002} = \boxed{10\ kN}$

ex3 (inelastic)

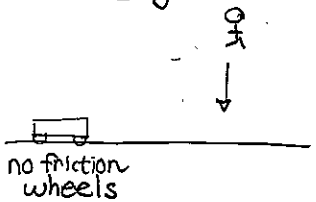
10 000 kg 10 000 kg

$$p_i = p_f$$

$$m_A v_A + 0 = (m_A + m_B) v_f$$

$$v_f = \frac{10 \times 24}{20} = \boxed{12.0 \text{ m/s}}$$

ex4 CISE frog NTQ

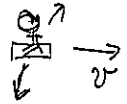


a) Land, sled speed ↓

(M bigger, ★momentum same horizontally ($\Sigma F_{x\,ext.} = 0$))

b) Susan falls off later sideways

speed same!

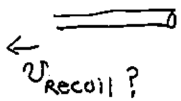


c) impulse on frog to speed up vs. impulse on skateboard to slow down

(F equal & opposite. same!)

ex5 Rifle recoil

$M_R = 5.0$ kg $m_B = 0.020$ kg



$\rightarrow v_B = 620$ m/s

a) v_R ?

b) Which more force? same

c) Impulse same Δp same

d) acceleration? bullet

$$p_i = p_f$$

$$0 = m_B v_B + M_R v_R$$

$$M_R v_R = -m_B v_B$$

$$v_R = \boxed{-2.5 \text{ m/s}}$$

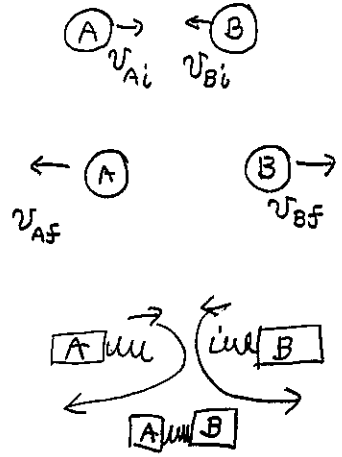
(left

7.5 ~ 7.7 Types of Collisions

• All ① $\boxed{\vec{p}_i = \vec{p}_f}$ ($\Sigma \vec{F}_{ext} = 0$)

I. elastic – ② $\boxed{K_i = K_f \quad \text{OR} \quad \vec{v}_{Ai} - \vec{v}_{Bi} = -(\vec{v}_{Af} - \vec{v}_{Bf})}$

relative velocities before & after same magnitude



• billiard ball

• no deformation / heat / friction

★ all conservative forces

$$W_{tot\ system} = \Delta K$$

$\underbrace{-\Delta U}_{0}$ since position rebound

★ Same mass... switch velocities!

★ different mass...

Proof: See APC

$$\boxed{\begin{aligned} v_{Af} &= \frac{m_A - m_B}{m_A + m_B} v_{Ai} + \frac{2m_B}{m_A + m_B} v_{Bi} \\ v_{Bf} &= \frac{2m_A}{m_A + m_B} v_{Ai} + \frac{m_B - m_A}{m_A + m_B} v_{Bi} \end{aligned}}$$

sometimes useful

APC See neutron carbon moderator

Given: m_A, m_B, $\vec{v}_{Ai}$, $\vec{v}_{Bi}$

Find: $\vec{v}_{Af}$, $\vec{v}_{Bf}$

• Unknowns: 4 x, y

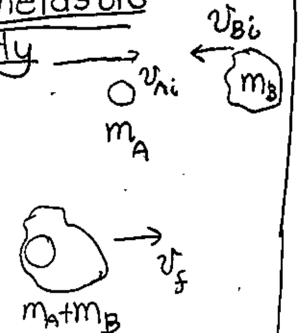
• Eqns: 3 ① $p_{xi} = p_{xf}$

② $p_{yi} = p_{yf}$

③ $K_i = K_f$

need know a mag or direction for only 3 unknowns

II. perfectly inelastic



Q1) $p_i = p_f$

$$\boxed{m_A v_{Ai} + m_B v_{Bi} = (m_A + m_B) v_f}$$

Q2) Show $K_f < K_i$ always

$$K_i = \frac{1}{2} m_A v_{Ai}^2 + \frac{1}{2} m_B v_{Bi}^2$$

$$K_f = \frac{1}{2}(m_A + m_B)\left(\frac{m_A v_{Ai} + m_B v_{Bi}}{m_A + m_B}\right)^2$$

$$= \frac{1}{2} \frac{m_A^2 v_{Ai}^2 + 2 m_A m_B v_{Ai} v_{Bi} + m_B^2 v_{Bi}^2}{m_A + m_B}$$

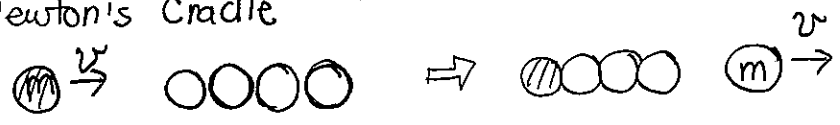
$$K_f < \underbrace{\frac{1}{2} \frac{m_A^2 v_{Ai}^2}{m_A} + \frac{1}{2} \frac{m_B^2 v_{Bi}^2}{m_B}}_{K_i}$$

inelastic: loses some KE but bounces a bit

APC: classify TYU

<u>elastic collisions</u>

① Newton's Cradle

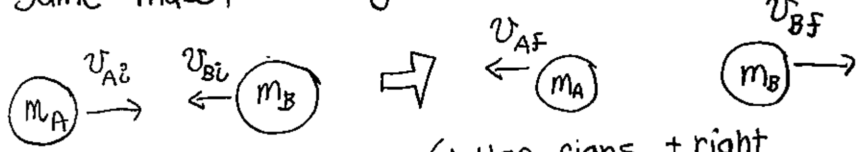


$mv \quad = \quad mv$ Not $(2m)(\frac{1}{2}v)$

$\frac{1}{2}mv^2 \quad = \quad \frac{1}{2}mv^2$ match

② Prove Same mass, exchange velocities

HW



(★ Use signs + right include within v values)

(see CW③ first)

$p_i = p_f$

$K_i = K_f$

$\cancel{m} v_{Ai} + \cancel{m_B} v_{Bi} = \cancel{m} v_{Af} + \cancel{m} v_{Bf}$

$v_{Ai} - v_{Bi} = - v_{Af} + v_{Bf}$ ②

$2v_{Ai} + 0 = 2\, v_{Bf}$

$\therefore v_{Ai} = v_{Bf}$

$v_{Af} = \cancel{v_{Bf}} - \cancel{v_{Ai}} + v_{Bi}$

OR ① $\begin{cases} p: \cancel{m} v_{Ai} + \cancel{m} v_{Bi} = \cancel{m} v_{Af} + \cancel{m} v_{Bf} \\ K: \cancel{\tfrac{1}{2}m} v_{Ai}^2 + \cancel{\tfrac{1}{2}m} v_{Bi}^2 = \cancel{\tfrac{1}{2}m} v_{Af}^2 + \cancel{\tfrac{1}{2}m} v_{Bf}^2 \end{cases}$

$\begin{cases} v_{Ai} - v_{Af} = v_{Bf} - v_{Bi} \\ (v_{Ai} - v_{Af})(v_{Ai} + v_{Af}) = (v_{Bf} - v_{Bi})(v_{Bf} + v_{Bi}) \end{cases}$

$v_{Ai} + v_{Af} = v_{Bf} + v_{Bi}$

③ Prove eqns on P. 2

CW

a) $v_{Ai} - v_{Bi} = v_{Bf} - v_{Af}$

b) $\begin{cases} v_{Af} = \dfrac{m_A - m_B}{m_A + m_B} v_{Ai} + \dfrac{2m_B}{m_A + m_B} v_{Bi} \\ v_{Bf} = \dfrac{2m_A}{m_A + m_B} v_{Ai} + \dfrac{m_B - m_A}{m_A + m_B} v_{Bf} \end{cases}$

(Hint: $\begin{bmatrix} a & b \\ c & d \end{bmatrix}^{-1} = \dfrac{\begin{bmatrix} d & -b \\ -c & a \end{bmatrix}}{ad - bc}$)

$p_i = p_f$

$K_i = K_f$

$\begin{cases} m_A v_{Ai} + m_B v_{Bi} = m_A v_{Af} + m_B v_{Bf} \\ \cancel{\tfrac{1}{2}} m_A v_{Ai}^2 + \cancel{\tfrac{1}{2}} m_B v_{Bi}^2 = \cancel{\tfrac{1}{2}} m_A v_{Af}^2 + \cancel{\tfrac{1}{2}} m_B v_{Bf}^2 \end{cases}$

$\begin{cases} m_A (v_{Ai} - v_{Af}) = m_B (v_{Bf} - v_{Bi}) \\ m_A (v_{Ai} - v_{Af})(v_{Ai} + v_{Af}) = m_B (v_{Bf} - v_{Bi})(v_{Bf} + v_{Bi}) \end{cases}$

$\therefore$ ⓐ $v_{Ai} + v_{Af} = v_{Bf} + v_{Bi}$

$\begin{cases} m_A (v_{Ai} - v_{Af}) = m_B (v_{Bf} - v_{Bi}) \\ m_A [\; v_{Ai} + v_{Af} = v_{Bf} + v_{Bi} \;] \end{cases}$ 2 eqns, 2 unknowns

OR $-m_A v_{Af} - m_B v_{Bf} = -m_A v_{Ai} - m_B v_{Bi}$

$v_{Af} - v_{Bf} = -v_{Ai} + v_{Bi}$

$\begin{bmatrix} -m_A & -m_B \\ 1 & -1 \end{bmatrix} \begin{bmatrix} v_{Af} \\ v_{Bf} \end{bmatrix} = \begin{bmatrix} \quad \\ \quad \end{bmatrix}$

$\begin{bmatrix} v_{Af} \\ v_{Bf} \end{bmatrix} = \dfrac{\begin{bmatrix} -1 & m_B \\ -1 & -m_A \end{bmatrix}}{m_A + m_B} \begin{bmatrix} -m_A v_{Ai} - m_B v_{Bi} \\ -v_{Ai} + v_{Bi} \end{bmatrix} = \dfrac{1}{m_A + m_B} \begin{bmatrix} m_A v_{Ai} + m_B v_{Bi} - m_B v_{Ai} + m_B v_{Bi} \\ m_A v_{Ai} + m_B v_{Bi} + m_A v_{Ai} - m_A v_{Bi} \end{bmatrix}$

$\therefore$ ⓑ

7.5 elastic collisions

- Carbon moderator vs. water moderator. See APC

ex relative velocities

cise applet

1kg $\xrightarrow{3\,m/s}$ 2kg $\xrightarrow{1\,m/s}$

$\frac{2}{3}\times 3 - \frac{1}{3}\times 1 = \frac{2}{3}\,m/s$

$-\frac{3}{3} + \frac{4}{3}\times 1 = \frac{1}{3}$

1kg $\xrightarrow{3\,m/s}$ 9kg $\xleftarrow{1\,m/s}$

$\frac{2}{10}\cdot 3 + \frac{8}{10}(-1) = 0.2\,m/s$

$-\frac{8}{10}\times 3 + \frac{18}{10}(-1)$

$= -42/10 = -4.2\,m/s$

ex Using the equations. No calculator.

Carbon vs. water moderator

$\frac{K_{nf}}{K_{ni}}$ (which takes energy away better per collision?)

a) neutron 1u $\xrightarrow{v_{ni}}$ carbon 12u At rest

$v_{nf} = \frac{-11}{13} v_{ni}$

$\vec{v}_{cf} = \frac{2}{13} v_{ni}$

$\frac{K_{nf}}{K_{ni}} = \left(\frac{11/13}{1}\right)^2 = 0.72$

b) neutron 1u $\xrightarrow{v_{ni}}$ 18u

$v_{nf} = -\frac{17}{18} v_{ni}$

$\frac{K_{nf}}{K_{ni}} = \left(\frac{17}{18}\right)^2$ closer to 1 than carbon

$\therefore$ water $\boxed{\text{worse}}$ so why water?

- To have neutron lose energy in collision use about same mass
- elastic baseball bat

m, M, $\vec{v}$

$K_m \uparrow \Leftarrow K_M \downarrow$

M imparts KE to m & m speeds up a lot!

ex8 NOT using the equations (HW: 1 with eqns v_{Af}, v_{Bf}; 1 w/o eqns)

NO CALC ($1u = 1.66\times 10^{-27}\,kg$)

neutron 1u $\longrightarrow$ $v_{ni} = 3.60\times 10^4\,m/s$; v_{nf}?

Helium 4.00u at rest; $\rightarrow v_{Hf}$?

$p_i = p_f$ $\quad 1v_{ni} + 0 = 1v_{nf} + 4v_{Hf}$

$K_i = K_f$ $\quad v_{ni} = v_{Hf} - v_{nf}$

$2v_{ni} = 5v_{Hf}$

$v_{Hf} = \frac{2}{5}(3.60\times \underbrace{10^4}_{2\times 10^3}) = \boxed{1.44\times 10^4\,m/s}$

$v_{nf} = v_{Hf} - v_{ni} = (1.44 - 3.6)\times 10^4$

$= \boxed{-2.16\times 10^4\,m/s}$

ex cise Applet Demo

- 9.0kg $\xrightarrow{10\,m/s}$ 1kg $\xrightarrow{3\,m/s}$

$\frac{18}{10}\times 10 + \frac{-8}{10}\times 3 = 15.6\,m/s$

$\vec{v}_f = \frac{8}{10}\times 10 + \frac{2}{10}\times 3 = 8.6\,m/s$

- 9.0kg $\xrightarrow{10.0\,m/s}$ 1kg $\xleftarrow{5.0\,m/s}$

$\frac{18}{10}\times 10 + \frac{-8}{10}\times(-5) = 22.0\,m/s$

$\frac{8}{10}\times 10 + \frac{2}{10}(-5)$

$= 7.0\,m/s$

- 4.0kg $\xrightarrow{10\,m/s}$ 1kg $\xleftarrow{-10\,m/s}$

$\frac{8}{5}\times 10 + \frac{-3}{5}(-10) = 22$

$\rightarrow \frac{3}{5}\times 10 + \frac{2}{5}(-10) = 2$

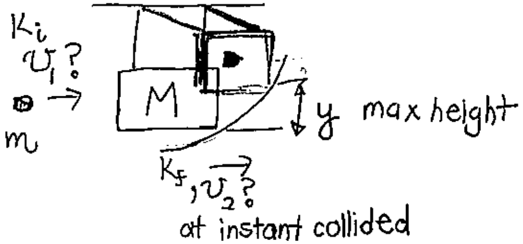
7.6 Inelastic Collisions (completely)

ex9 Ballistic Pendulum – See APC

(measures speed of bullet) a) Two parts

$y = 3.00\,cm$, $m_B = 5.00\,g$, $M = 2.00\,kg$ — Numbers

b) How much energy was converted to thermal energy?



a) expression in m, M, y

$v_1 = ?$

$v_2 = ?$

Collision

$m v_1 = (m+M) v_2$

$v_1 = \boxed{\frac{m+M}{m}\sqrt{2gy}}$

@. momentum conserved? yes. Why? $\Sigma F_{ext} = 0$

· energy conserved? No

why? $K_f < K_i$ (deformation) heat. friction eats up in splintering

$K_B \to$ OPE

K_{B+W} ↓ slower

Going up

$K_i \to U_g$

$\frac{1}{2}\cancel{(m+M)}v_2^2 = \cancel{(m+M)}gy$

$\boxed{v_2 = \sqrt{2gy}}$

@ mechanical energy conserved? Yes

why? only conservative force. gravity. Tension - no work

· momentum conserved? Yes

why? $(m+M)\ v \to 0$

$\Sigma F_{ext} \neq 0$ — Tension $\neq 0$, weight $\neq 0$ } don't cancel, decelerates

- block & bullet
- then also gravity, tension

(No calc)

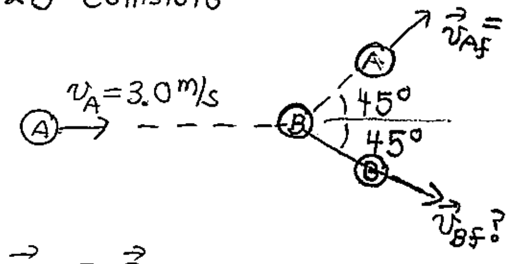
b) $K_i = \frac{1}{2}m_B v_1^2 = \frac{1}{2}\times 0.005 \times \left(\frac{2.005}{0.005}\right)^2 2\times 9.8\times 0.03$

$\approx \frac{40\times 3}{100}\cdot\frac{1000}{5} = \frac{6}{5}\ \frac{1000}{5} = \boxed{240\,J}$

$K_f = \frac{1}{2}(m+M)v_2^2 = \frac{1}{2}(m+M)\cdot 2gy = 2.005\times 9.8\times 0.03 \approx \boxed{0.6\,J}$

(7-7) 2D Collision

ex11



Speeds? same mass

$\vec{p}_i = \vec{p}_f$

$\langle m3, 0\rangle = \langle m v_{Af}\cos 45° + m v_{Bf}\cos 45°,\ \underbrace{m v_{Af}\sin 45° - m v_{Bf}\sin 45°}_{v_{Af} = + v_{Bf}}\rangle$

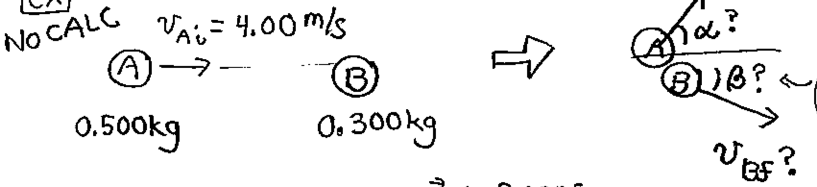
⇓

$\cancel{2} v_f \frac{\sqrt{2}}{\cancel{2}} = 3$

$v_f = \frac{3\sqrt{2}}{2} = \boxed{2.1\,m/s}$ both

APC 2D elastic collision

ex NO CALC



(use CALC at end for β only)

3 unknowns, 3 eqns $\vec{p} \leftarrow$ 2 eqns, $K \leftarrow$ 1 eqn

x: $0.5\times 4 + 0 = 0.5\times 2\cos\alpha + 0.3\times v_{Bf}\cos\beta$

y: $0 = 0.5\times 2\sin\alpha - 0.3\times v_{Bf}\sin\beta$

· $K_i = K_f$: $\frac{1}{2}0.5\times 4^2 = \frac{1}{2}0.5\times 2^2 + \frac{1}{2}0.3 v_{Bf}^2$

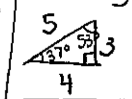
$v_{Bf}^2 = \frac{1}{3}(5\times 16 - 5\times 4) = 5\times\frac{12}{3} = 5\times 4$

$v_{Bf} = \boxed{2\sqrt{5}\ m/s}$

· x: $2 = \cos\alpha + \frac{3\sqrt{5}}{5}\cos\beta$

y: $0 = \sin\alpha - \frac{3\sqrt{5}}{5}\sin\beta$

$\cos\alpha = \frac{4}{5}$



$\boxed{\alpha \approx 37°}$

$\sin\beta = \frac{\sqrt{5}}{3}\cdot\frac{3}{5}$

$\beta = \sin^{-1}\left(\frac{1}{\sqrt{5}}\right) = \boxed{27°}$

$\sin^2\beta + \cos^2\beta = 1$

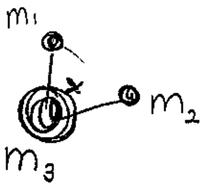
$\left(\frac{\sqrt{5}}{3}\sin\alpha\right)^2 + (2-\cos\alpha)^2\left(\frac{\sqrt{5}}{3}\right)^2 = 1$

$\sin^2\alpha + 4 - 4\cos\alpha + \cos^2\alpha = 9/5$

$5 - 4\cos\alpha = 9/5$

$4\cos\alpha = 16/5$

7-8 Center of Mass, 7-10



weighted average

$$x_{cm} \equiv \frac{\sum_i m_i x_i}{\sum_i m_i} = \frac{m_1}{m_1+\dots+m_N} x_1 + \frac{m_2}{m_1+\dots+m_N} x_2 + \dots$$

$$y_{cm} = \dots$$

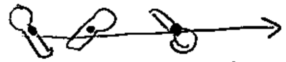
$$v_{cm} = \frac{m_1}{m_1+\dots+m_N} v_1 + \frac{m_2}{M} v_2 + \dots$$

$$a_{cm} = \frac{m_1}{M} a_1 + \frac{m_2}{M} a_2 + \dots$$

As if M concentrated at point $\vec{r}_{cm}$

$\vec{v}_{cm}$, $\vec{a}_{cm}$

- $\sum \vec{F}$ on that point particle
- Translational motion - show diagram



Where from?

$$\sum \vec{F}_1 + \sum \vec{F}_2 + \dots = m_1 \vec{a}_1 + m_2 \vec{a}_2 + \dots$$

$$= \underbrace{(m_1+\dots+m_N)}_{M}\left(\frac{m_1}{M}\vec{a}_1 + \frac{m_2}{M}\vec{a}_2 + \dots\right)$$

also

$$= \frac{\Delta \vec{p}_1}{\Delta t} + \frac{\Delta \vec{p}_2}{\Delta t} + \dots$$

$$= \frac{\Delta(\vec{p}_1 + \vec{p}_2 + \dots)}{\Delta t}$$

$$= \frac{\Delta \vec{p}_{total}}{\Delta t}$$

$$\therefore \boxed{\sum \vec{F}_{system} = M \vec{a}_{cm} = \frac{\Delta \vec{p}_{tot}}{\Delta t}}$$

$$\vec{p}_{tot} = m_1 \vec{v}_1 + m_2 \vec{v}_2 + \dots$$

$$= (m_1+\dots+m_N)\left(\frac{m_1}{M}\vec{v}_1 + \frac{m_2}{M}\vec{v}_2 + \dots\right)$$

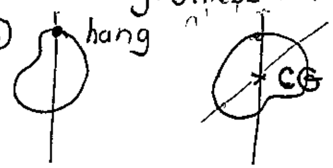
$$\boxed{\vec{p}_{tot} = M \vec{v}_{cm}}$$

OK for linear relations (but not moment of inertia $I = \sum m_i r_i^2 \neq M \vec{r}_{cm}^{\,2}$)

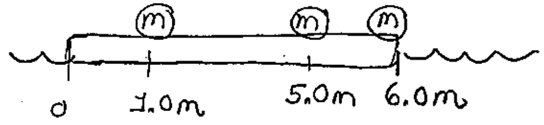
- How to find CM (same as center of Gravity. Unless mass so big, g varies)

① geometric center for uniform density

② hang

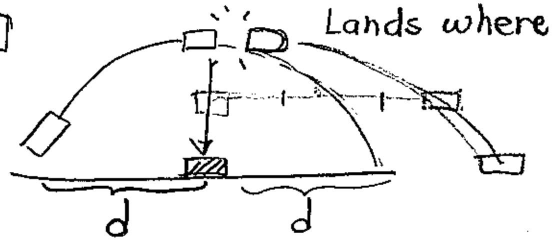


ex12



$$x_{cm} = ? = \frac{1}{3}1 + \frac{1}{3}\cdot 5 + \frac{1}{3}6 = \boxed{4.0m}$$

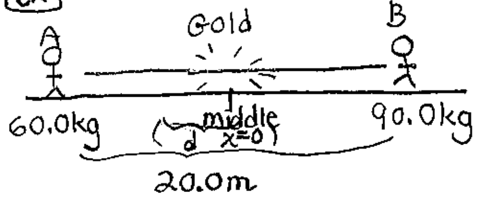
ex14 Lands where?



- 3d from start
- ΣF = gravity on CM parabola

NO CALC
APC **ex**

- On ice
- Tug at rope



a) who gets the gold? A. action-reaction, $m_A < m_B$, $a_A > a_B$

b) B moves 6.0m left.
A moves how much?

$\Sigma F = 0$ $a_{cm} = 0$, x_{cm} = same

$$x_i = x_f$$

$$\frac{6}{15}(-10) + \frac{9}{15}\times 10 = \frac{-6}{15}d + \frac{9}{15}\times 4$$

$$-20 + 30 = -2d + 12$$

$$d = 1.0m$$

A moved right $\boxed{9m}$