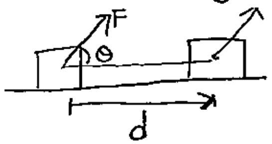
APB Ch6 : work and Energy
Giancoli (YF reference)

6-1 • Work done by 1 force on object



$$\boxed{W_F = F\cos\theta\, d = \vec{F}\cdot\vec{d}}$$

- >0 $\theta \in [0, 90°)$ speed up $\vec{F}, \vec{a} \rightarrow$ $\vec{v} \rightarrow$
- $=0$ $\theta = 90°$ const. speed $\vec{a} \uparrow \vec{v} \rightarrow$
- <0 $\theta \in (90°, 180°]$ slow down $a \leftarrow$ $\vec{v} \rightarrow$

Joule = $N \cdot m$ = $kg \cdot m^2/s^2$

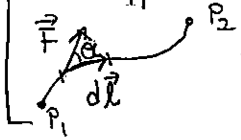
• Work = transfer of energy into / out of system

• Net work done by all forces on object

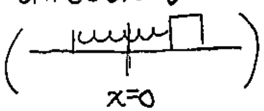
$$\boxed{W_{tot} = (\Sigma\vec{F})\cdot\vec{d}} \Leftarrow$$

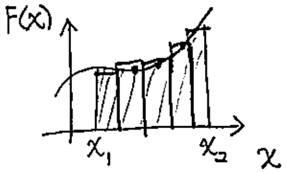
$$W_1 + W_2 + \ldots + W_N = \vec{F}_1\cdot\vec{d} + \vec{F}_2\cdot\vec{d} + \ldots \vec{F}_N\cdot\vec{d} = (\vec{F}_1 + \ldots + \vec{F}_N)\cdot\vec{d}$$

General $\left[W_{tot} = \int_{P_1}^{P_2} \vec{F}\cdot d\vec{l} \right.$ ← can vary ← curved



6-2
• Work_F when F & d same direction but F varies



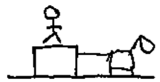


$$W = F_1\Delta x_1 + F_2\Delta x_2 + \ldots = \boxed{\text{area under F - x curve}}$$

$$= \lim \sum_1 F\,\Delta x \Rightarrow \int_{x_1}^{x_2} F(x)\,dx$$

Q • lift barbell up. Hand does (ask?) positive work, gravity does negative work

• Push the wall, $W = 0$



• ⇨ horse does positive work
friction does negative work
you : $W = 0$
$\vec{n}$: $W = 0$

$F \perp \text{motion} \Leftrightarrow W_F = 0$

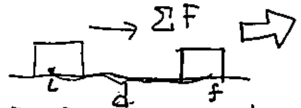
ex3 • Does Earth do work on Moon in orbit? (No! $\vec{F}_g \perp \vec{v}$)
gravity keeps in   (⇒ see ex1 & 2)

6-3 W-KE Thm
($\Sigma\vec{F} = m\vec{a}$) always true

$$\boxed{W_{tot} = \Delta K}$$
on object   of object

(Work ⇒ ΣF ⇒ a ⇒ $K\uparrow$) interpret
* Use this (WKE) if F varies
* Use F=ma if time involved

Proof of special case:
Constant acceleration (ΣF = constant in dir of motion)



$$W_{tot} = \Sigma\vec{F}\cdot\vec{d} = \Sigma F_x d_x = mad = m\frac{v_f^2 - v_i^2}{2d}d = \frac{1}{2}mv_f^2 - \frac{1}{2}mv_i^2$$

• General

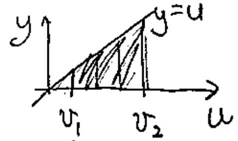
$$W_{tot} = \int_{P_1}^{P_2}\vec{F}\cdot d\vec{l} = \int_{P_1}^{P_2} m\frac{d\vec{v}}{dt}\cdot d\vec{l} = m\int_{P_1}^{P_2}\vec{v}\cdot d\vec{v}$$

$\int v_x dv_x$

$$= \frac{m}{2}\left[\vec{v}\cdot\vec{v}\right]_i^f = \frac{1}{2}m(v_f^2 - v_i^2) = \Delta K$$

• In same direction, but F varies

$$W_{tot} = \sum_i F_i\Delta x_i = \sum m\frac{\Delta v_i}{\Delta t}\Delta x_i = \sum m v_i \Delta v_i = m\frac{1}{2}(v_1 + v_2)(v_2 - v_1) = \Delta K$$

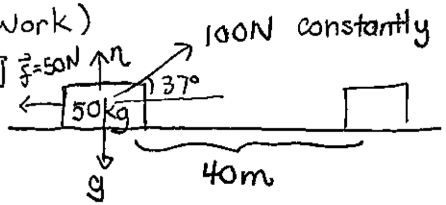


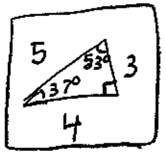
• KE = energy of motion
(mathematically defined / interpreted from 2nd Law)

see ex 1 & 2
⇒ See ex 4 ~ 5

(Work)

ex 1 (No calc)



★ 

a) W of each force

b) W_{net}? First ask =0, >0? <0?
ΣF accelerates it

a) $\star W_{Pull} = 100\cos 37° \times 40 = 100 \cdot \frac{4}{5} \cdot 40 = 3200\text{ J}$

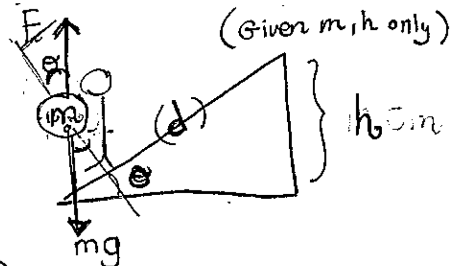
$W_n = 0$ $\qquad W_f = 50\cos 180°\ 40 = -2000\text{ J}$

$W_g = 0$

b) $W_{net} = 3200 - 2000 = \boxed{1200\text{J}}$

OR $W_{net} = \Sigma\vec{F}\cdot\vec{d} = (100\cos 37° - 50)\cdot 40$

ex2



(Given m, h only) constant velocity

a) Work hiker does on m backpack to carry up the hill? h

b) $W_{gravity}$?

c) W_{net}? ($\overset{\text{right off}}{=0}$, $\vec{a}=0$ $\Sigma F=0$)
$\Delta K=0$

a) equilibrium $W_F = (F\sin\theta)d = F\cancel{\sin\theta}\frac{h}{\cancel{\sin\theta}} = F h$
$= \boxed{mg\ h}$

F=mg $\qquad \sin\theta = \frac{h}{d}$

★ (recall U_g)
★ same work as straight up ignoring friction on slope

b) $W_g = -mg\sin\theta \underset{\left(\frac{h}{\sin\theta}\right)}{d} = \boxed{-mgh}$

c) $W_{net} = 0$

(W-KE)

ex 4 No calc
Net Force to accelerate car from 20m/s to 30m/s ? (1000kg) in distance of 5m

• $W_{tot} = \Delta K$

$F_{net}\ d = \frac{1}{2}mv_f^2 - \frac{1}{2}mv_i^2$

$F_{net}\cdot 5 = 500(900 - 400) = 25.0000$

$F_{net} = \boxed{5.0\times10^4\text{ N}}$

• OR if constant a

$v_f^2 = v_0^2 + 2a\Delta x$

$900 = 400 + 2a5$

$\Rightarrow a = 50\text{ m/s}^2 \Rightarrow \Sigma F = \underset{1000\times 50}{ma} = 5.0\times10^4\text{N}$

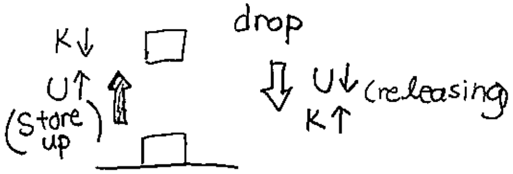
ex5

 $\rightarrow$ 60km/h $\quad d = 20m$ to stop

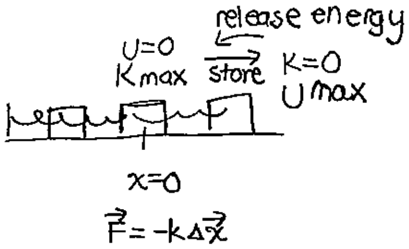
120km/h $\quad d = ?\ 20\times 4 = 80m$

$\Delta K\times 4 = \underbrace{f_k}_{\text{same}}(4d)$

6-5 Conservative Force F (4 equivalent conditions)



① Reversible
Storage Bank
U (potential energy)



② $W_{F_c} = -\Delta U$ $(= \Delta K)$ if $\Sigma F = F_c$

bank

$= \left(-W_{\text{external force}} \right.$ with 0 acceleration
e.g. hand lifts marker)

③ U by position

$W = -\Delta U$ independent of path

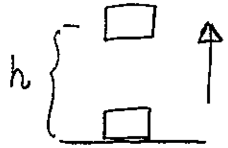
④ $W = 0 \Leftrightarrow$ displacement $= 0$

Start and end at same place, 0 work done.

• friction not any of the 4

6-4 Potential Energy Function

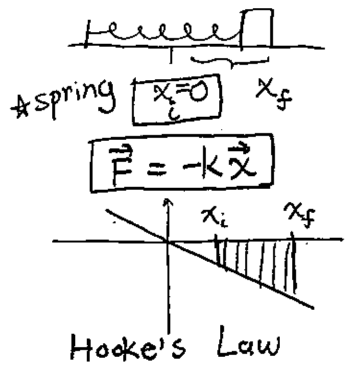
U by position



$$W_g = -mg(h-0) = -\Delta U$$
$$= -[U_f - \cancel{U_0}]$$

$\boxed{U_g = mgh}$

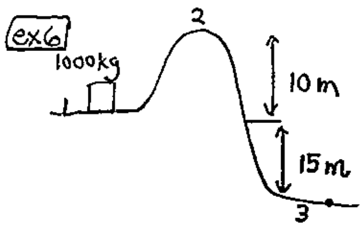
ΔU_g : $x=0$ does not matter
ΔU_{spring} : $x=0$ at equilibrium



$$W = \int_{x_i}^{x_f} -F_x\,dx$$
$$= \frac{1}{2}(-kx_i - kx_f)(x_f - x_i)$$
$$= -\frac{1}{2}k\left(x_f^2 - x_i^2\right) = -\Delta U$$

$\boxed{U_s = \frac{1}{2}kx^2}$

ex6



wrt ① y=0

a) $U_2 ? = mg(h_2 - h_1) = 9.8 \times 10^4$ J

$U_3 ? = mg(h_3 - h_1) = -1.5 \times 10^5$ J

b) $\Delta U_{2 \text{ to } 3} = -2.5 \times 10^5$ J

* y=0 doesn't matter for ΔU_g

*but x=0 for spring at equilibrium

c) wrt ③ y=0

$U_2 = 2.5 \times 10^5$ J

$U_3 = 0$

$\Delta U = -2.5 \times 10^5$ J

6-6 Conservation

① of Mechanical Energy

$$W_{tot} = \Delta K$$

$$W_g + W_s = \Delta K$$

$$-\Delta U_g - \Delta U_s$$

$$-\Delta U = \Delta K$$

$$0 = \Delta K + \Delta U$$

$$U_i + K_i = U_f + K_f$$

all forces are conservative

Mechanical Energy = constant

③ General.

conservative or not

$$\boxed{W_{tot} = \Delta K}$$ ← Use when

★ ΣF not constant

$$\underbrace{W_g + W_s}_{-\Delta U} + W_{friction}$$

$$W_{friction} = \Delta K + \Delta U$$

↑

nonconservative

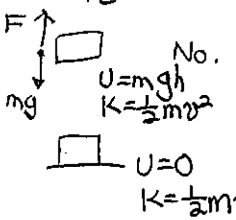
force puts energy in

takes out

② free fall

a) ME conserved? yes

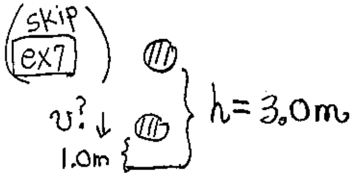
b) Push box up at constant speed, ME conserved?



No. F not conservative

ME↑ (F puts energy into system)

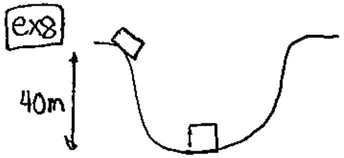
(skip) ex7



$$mg\Delta h \rightarrow \frac{1}{2}mv^2$$

$$v = \sqrt{2g\Delta h} = \sqrt{2 \times 9.8 \times 2} = \boxed{6.3\ m/s}$$

ex8



a) v_{bottom}?

b) $h = ?$ for $\frac{1}{2}v_{max}$

a) $\cancel{m}g\Delta h = \frac{1}{2}\cancel{m}v^2$

$$v = \sqrt{2g\Delta h} = \boxed{28\ m/s}$$

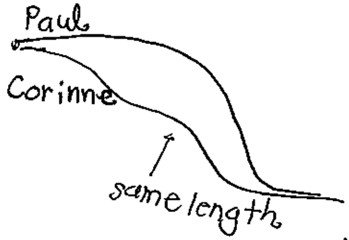
$$\sqrt{2 \times 9.8 \times 40}$$

b) $g(40-h) = \frac{1}{2} \cdot 14^2$

$$h = \boxed{30m}$$

tall 10m above ground

ex9



Water slides

a) Whose faster at bottom?

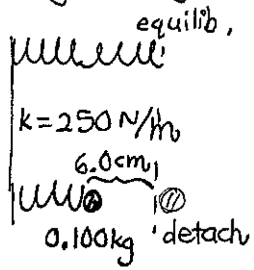
Same $mgh \rightarrow \frac{1}{2}mv^2$

b) Who reaches bottom first?

Corinne. v higher throughout (h lower)

Corinne converted her PE to KE earlier

ex10 Toy dart gun



$v = ?$

$$\frac{1}{2}kx^2 \rightarrow \frac{1}{2}mv^2$$

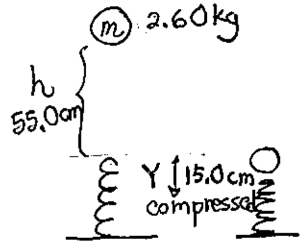
$$v = \sqrt{\frac{k}{m}}\ x$$

$$= \sqrt{\frac{250}{0.1}}\ 0.06$$

$$= 50 \times 0.06 = \boxed{3.0\ m/s}$$

ex11 Two kinds of PE

$k = ?$



$$U_g \rightarrow K_{+U_g} \rightarrow U_s$$

$$mgh = \cancel{\frac{1}{2}kY^2}$$

$$\cancel{k = \frac{2mgh}{Y^2} = \frac{2 \times 2.6 \times 9.8 \times 0.55}{0.15^2}}$$

★ $mgh + mgY = \frac{1}{2}kY^2$

$$k = \frac{2mg(h+Y)}{Y^2} = \boxed{1590\ N/m}$$

OR (careful signs!)

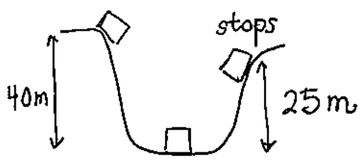
during compression

★ $W_{tot} = \Delta K$

$$W_g + W_s = \frac{1}{2}m(0^2 - v_i^2)$$

$$mg(0.15) - \frac{1}{2}k\,0.15^2 = -mg(0.55) \Rightarrow k = \frac{2mg(0.15+0.55)}{0.15^2} = 1590\ N/m$$

$\boxed{\text{ex12}}$ Friction on roller coaster



$m = 1000\,kg$

a) How much thermal energy?

b) friction = ?

$d = 400m$ distance

(Conceptually: energy lost → friction)

$W_{tot} = \Delta K$

$W_g + W_f$

$\underset{\text{positive}}{mg(40-25)} + \underset{\text{negative}}{\underbrace{W_f}} = 0$

$W_f = mg\,15 = 9.8 \times 15 \times 10^3 = \boxed{1.47 \times 10^4\,J}$

$fd =$

$f = \dfrac{1.47 \times 10^4}{400} = \boxed{370\,N}$

$\boxed{\text{ex}}$ See APC elevator question

$\boxed{\text{6-10}}$ Power

- $\boxed{P_{avg} = \dfrac{\Delta W}{\Delta t} = F\bar{v}}$ $P = \dfrac{dW}{dt}$ $\boxed{E = P\Delta t}$

$1\,Watt = 1\,J/s$

$\boxed{\text{efficiency} = \dfrac{W_{out}}{W_{in}} = \dfrac{P_{out}}{P_{in}}}$

- $1\,hp = \dfrac{550\,ft \cdot lb}{1s} = 746\,W$ $(1kW \approx 1\frac{1}{3}hp)$

work of horse at coal mine

machine

$W_{in} = W_{out} + W_{wasted}$

$\boxed{\text{ex13}}$ 60 kg jogger runs up stairs 4.5m height in 4.0s

a) P (W & hp)?

b) energy needed?

a) If constant speed, $F = mg$

$P = \dfrac{mgh}{t} = \dfrac{60 \times 9.8 \times 4.5}{4} = 660\,W$

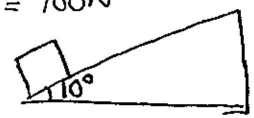
$\approx 0.88\,hp$

(human cannot maintain long)

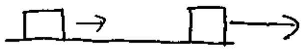
b) $mgh = \boxed{2600\,J}$

ex14 $m = 1400\,kg$ $v = 80\,km/h$ steady

a) $F_R = 700\,N$ Power ?



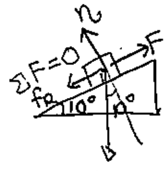
b) $F_R = 700\,N$



90 km/h 110 km/h

in 6.0 s

a)



$\Sigma F_x = 0$

$F = f_R + mg\sin\theta$

$$P = \frac{F_{engine}\,\Delta x}{\Delta t} = F_{engine}\, v$$

$$= (f_R + mg\sin\theta)\, v$$

$$= (700 + 1400 \times 9.8 \sin 10^\circ)\frac{80\times10^3}{3600}$$

$$= \boxed{68\,kW} = 91\,hp$$

b) $\Sigma F = ma$

$(v_f^2 = v_0^2 + 2a\Delta x)$

$F - F_R = m\dfrac{\Delta v}{\Delta t}$ $\dfrac{\Delta x}{\Delta t} = \dfrac{v_0 + v_f}{2}$

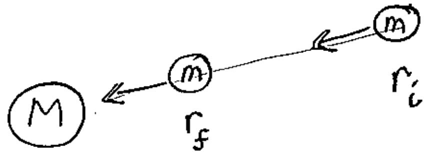
$$\left(\frac{F\Delta x}{\Delta t} = \left(F_R + m\frac{\Delta v}{\Delta t}\right)\left(\frac{v_0+v_f}{2}\right)\frac{\Delta t}{\Delta t} = \left(F_R + m\frac{\Delta v}{\Delta t}\right)\left(\frac{v_0+v_f}{2}\right)\right)$$

$$F = F_R + m\frac{\Delta v}{\Delta t} = 700 + 1400 \times \frac{20\times10^3}{3600\times 6} = 2000\,N$$

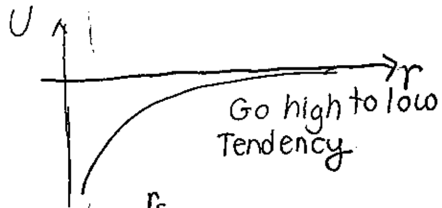
$$P_{avg} = F\bar{v} = 2000 \times \frac{100\times10^3}{3600} = \boxed{55\,kW}$$

Bonus for Gravity

$U_\infty = 0$



- $K\uparrow \Rightarrow U\downarrow$
  $U < 0$
- $W_F > 0 \Rightarrow \Delta U < 0$
  $\Delta K > 0$



- $\Delta U = -W_g = W_{external\ force}$ to bring from ∞ to r

$$W_{F_g} = -\int_{r_i}^{r_f} G\frac{mM}{\tilde{r}^2}\,d\tilde{r} = GmM\left[+\frac{1}{r}\right]_{r_i}^{r_f}$$

$(-\ \hat{r})$

$$= +GmM\left[\frac{1}{r_f} - \frac{1}{r_i}\right]$$

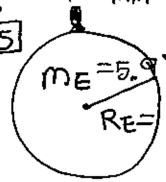
$$= -\Delta U$$

$$\therefore U(r) \equiv -\frac{GmM}{r}$$

$$U(\infty) = 0$$

APC ex13.5

$\uparrow v_{min} = ?$ to escape Earth?



$E_i = E_f$

$K \rightarrow U$

$$-G\frac{\cancel{m}M}{R} + \frac{1}{2}\cancel{m}v^2 = 0 + 0$$

v_{escape}

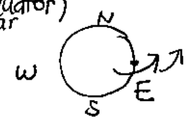
* does not depend on mass or direction of launch!

$$v_{escape} = \sqrt{\frac{2GM}{R}} = 1.12\times10^4\,m/s$$

$= 25000\,mph$

$= 40200\,kph$

* Florida (near equator) is moving east 410 m/s already



Launch this way "free" contribution