APB Giancoli
ref APC Notes

Ch11 : Oscillations & Waves

11-1 ~ 11.3 Simple Harmonic Motion / oscillator

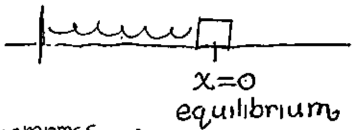
1 frequency — Simple
sinusoidal — Harmonic

① Condition

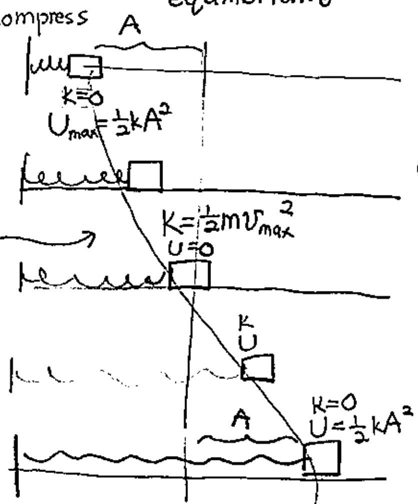
$$\boxed{\begin{array}{l} F = -kx \quad (\text{elastic region, small amplitude}) \\ x(t) = A\cos(\omega t + \phi) \end{array}}$$

Hooke's Law
★ Restoring force
Stable equilibrium
phase angle (starting)
displacement ~ sinusoid

② Show spring example



④ energy (see ex3~5)

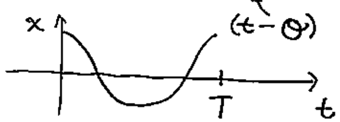


$\frac{1}{2}mv_{max}^2 = \frac{1}{2}kA^2$
$v_{max} = \sqrt{\frac{k}{m}}\,A$

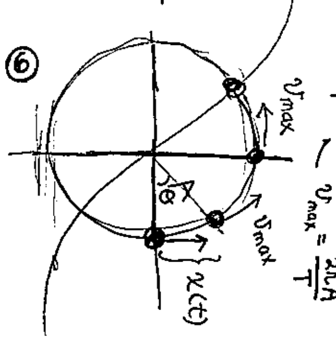
③
(m) A = amplitude
(s) T = time for one cycle = period
$f = \frac{1}{T}$ = frequency (Hz)
$\omega = 2\pi f$ (rad/s) = angular frequency

⑤ Can intuitively see it's sinusoidal

$x(t) = A\cos\left(\frac{2\pi}{T}t\right)$ $(t-\theta)$



show applet ⑥



$\theta(t) = \omega t$
$\omega = \frac{2\pi}{T} = \frac{v_{max}}{A}$
$v_{max} = \frac{2\pi A}{T} = \boxed{\sqrt{\frac{k}{m}}}A$

⑦ OR

$$F_x = -kx$$
$$m\frac{d^2x}{dt^2} = -kx$$
$$\frac{d^2x}{dt^2} = -\frac{k}{m}x$$

whose second derivative is itself ?

$x(t) = A\cos(\omega t + \phi)$
$v(t) = -\omega A\sin(\omega t + \phi)$
$a(t) = -\omega^2 A\cos(\omega t + \phi) = -\omega^2 x$

★ makes sense
k↑, stiff, faster
m↑, inertia, slower

$$\Rightarrow \boxed{\omega = \sqrt{\frac{k}{m}}} \qquad T = 2\pi\sqrt{\frac{m}{k}}$$

T, f independent of A!

- musical instrument,
- pendulum clock
friction countered by weight dropped
grandfather clock

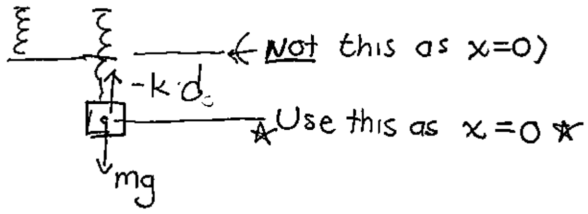
④b Show

$$v = \pm v_{max}\sqrt{1 - \frac{x^2}{A^2}}$$

$\because$ ★ $\frac{1}{2}kx^2 + \frac{1}{2}mv^2 = \frac{1}{2}kA^2$

$$v = \pm\sqrt{\frac{k}{m}(A^2 - x^2)} = \pm\sqrt{\frac{k}{m}A^2}\sqrt{\left(1 - \frac{x^2}{A^2}\right)}$$
$$= \pm v_{max}\sqrt{1 - \frac{x^2}{A^2}}$$

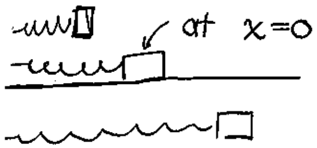
③b Vertical Spring



(see ex to ex2)

11.1

QA) mass on horizontal spring.
Where, if any, is $a = 0$?



$x = $ -A / $\boxed{0}$ / +A / both -A & +A / nowhere

QB) $f = 1.25\,Hz \Rightarrow$ 100 oscillations in <u>80</u> seconds

$\frac{5}{4}\frac{\text{cycles}}{\text{sec}} \cdot \frac{\times 20}{\times 20}$

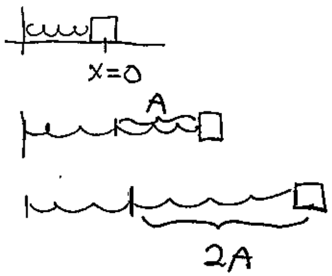
ex1 family of four: 200kg
car: 1200kg
When step in, car spring compressed 3.0cm
$k = ?\quad \frac{F}{\Delta x} = \frac{200 \times 9.8}{0.03} = \boxed{6.5 \times 10^4\ N/m}$

ex2 which is SHM?

a) $F = -0.5x^2$ ↑ x
b) $F = -2.3y$ ✓
c) $F = 8.6x$ ↑ + x
d) $F = -4\theta$ ✓

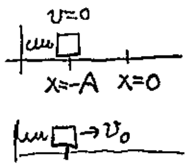
11.2

ex3



If stretched twice as far, compare
a) energy $\quad \frac{1}{2}kA^2 \rightarrow \times 4$
b) $v_{max} \quad \frac{1}{2}mv_{max}^2 \rightarrow \times 2$
c) $a_{max} \quad F = -kA \rightarrow \times 2$ at end

QC)

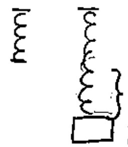


extra push when at x= -A

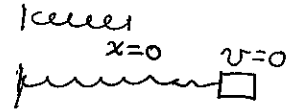
Compare effect of push on
a) energy $\leftarrow \frac{1}{2}kA^2 + \frac{1}{2}mv_0^2$
b) $v_{max} \leftarrow \frac{1}{2}mv_{max}^2$ (bigger)
c) $a_{max} \leftarrow$ bigger! stretches more at other side! (recall
>A, $F = -k(A + \Delta A) =$

ex4,5

Lab?



$\Delta x = 0.150m$
$m = 0.300kg$



$x = 0.100m$

a) spring stiffness constant k
b) amplitude of oscillation A
c) v_{max}
d) v when $x = \pm 0.050m$
e) a_{max}
f) E NO CALC
g) $x = \pm A/2 \Rightarrow K = ?\ U = ?$

Soln Show work from basics

a) $\Sigma F_y = 0$
$mg = k\Delta x$
$k = \frac{mg}{\Delta x} = \frac{0.3 \times 9.8}{0.15} = \boxed{19.6\,N/m}$

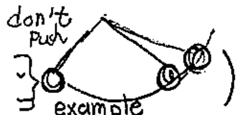
b) $A = \boxed{0.100m}$

c) $\frac{1}{2}kA^2 = \frac{1}{2}mv_{max}^2$
$v_{max} = \sqrt{\frac{k}{m}}A$
$= \sqrt{\frac{19.6}{0.3}} \times 0.1$
$= \boxed{0.808\,m/s}$

d) $\frac{1}{2}mv^2 + \frac{1}{2}kx^2 = \frac{1}{2}kA^2$
$v = \pm\sqrt{\frac{k}{m}(A^2 - x^2)}$
$= \pm\sqrt{\frac{19.6}{0.3}(0.1^2 - 0.05)^2}$
left
$= \boxed{-0.700\,m/s}$

e) $a_{max} = \frac{F_{max}}{m} = \frac{kA}{m}$
$= \frac{19.6}{0.3} \cdot 0.1$
$= \boxed{6.53\,m/s^2}$

NO CALC
f) $E = \frac{1}{2}kA^2 = \frac{1}{2} \times 19.6 \times 0.1^2 = \boxed{0.098J}$
g) $U = \frac{1}{2}k\left(\frac{A}{2}\right)^2 = \frac{1}{4}E = 0.0245J = \boxed{2.45 \times 10^{-2}J}$
$K = E - U = \frac{3}{4}E = 0.0735J = \boxed{7.35 \times 10^{-2}J}$



11.3

ex6 Spider web

$m_{spider} = 0.30$ grams

web negligible mass

motion ⇒ $f = 15\,Hz$ a) $k = ?$

b) If $m_{insect} = 0.10$ grams with spider,

freq of vibration = ? ↑ ↓ same

Show $\boxed{\begin{aligned} f &= f_0 \times ? \\ &= 15 \times \sqrt{3/4} \end{aligned}}$

$= 13\,Hz$

a) $\omega = \sqrt{\frac{k}{m}}$ (Pluck & let go T, f indep of A!)

$k = \left(\frac{2\pi}{T}\right)^2 m = 4\pi^2 \times 15^2 \times 0.30 \times 10^{-3}$

$= \boxed{2.7\ N/m}$

b) less, more inertia
   f

$f = \frac{1}{2\pi}\sqrt{\frac{k}{m_2}} = \frac{f_0}{\sqrt{4/3}} = \sqrt{\frac{3}{4}}\, f_0$

$\frac{m_0}{m_2} = \frac{3}{4}$

ex7 Large motor on factory floor

floor amplitude near motor ≈ 3.0 mm

frequency 10 Hz

Max acceleration of floor near motor?

Soln:

$\omega = \sqrt{\frac{k}{m}} \Rightarrow k = (2\pi f)^2 m$

$a_{max} = \frac{F_{max}}{m} = \frac{kA}{m} = \frac{(2\pi f)^2 \cancel{m}\, A}{\cancel{m}} = (2\pi\, 10)^2 \times 3 \times 10^{-3}$

$\approx 4 \times \pi^2 \times 3 \times 10^{-1} = 11.8 \approx \boxed{12\ m/s^2} > g$

objects on floor lose contact

Q Can we use the kinematic eqns for SHM?

No! $a \neq$ constant

$F = -kx \neq$ constant

ex8 $x = 0.30 \cos(8.0\, t)$

displacement meters seconds

a) $A = \boxed{0.30\,m}$

b) $f = \frac{\omega}{2\pi} = \frac{8}{2\pi} = \boxed{1.27\,Hz}$

c) $T = \frac{1}{f} = \boxed{0.79\,s}$

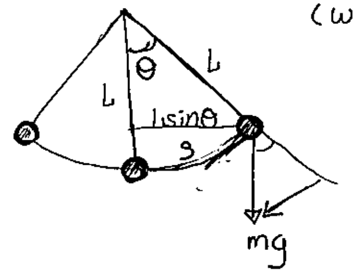
d) $v_{max} = \sqrt{\frac{k}{m}}\, A$ $\frac{1}{2}kA^2 = \frac{1}{2}mv_{max}^2$

$= \omega A$

$= 8 \times 0.3 = \boxed{2.4\,m/s}$

e) $a_{max} = \frac{F_{max}}{m} = \frac{kA}{m} = \omega^2 A = 8^2 \times 0.3 = 19.2 \approx \boxed{19\,m/s^2}$

11.4 Simple Pendulum is also SHM!
(when θ small ≲ 15°)



$F = -mg\sin\theta$

$\approx -mg\,\theta$ (Taylor)

$\approx -mg\left(\frac{s}{L}\right)$ OR $s \approx L\sin\theta$, $\sin\theta \approx \frac{s}{L}$

$= -\underbrace{\frac{mg}{L}}_{k}\, s$

$s = L\theta$

θ°	θ rad	sin θ	% diff
0	0	0	0
1°	0.01745	0.01745	0.005%
5°	0.087		0.1%
10°	0.17453		0.5%
15°	0.26180		1.1%
20°	0.34907		2.0%

$\omega = \sqrt{\frac{k}{m}} = \boxed{\sqrt{\frac{g}{L}}}$ no mass! or amplitude
★ pendulum clock
kid on swing

Q(E)  at 10° instead, frequency ↑ ↓ same

(F) Let go at 5°. Go to top of mountain, compared to sea level

f ↑ ↓ same

Less restoring $\omega = \sqrt{\frac{g}{l}}$

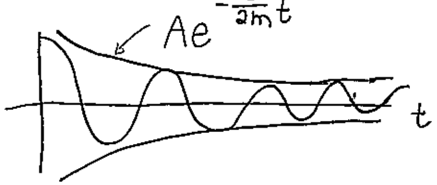
ex9 Lab

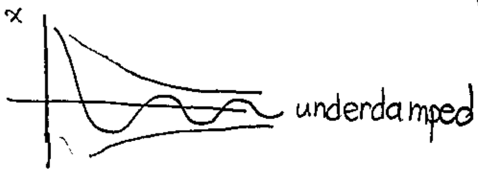
$l = 37.10\,cm$ Use pendulum to measure g

$f = 0.8190\,Hz$ at some place on Earth

$g = ?$ $l\omega^2 = 37.10\times10^{-2}\,(2\pi\, 0.819)^2 = \boxed{9.824\,m/s^2}$

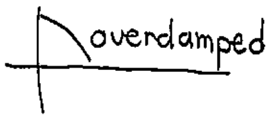
11.5 Damped Harmonic Motion



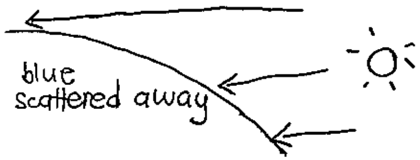


$x(t) = Ae^{-\frac{b}{2m}t}\cos(\omega' t + \phi)$

- shock absorber in car: critically or slightly underdamped
- building dampers



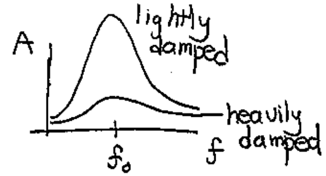
③ Sunsets red

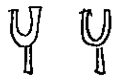


11.6 Forced Oscillation

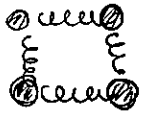
(Push at any frequency & force to move)

• Resonance - push at resonant frequency
  (no damping - amplitude builds up)
  with damping - amplitude is big

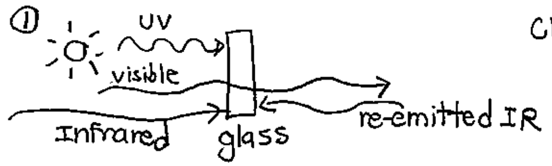


- See CISE & book pix in ppt
  - ~bridge P443 (break step)
  - ~goblet & trumpet P 547
  - ~radio receives freq when tuned
  - ~tuning forks matching  show CISE
    Push at right time
  - ~walking with coffee
  - ~ kid on swing

★ molecules like atoms vibrating at SHM when object dropped



⇒ natural, resonant frequency



CISE -gobble-gulp!
- atoms resonate at UV freq hold energy longer/dissipate as thermal E easily
- molecules resonate at IR freq

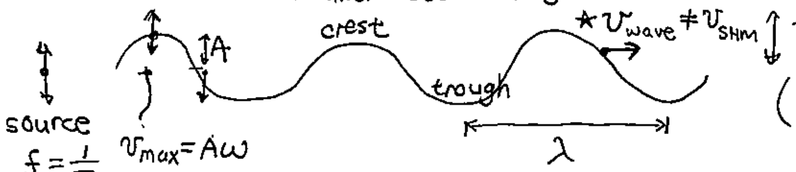
② Sky is blue. Resonate at blue (shorter) & scatter
gulp & re-emit (not like glass. gulp/collide/heat)
(∵ gas further apart

Smog ~ white - diff size re-emit all colors

pollution ~ brown - globs of stuff absorb colors (not scatter)

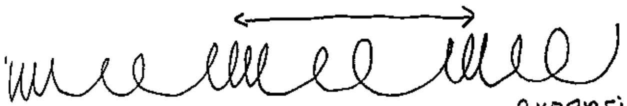
11-7 Wave Motion

- Mechanical wave - propagates by oscillation of matter
- CISE
  - energy travels v
  - matter does NOT go far, but oscillates SHM

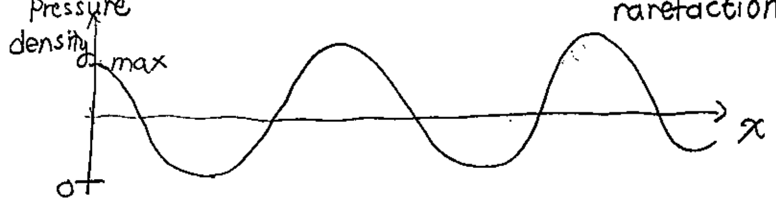


$\updownarrow$ transverse

$(T=\frac{1}{f} \updownarrow$ & $\sim)$



$\leftrightarrow$ Longitudinal



CISE bird $\boxed{v=\frac{\lambda}{T}=\lambda f}$

$y(x) = A\cos\left(\frac{2\pi}{\lambda}x\right)$

$y(x,t) = A\cos\left(\frac{2\pi}{\lambda}(x-vt)\right)$

$= \boxed{A\cos(kx-\omega t)}$

$\rightarrow v$

λ in time T

moving right

$\frac{2\pi}{\lambda}\frac{\lambda}{T}=\omega$

$\star$ T $\leftrightarrow$ = T $\updownarrow$

$y(x,t) = A\cos(kx+\omega t)$ moving left $Re\{Ae^{i(kx\pm\omega t)}\}$

Phasors engineering

- $A\uparrow$ $\Rightarrow$ energy $\uparrow$
  source
- $f\uparrow$ $\Rightarrow$ energy $\uparrow$
  source shakes faster

(• $T\uparrow$ $\Rightarrow$ energy $\downarrow$)
source shakes slower

- v by medium

λ by shake f
$v=\lambda f$
medium

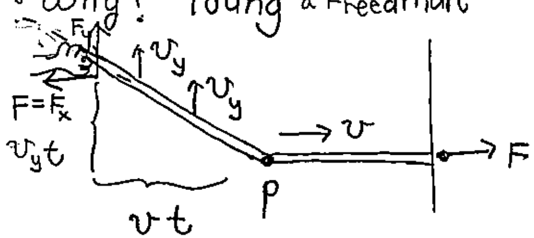
- Loud / soft voice travel at same speed
(though loud gets further past absorption)

- On a rope

$$\boxed{v=\sqrt{\frac{T}{\mu}}} \begin{array}{l}\leftarrow \text{tension} \\ \leftarrow \text{mass/length}\end{array}$$

sensible

- Why? Young & Freedman



F_x because string does not move $\leftrightarrow$
$=F$

why similar triangles? tension along rope?

$$\frac{F_y}{F} = \frac{v_y t}{vt}$$

($\Delta p_x = 0$ mv) always

impulse = Δp

$F_y\, t = m v_y$

- momentum since - mass increases $\uparrow$

$\uparrow$ string starts going up

$\Downarrow$

$$F\frac{v_y}{v}\cancel{t} = (\mu v \cancel{t})\, v_y$$

$$\therefore \boxed{v=\sqrt{\frac{F}{\mu}}}$$

- $v = \sqrt{\dfrac{\text{elastic force factor}}{\text{inertia factor}}} = \sqrt{\dfrac{E}{\rho}} = \sqrt{\dfrac{B}{\rho}}$

elastic modulus (E), bulk (B)

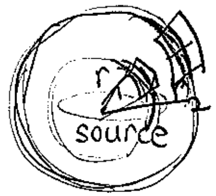
rod, fluid density

Other Waves:

1D 2D: surface waves 3D: earthquake

rope 

11-9 energy transported by waves



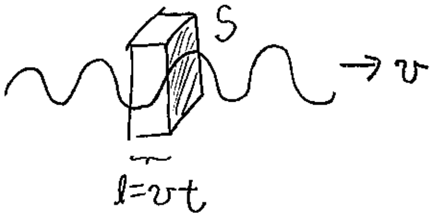
$$\text{Intensity} = \frac{\text{Power}}{\text{Area}} \sim \boxed{\frac{1}{r^2}}$$

$$= \frac{\text{energy/time}}{\text{area}} \quad \left(U = \tfrac{1}{2}kA^2\right)$$

$16J\ \dfrac{\frac{1}{2}kA^2}{1\text{square}} \rightarrow \dfrac{\frac{1}{2}k\tilde{A}^2\ 16J}{4\text{ squares}}$ $\quad \dfrac{4J}{1\text{square}} \leftarrow A \sim 2$

$A \sim 4$ $\quad \tilde{A} \sim$

$$\frac{\frac{1}{2}kA^2}{(r^2)} \qquad \boxed{A \sim \frac{1}{r}}$$



$$\omega = \sqrt{\frac{k}{m}}$$

$$k = m\omega^2 = \rho v t S (2\pi f)^2$$

$$I = \frac{P}{S} = \frac{E}{tS} = \frac{\frac{1}{2}kA^2}{tS} = \frac{\frac{1}{2}\left[(\rho v \cancel{t} \cancel{S})(2\pi f)^2\right]A^2}{\cancel{t}\cancel{S}}$$

$$= \boxed{2\pi^2 \rho v f^2 A^2} \qquad I \sim f^2, A^2$$

11-10 Reflection, transmission

fixed NOT

$\boxed{\text{ex14}}$ String 1.10 m long

$m = 9.00\,g$

a) $T = ?$ $\quad f_1 = 131\,Hz$

b) $f_{2,3,4}$?

$f_2 = 2f_1 = 262\,Hz$

$f_3 = 3f_1 = 393\,Hz$

$f_4 = 4f_1 = 524\,Hz$

$\frac{\lambda}{2} = l$

$v = \sqrt{\frac{T}{\mu}} \qquad v = \lambda f$

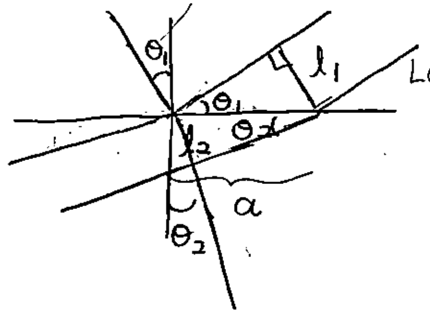
$T = \mu v^2$

$= \frac{m}{l}(\lambda f_1)^2 = 679\,N$

11-13 Refraction

Snell's $(n_1 \sin\theta_1 = n_2 \sin\theta_2)$

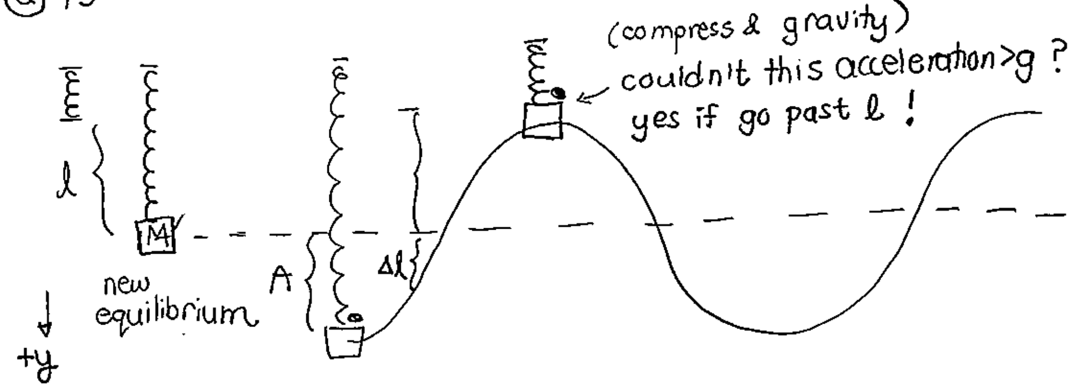
Law of refraction



$\sin\theta_1 = \frac{l_1}{a} = \frac{v_1 t}{a} \Rightarrow \frac{\sin\theta_1}{\sin\theta_2} = \frac{v_1}{v_2}$

$\sin\theta_2 = \frac{l_2}{a} = \frac{v_2 t}{a}$

Q 75



$\Sigma F = \cancel{Mg} - k(\cancel{l} + \Delta y) = Ma$

$Mg = kl$

$g = \frac{k}{M} l$

$\Sigma F = \underbrace{-k\Delta y}_{\text{still SHM}} = Ma \Rightarrow a = -\frac{k}{M}\Delta y$

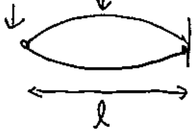
11-14 Diffraction

11-11 superposition

11-12 Standing waves

Resonance at 1st Harmonic

node antinode



$v = \lambda f$

$\frac{\lambda}{2} = \ell$

$\boxed{f_1 = \frac{v}{2\ell}} = \leftarrow f_n = \frac{v}{\lambda_n}$

$f_2 = \frac{v}{\lambda_2}$

$= n\left(\frac{v}{2\ell}\right)$

$= \boxed{n f_1}$

$\frac{\lambda_2}{2} \cdot 2 = \ell$

$\frac{\lambda_3}{2} \cdot 3 = \ell$

$\lambda_n = \frac{2\ell}{n}$

Math. Why nodes there