TI-89

real mode $(-1)^{\frac{1}{6}}$ not real, $(3+2i)(4-2i) = 16+2i$

rectangular mode $(-1)^{\frac{1}{6}} = 0.866025 + 0.5i$

Polar mode $2+2i \rightarrow \sqrt{2}\, e^{i\frac{\pi}{4}}$

cSolve(x^4 = -1 , x) $\Rightarrow$ real mode exact

$$\begin{cases} \frac{\sqrt{2}}{2}(1+i) \\ \text{or } \frac{\sqrt{2}}{2}(1-i) \\ \frac{\sqrt{2}}{2}(-1+i) \\ \frac{\sqrt{2}}{2}(-1-i) \end{cases}$$

cSolve(x^4 = 1 , x) $\Rightarrow$ $i, -i, 1, -1$ ✓ Cⓒ?

Kreyzig 12.6

complex exponential function $z = x + iy$

$$e^{z} \triangleq e^{x}(\cos y + i \sin y)$$

Properties $(e^{z})' = e^{z}$

$e^{z_1+z_2} = e^{z_1} e^{z_2}$ by trig ID

$$\boxed{z = x+iy = r(\cos\theta + i\sin\theta) = re^{i\theta}}$$

standard Polar exponential polar form

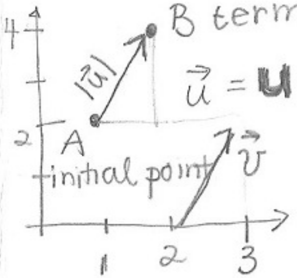
$e^{2\pi i} = 1$  $e^{-i\pi/2} = -i$

$e^{\pi i/2} = i$  $e^{-i\pi} = -1$

$e^{i\pi} = -1$ $\arg(e^{iy}) = y \pm 2k\pi$

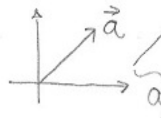
$|e^{iy}| = 1 \because |\cos y + i\sin y| = \sqrt{\cos^2 y + \sin^2 y} = 1$

8.4 Vectors	Scalars
directed qty	no direction, mass, time
displacement	distance
velocity	speed
acceleration	
force	
magnitude + direction	



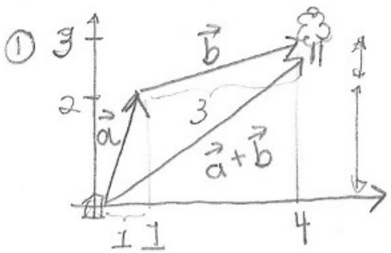
$\vec{u} = \mathbf{u} = \vec{AB} = \langle 1, 2 \rangle$

$\vec{v} = \vec{u}$ if direction & magnitude same (length) norm absolute value



$\boxed{\vec{a} = \langle a_x, a_y \rangle = a_x\hat{i} + a_y\hat{j}}$

Vector Addition

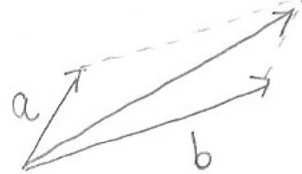
① 

② Notice.

$\vec{a} + \vec{b} \triangleq \langle a_x, a_y \rangle + \langle b_x, b_y \rangle$

$= \langle a_x + b_x, a_y + b_y \rangle$

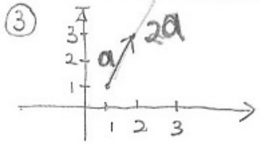
fits geometric interpretation

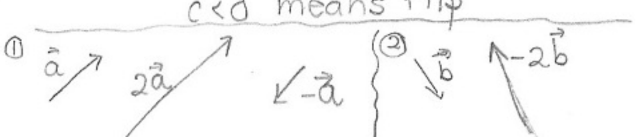
③ 

Parallelogram method

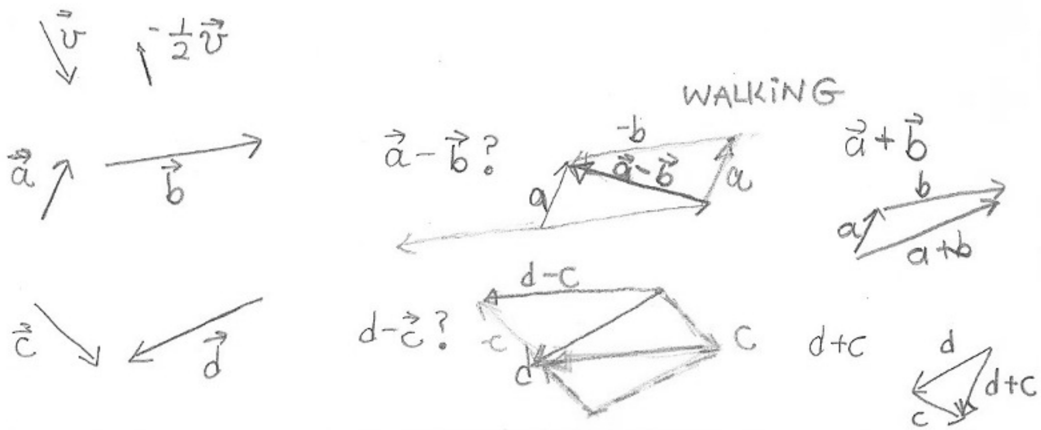
Scalar Multiplication $c\vec{a} = \langle ca_x, ca_y \rangle$

$c<0$ means flip

③ 

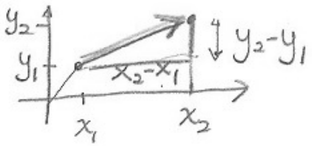


Vector Subtraction $\vec{a}-\vec{b} = \langle a_x - b_x, a_y - b_y \rangle$

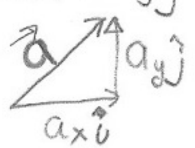


- $\vec{v}$ $P(x_1, y_1) \longrightarrow Q(x_2, y_2)$

  $\vec{v} = \langle x_2 - x_1, y_2 - y_1 \rangle$



- $\vec{a} = a_x \hat{i} + a_y \hat{j}$ Addition makes sense



$\boxed{\text{ex1}}$ $P(-2,5)$ initial pt $\quad$ $Q(3,7)$ terminal Pt

a) vector $\vec{u} = \underline{\langle 5, 2 \rangle} = 5\hat{i} + 2\hat{j}$

b) $\vec{V} = \langle 3, 7 \rangle$

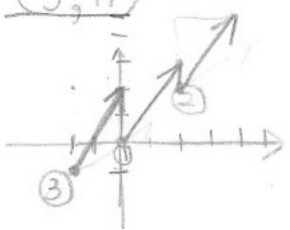
initial $(2,4)$ terminal at $\underline{(5,11)}$

c) $\vec{w} = \langle 2, 3 \rangle$

Sketch with initial pts

$(0,0)$, $(2,2)$, $(-2,-1)$, $(1,4)$

1 $\quad$ 2 $\quad$ 3



$$\left\{\begin{array}{l} \vec{v} = \langle v_1, v_2 \rangle = v_1\hat{i} + v_2\hat{j} \\ |\vec{v}| = \sqrt{v_1^2 + v_2^2} \; (= \vec{v}\cdot\vec{v}) \\ \vec{u} \pm \vec{v} = \langle u_1 \pm v_1, u_2 \pm v_2 \rangle \\ c\vec{u} = \langle cu_1, cu_2 \rangle \\ \text{See properties on Pg 611 (6th 583)} \end{array}\right.$$

unit vector $\left\{\begin{array}{l} \hat{i} = \langle 1, 0, 0 \rangle \\ \hat{j} = \langle 0, 1, 0 \rangle \\ \hat{k} = \langle 0, 0, 1 \rangle \end{array}\right.$

$|\hat{i}| = |\hat{j}| = |\hat{k}| = 1$

$\boxed{\text{ex3}}$ $\vec{u} = \langle 2, -3 \rangle$

$\vec{v} = \langle -1, 2 \rangle$

$\vec{u} + \vec{v} = \langle 1, -1 \rangle \quad 2\vec{a} - 3\vec{v} = \langle 4, -6 \rangle - \langle -3, 6 \rangle = \langle 7, -12 \rangle$

$\vec{u} - \vec{v} = \langle 3, -5 \rangle \quad 2\vec{u} + 3\vec{v} = \langle 1, 0 \rangle$

$\boxed{\text{ex4}}$ a) $\vec{u} = \langle 5, -8 \rangle = 5\hat{i} - 8\hat{j}$

b) $\vec{u} = 3\hat{i} + 2\hat{j}$

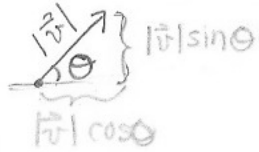
$\vec{v} = -\hat{i} + 6\hat{j}$

$$\begin{aligned} 2\vec{u} + 5\vec{v} &= 2(3\hat{i} + 2\hat{j}) + 5(-\hat{i} + 6\hat{j}) \\ &= 6\hat{i} + 4\hat{j} - 5\hat{i} + 30\hat{j} \\ &= \hat{i} + 34\hat{j} \end{aligned}$$

Horizontal & Vertical Components "DIRECTION"

when $|\vec{v}|$ & θ are known (θ = from $+x$axis)

(Force, geometrically)



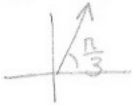
like polar

$$\begin{aligned} \Rightarrow \vec{v} &= \langle |\vec{v}|\cos\theta, |\vec{v}|\sin\theta \rangle \\ &= \underbrace{|\vec{v}|\cos\theta}_{\text{"Projection" onto x-axis, Horizontal component}}\hat{i} + |\vec{v}|\sin\theta\,\hat{j} \end{aligned}$$

ex5 a) $\vec{v} = 4\langle 1, \sqrt{3} \rangle = 4\hat{i} + 4\sqrt{3}\,\hat{j}$

Length $= 8$

Direction $= \frac{\pi}{3}$



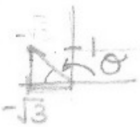
Horiz Vertical Components?

$v_x = |\vec{v}|\cos\frac{\pi}{3} = 8 \times \frac{1}{2}$

$v_y = |\vec{v}|\sin\frac{\pi}{3} = 8\,\frac{\sqrt{3}}{2}$

b) $\vec{u} = -\sqrt{3}\,\hat{i} + \hat{j}$

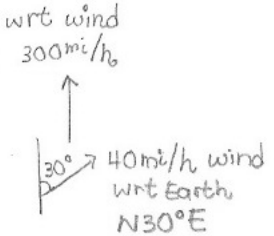
Direction?



$\alpha = \tan^{-1}\left(\frac{1}{\sqrt{3}}\right) = 30^\circ = \frac{\pi}{6}$

$\theta = \boxed{\frac{5\pi}{6}}$

ex6

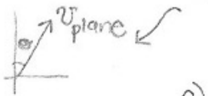


a) $\vec{v}_{\text{plane wrt air}} = \langle 0, 300 \rangle$

$\vec{u}_{air} = \langle 20, 20\sqrt{3} \rangle$

b) $\vec{v}_{plane} = \vec{v}_{\text{p wrt air}} + \vec{u}_{air}$

$= \langle 20, 300 + 20\sqrt{3} \rangle$



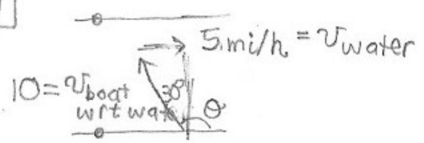
c) $|\vec{v}_{plane}| = 335.2$ mi/h

$\theta = \tan^{-1}\left(\frac{20}{300 + 20\sqrt{3}}\right)$

$= 3.4^\circ$

$\boxed{N\,3.4^\circ E}$

ex7



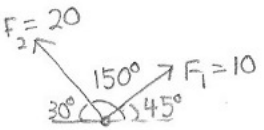
$v_{boat}\cos\theta = -5$

$\cos\theta = -\frac{1}{2}$

$\theta = 120^\circ$

$\boxed{N\,30^\circ W}$

ex8 Resultant Force



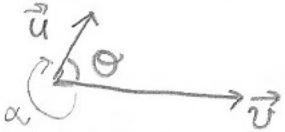
$\vec{F} = \langle 10\cos 45^\circ - 20\cos 30^\circ, 5\sqrt{2} + 10 \rangle$

$= \langle 5\sqrt{2} - 10\sqrt{3}, 5\sqrt{2} + 10 \rangle$

8.5 Dot Product

$\vec{u} = \langle u_1, u_2 \rangle$

$\vec{v} = v_1 \hat{i} + v_2 \hat{j}$

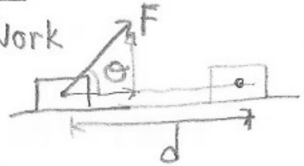


$$\boxed{\underset{\text{vectors}}{\vec{u} \cdot \vec{v}} = u_1 v_1 + u_2 v_2 \quad \text{scalar}} = |\vec{u}||\vec{v}| \cos\theta$$

$\theta \in [0, 180°]$ smaller angle between NOT α

- Uses

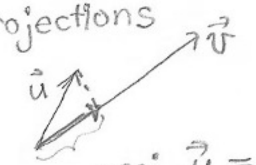
① Work



$W = \vec{F} \cdot \vec{d}$

$= (F\cos\theta) d$ Nm Joules

② Projections



$$\text{proj}_{\vec{v}} \vec{u} = \underbrace{\frac{\vec{u} \cdot \vec{v}}{|\vec{v}|}}_{|\vec{u}|\cos\theta} \underbrace{\frac{\vec{v}}{|\vec{v}|}}_{\text{unit vector}}$$

- Project $\vec{u}$ onto $\vec{v}$ & $\perp \vec{v}$

(every vector can be decomposed to orthogonal components)

ex1

b) $\vec{u} = 2\hat{i} + \hat{j}$

$\vec{v} = 5\hat{i} - 6\hat{j}$

$\vec{u} \cdot \vec{v} = 2 \times 5 + 1(-6) = 4$ NOT a vector anymore!

Properties in HW prove

$\vec{u} \cdot \vec{v} = \vec{v} \cdot \vec{u}$

$(c\vec{u}) \cdot \vec{v} = \vec{u} \cdot c\vec{v} = c(\vec{u} \cdot \vec{v})$

$\langle cu_1, cu_2 \rangle \cdot \langle v_1, v_2 \rangle$

$= cu_1 v_1 + cu_2 v_2$

$= u_1 cv_1 + u_2 cv_2 = \langle u_1, u_2 \rangle \cdot \langle cv_1, cv_2 \rangle$

$= c(u_1 v_1 + u_2 v_2) = c(\vec{u} \cdot \vec{v})$

$(\vec{u} + \vec{v}) \cdot \vec{w} = \vec{u} \cdot \vec{w} + \vec{v} \cdot \vec{w}$

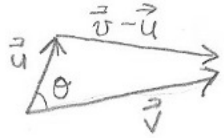
$|\vec{u}|^2 = \vec{u} \cdot \vec{u} \leftarrow |\vec{u}|^2 = u_1^2 + u_2^2 = \vec{u} \cdot \vec{u}$

DOT Product Thm.

$$\boxed{\vec{u} \cdot \vec{v} = |\vec{u}||\vec{v}| \cos\theta}$$

Find the angle θ? solve $\cos\theta = \frac{\vec{u} \cdot \vec{v}}{|\vec{u}||\vec{v}|}$

Proof



Law of Cosines

$|\vec{v} - \vec{u}|^2 = |\vec{u}|^2 + |\vec{v}|^2 - 2|\vec{u}||\vec{v}| \cos\theta$

$(\vec{v} - \vec{u}) \cdot (\vec{v} - \vec{u})$

$= (\vec{v} - \vec{u}) \cdot \vec{v} - (\vec{v} - \vec{u}) \cdot \vec{u}$ Prop 3

$= \vec{v} \cdot \vec{v} - \vec{u} \cdot \vec{v} - \vec{v} \cdot \vec{u} + \vec{u} \cdot \vec{u}$

$\therefore = \cancel{|\vec{v}|^2} - \cancel{2}\vec{u} \cdot \vec{v} + \cancel{|\vec{u}|^2}$

$= \cancel{|\vec{v}|^2} + \cancel{|\vec{u}|^2} - \cancel{2}|\vec{u}||\vec{v}| \cos\theta$

$\therefore \vec{u} \cdot \vec{v} = |\vec{u}||\vec{v}| \cos\theta$

ex2 $\vec{u} = \langle 2, 5 \rangle$

$\vec{v} = \langle 4, -3 \rangle$

$$\cos\theta = \frac{\vec{u} \cdot \vec{v}}{|\vec{u}||\vec{v}|} = \frac{8 - 15}{\sqrt{29 \cdot 25}} = \frac{-7}{5\sqrt{29}}$$

$$\theta = \cos^{-1}\left(\frac{-7}{5\sqrt{29}}\right) \approx \boxed{105.1°}$$

Corrollaries

① ORTHOGONAL (NORMAL) PERPENDICULAR

$$\boxed{\vec{u} \perp \vec{v} \iff \vec{u} \cdot \vec{v} = 0}$$

Proof "$\Rightarrow$" $\theta = 90° \Rightarrow \vec{u} \cdot \vec{v} = |\vec{u}||\vec{v}| \cos 90° = 0$

"$\Leftarrow$" $|\vec{u}||\vec{v}| \cos\theta = 0 \Rightarrow \theta = 90°$

ex3 a) $\vec{u} = \langle 3, 5 \rangle$ ORthogonal?

$\vec{v} = \langle 2, -8 \rangle$ $\quad \vec{u} \cdot \vec{v} = -34 \neq 0$ NO

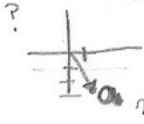
b) $\langle 2, 1 \rangle \cdot \langle -1, 2 \rangle = 0$ yes

★ ex ② Finding $\perp$ vector

$\vec{a} = \langle 1, -3 \rangle$

$\vec{b} = \langle 3, +1 \rangle$ $\quad \vec{b} \perp \vec{a}$ switch x, y flip one sign

Why?



$m_b = \frac{+1 \text{ up}}{3 \text{ right}} \rightarrow \vec{b} = \langle 3, 1 \rangle$

$m_a = -\frac{3}{1}$

ex $\vec{a} = \langle 2, 3\rangle$. Find $\perp$ vector

$\vec{b} = \underline{\langle -3, 2\rangle}$ DOT $= 0$ too

Find $\perp$ UNIT vector

$\vec{c} = \left\langle -\frac{3}{\sqrt{13}}, \frac{2}{\sqrt{13}} \right\rangle = \frac{\vec{b}}{|\vec{b}|}$ normalized

ex

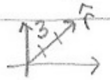
$\vec{a} = \langle -3, 4\rangle$

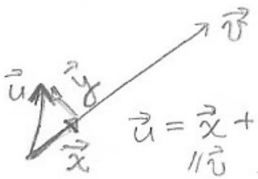
find a unit vector $\perp \vec{a}$

$\vec{b} = \frac{\langle -4, -3\rangle}{5} = \langle -\frac{4}{5}, -\frac{3}{5} \rangle$

---

③ 





$\vec{u} = \vec{x} + \vec{y}$

$\quad\ \ \parallel\vec{v} \quad \perp\vec{v}$

component of $\vec{u}$ along $\vec{v}$
(scalar length)

$\frac{|\vec{u}\cdot\vec{v}|}{|\vec{v}|} = |\vec{u}|\cos\theta$

PROJECTION of $\vec{u}$ onto $\vec{v}$
VECTOR

$$\boxed{\begin{gathered}\text{proj}_{\vec{v}}\vec{u} = \frac{\vec{u}\cdot\vec{v}}{|\vec{v}|}\,\frac{\vec{v}}{|\vec{v}|} = \vec{x} \\ \vec{y} = \vec{u} - \vec{x}\end{gathered}}$$

ex6 $\vec{u} = \langle -2, 9\rangle$

$\vec{v} = \langle -1, 2\rangle$

ASK ① draw
② $\text{proj}_{\vec{v}}\vec{u}$ is what in pic? ($\vec{x}$)
③ component of $\vec{u} \perp \vec{v}$ is?

a) $\text{proj}_{\vec{v}}\vec{u} = \frac{\vec{u}\cdot\vec{v}}{|\vec{v}|^2}\vec{v} = \frac{2+18}{5}\langle -1, 2\rangle = \langle -4, 8\rangle$

b) Resolve $\vec{u}$ into $\vec{x}$ parallel to $\vec{v}$
& $\vec{y}$ orthogonal to $\vec{v}$

$\vec{x} = \langle -4, 8\rangle$

$\vec{y} = \vec{u} - \vec{x} = \langle -2, 9\rangle - \langle -4, 8\rangle$

$= \langle 2, 1\rangle$

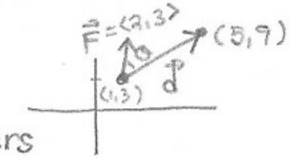
CW # 3, 12, 18, 20, 25

Work done by force in moving along $\vec{D}$

$W = \vec{F}\cdot\vec{d} = |\vec{F}||\vec{d}|\cos\theta$

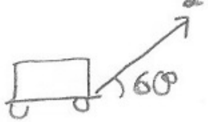
ex7 $\vec{F} = \langle 2, 3\rangle$ Newtons

move from $(1, 3)$ to $(5, 9)$ meters



$W = \vec{F}\cdot\vec{d} = \langle 2, 3\rangle \cdot \langle 4, 6\rangle = 8 + 18 = \boxed{26\text{ J}}$
N·m

ex8



move 100 m

$W = (20\cos 60°)\text{N}\,100\text{m}$

$= 20 \times \frac{1}{2} \times 100$ Nm

$= 1000$ J

OR $W = \vec{F}\cdot\vec{d}$

$= \langle 20\cos 60°, 20\sin 60° \rangle \cdot \langle 100, 0\rangle$

$= (10\hat{i} + 10\sqrt{3}\hat{j}) \cdot (100\hat{i})$

$= 10\hat{i}\cdot 100\hat{i} + 10\sqrt{3}\hat{j}\cdot 100\hat{i}$

$= 1000\underbrace{\hat{i}\cdot\hat{i}}_{|1|^2} + 1000\sqrt{3}\,\hat{j}\cdot\hat{i}$

90°
0

$= 1000$

Demo   CW: 30, 36, 38, 46, 47

36
41 $(\vec{u}-\vec{v})\cdot(\vec{u}+\vec{v}) \overset{\text{Proof}}{=} |\vec{u}|^2 - |\vec{v}|^2$

$(\vec{u}-\vec{v})\cdot\vec{u} + (\vec{u}-\vec{v})\cdot\vec{v}$

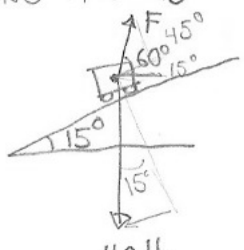
$= \vec{u}\cdot\vec{u} - \cancel{\vec{v}\cdot\vec{u}} + \cancel{\vec{u}\cdot\vec{v}} - \vec{v}\cdot\vec{v}$

38
44 $\vec{v}\cdot \text{proj}_{\vec{v}}\vec{u} = \vec{v}\cdot\left(\frac{\vec{u}\cdot\vec{v}}{|\vec{v}|^2}\vec{v}\right) = \frac{\vec{u}\cdot\vec{v}}{\cancel{|\vec{v}|^2}}\cancel{\vec{v}\cdot\vec{v}} = \vec{u}\cdot\vec{v}$

Guess: $\underbrace{|\vec{\text{proj}}|}_{|\vec{u}|\cos\theta}|\vec{v}|\cos 0^\circ$

$= \vec{u}\cdot\vec{v}$

46 Like HW 43
52



Keep from rolling down $|\vec{F}| = ?$

$|\vec{F}|\cos 45^\circ = 40\sin 15^\circ$ $\sin^2\theta = \frac{1-\cos 2\theta}{2}$

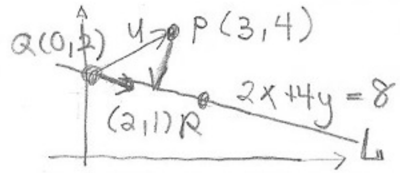
$|\vec{F}|\cdot\frac{\sqrt{2}}{2} = 40\,(+)\sqrt{\frac{1-\cos 30^\circ}{2}}$

$|\vec{F}|\frac{1}{\sqrt{2}} = 40\sqrt{\frac{1-\sqrt{3}/2}{\cancel{2}}}$

$|\vec{F}| = 40\sqrt{\frac{1-\sqrt{3}/2}{\cancel{2}}} = 40\sqrt{1-\frac{\sqrt{3}}{2}}$

$\approx 14.641\ ?$

53
47 distance between Point & Line (Hint: projection $\perp$)



$d(P, L) = \|\vec{u}_\perp\|$

$\vec{u}_\perp = \vec{u} - \vec{u}_{\parallel}$

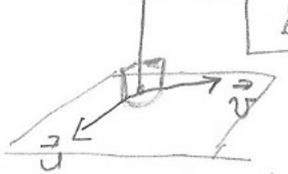
$\vec{u}_{\parallel} = \text{proj}_{\vec{v}}\vec{u}$

(6th ed Stewart 9.5)

Anton Calculus 11.4 Cross Product

$$\boxed{\underbrace{\vec{u}\times\vec{v}}_{\text{vector product}} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ u_1 & u_2 & u_3 \\ v_1 & v_2 & v_3 \end{vmatrix}}$$

★ $\boxed{\vec{u}\times\vec{v} \perp \vec{u} \ \& \ \vec{v}}$



Right-hand Rule

$\boxed{\text{ex1}}$ $\vec{u} = \langle 1, 2, -2\rangle$

$\vec{v} = \langle 3, 0, 1\rangle$

$$\vec{u}\times\vec{v} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & 2 & -2 \\ 3 & 0 & 1 \end{vmatrix} = \hat{i}(2+0) - \hat{j}(1+6) + \hat{k}(0-6)$$
$$= \langle 2, -7, -6\rangle$$

Properties

a) $\vec{u}\times\vec{v} = -(\vec{v}\times\vec{u})$ NOT COMMUTATIVE interchange 2 rows in determinant makes it negative!

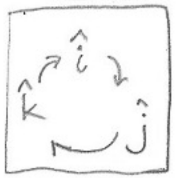
b) $\vec{u}\times(\vec{v}+\vec{w}) = (\vec{u}\times\vec{v}) + (\vec{u}\times\vec{w})$ distributive, Keep order

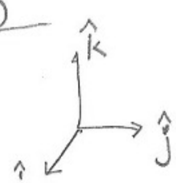
c) $(\vec{u}+\vec{v})\times\vec{w} = (\vec{u}\times\vec{w}) + (\vec{v}\times\vec{w})$

d) $k(\vec{u}\times\vec{v}) = (ku)\times\vec{v} = \vec{u}\times(k\vec{v})$

e) $\vec{u}\times\vec{0} = \vec{0}\times\vec{u} = \vec{0}$

f) $\vec{u}\times\vec{u} = \vec{0}$





$\hat{i}\times\hat{j} = \hat{k}$, $\hat{j}\times\hat{k} = \hat{i}$, $\hat{k}\times\hat{i} = \hat{j}$

$\hat{j}\times\hat{i} = -\hat{k}$, $\hat{k}\times\hat{j} = -\hat{i}$, $\hat{i}\times\hat{k} = -\hat{j}$

WARNING: not associative!

$$\hat{i}\times(\hat{j}\times\hat{j}) = \vec{0} \neq (\underbrace{\hat{i}\times\hat{j}}_{\hat{k}})\times\hat{j} = -\hat{i}$$

(ex1) $(1\hat{i}+2\hat{j}-2\hat{k})\times(3\hat{i}+\hat{k})$

$= 3\cancel{\hat{i}\times\hat{i}} + \hat{i}\times\hat{k} + 6\hat{j}\times\hat{i} + 3\hat{j}\times\hat{k} - 6\hat{k}\times\hat{i} - \cancel{2\hat{k}\times\hat{k}} = 3\hat{i} - 7\hat{j} - 6\hat{k}$

$-\hat{j} + -6\hat{k} + 3\hat{i} - 6\hat{j}$

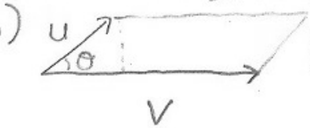
Thm $\begin{cases} \vec{u}\cdot(\underbrace{\vec{u}\times\vec{v}}_{\perp \vec{u}}) = 0 \\ \vec{v}\cdot(\vec{u}\times\vec{v}) = 0 \end{cases}$

$\boxed{\text{ex3}}$ find a vector orthogonal to both $\vec{u} = \langle 2, -1, 3\rangle$ and $\vec{v} = \langle -7, 2, -1\rangle$

$$\vec{u}\times\vec{v} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 2 & -1 & 3 \\ -7 & 2 & -1 \end{vmatrix} = -5\hat{i} - 19\hat{j} - 3\hat{k}$$

Thm

a) $\boxed{\|\vec{u}\times\vec{v}\| = |\vec{u}||\vec{v}|\sin\theta}$

b) 

$\boxed{\text{Area} = |\vec{u}\times\vec{v}|}$

c) $\boxed{\vec{u}\times\vec{v} = 0 \Leftrightarrow \vec{u} \parallel \vec{v}, \ \vec{u} = c\vec{v}}$ $\sin\theta = 0 \Rightarrow$

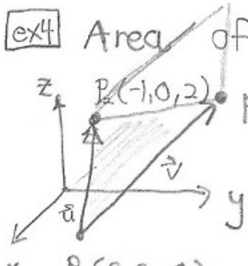
Proof (a) Bonus: Hint RHS → cos, proj → determinant

$$|\vec{u}||\vec{v}|\sin\theta = |u||V|\sqrt{1-\cos^2\theta}$$
$$= |u||v|\sqrt{1 - \frac{(\vec{u}\cdot\vec{v})^2}{|u|^2|v|^2}}$$
$$= \sqrt{|u|^2|v|^2 - (\vec{u}\cdot\vec{v})^2}$$
$$= \sqrt{(u_1^2+u_2^2+u_3^2)(v_1^2+v_2^2+v_3^2) - (u_1v_1+u_2v_2+u_3v_3)^2}$$
$$= \sqrt{(u_2v_3-u_3v_2)^2 + (u_1v_3-u_3v_1)^2 + (u_1v_2-u_2v_1)^2}$$

↓ matrix determinant

$$= |\vec{u}\times\vec{v}|$$

ex4 Area of triangle?

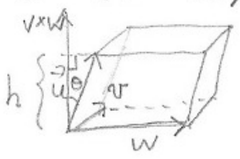


$$A = \frac{1}{2}\,|\vec{u} \times \vec{v}| = \frac{1}{2}\begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ -3 & -2 & +2 \\ -2 & +2 & +3 \end{vmatrix}$$

$$= \frac{15}{2}$$

<u>Scalar Triple Product</u>

$$\vec{u} \cdot (\vec{v} \times \vec{w}) = \vec{u} \cdot \underbrace{\vec{v} \times \vec{w}}_{1^{st} \text{ of course}} = \begin{vmatrix} u_1 & u_2 & u_3 \\ v_1 & v_2 & v_3 \\ w_1 & w_2 & w_3 \end{vmatrix}$$



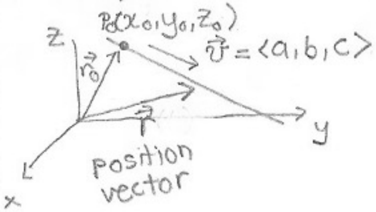
• instead of $\hat{i}$, component $\times u_1$, etc.

ex5 Calculate the scalar triple product $\vec{u} \cdot (\vec{v} \times \vec{w})$

Scan & 11.6 planes in 3space +HW

stewart

9.6 Equation of a line. Give point and parallel direction



$$\{\vec{r} : \vec{r} - \vec{r_0} = t\vec{v}\}$$

$$\boxed{\vec{r} = \vec{r_0} + \vec{v}\,t}$$
$$= \langle x_0, y_0, z_0 \rangle + \langle a, b, c \rangle t$$

Parametric equations for a line

$$\begin{cases} x(t) = x_0 + at \\ y(t) = y_0 + bt \\ z(t) = z_0 + ct \end{cases}$$

ex1 Find for line thru $(5,-2,3)$, $\vec{v} = \langle 3,-4,2 \rangle$

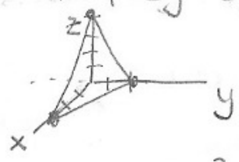
$$\vec{r} - \vec{r_0} = \vec{v}\,t$$

$$\vec{r} = \langle 5,-2,3 \rangle + \langle 3,-4,2 \rangle t$$

Ch8

Review: eqn of plane

$$4x + 6y + 3z = 12$$



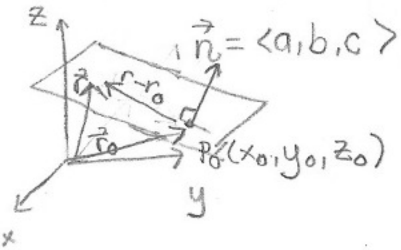
$y = z = 0 \Rightarrow x = 3$

$x = y = 0 \Rightarrow z = 4$

$x = z = 0 \Rightarrow y = +2$

Equation of Plane

$\begin{cases} \text{Given point} \\ \text{normal vector} \end{cases}$



$$\{\vec{r} : (\vec{r} - \vec{r}_0) \perp \vec{n}\}$$

$$\boxed{(\vec{r} - \vec{r}_0) \cdot \vec{n} = 0}$$

$$(\langle x, y, z \rangle - \langle x_0, y_0, z_0 \rangle) \cdot \langle a, b, c \rangle = 0$$

$$a(x - x_0) + b(y - y_0) + c(z - z_0) = 0$$

$\boxed{ex3}$ Eqn of plane $\vec{n} = \langle 4, -6, 3 \rangle$

$P(3, -1, -2)$

$$(\vec{r} - \vec{r}_0) \cdot \vec{n} = 0$$

$$4(x - 3) - 6(y + 1) + 3(z + 2) = 0$$

$$4x - 6y + 3z = 12$$