SAT Subject Test - Math Level 2

Lesson #1 Number Systems
Coordinate Systems

August 6, 2009 Thursday 10am~12pm

- Test Overview

I. $\mathbb{R}, \mathbb{Q}, \mathbb{Q}^c$, formulas, absolute values

III. Coordinates Rectangular in 3D §2.2 P.134~135
II. Exponents & Radicals §1.4 P.93 Part
IV. Pos & Angles §1.3 P.65-67 Part
V. Trig - unit circle, table, cofunction §1.3 P.61~65 Part
VI. Polar Coordinates §2.1 P.128~129
VII. Vector §3.5 P.158~160
VIII. Complex #s §3.2 P.142~145

HW ② booklet: show work. Give soln, do in lessons
★ which troubling

① HW #1

This Class

August
- 7 classes, Review exam topics
- Short quiz on previous lesson, timed like SAT 1min/question
- Homework - show work

Textbook: 1. Barron's SAT Subject Test Math Level 2
2008 8th Edition by Richard Ku and Howard Dodge

Homework: 2. Kaplan SAT Subject Test Mathematics Level 2
2007~2008 Edition

3. The Princeton Review, Cracking the SAT
Math 1 & 2 Subject Tests, 2007~2008 Edition
by Jonathan Spaihts

4. College Algebra & Trigonometry, 3rd ed. by Aufmann, Barker, Nation
Houghton Mifflin 1997

|Test Overview|

- 50 multiple-choice questions in 1 hour
  $\Rightarrow \approx 1$ minute/question

- Scoring 200~800 scale
  Raw Score = # correct $- \frac{1}{4}$ # incorrect rounded
  $R = C - \frac{1}{4} I$

correct +1 point
Incorrect $-\frac{1}{4}$ point
Blank 0 points

$\Rightarrow$
- Leave blank when really don't know the answer
- Guess wisely - if you can eliminate ≥ 2 answers

Total Scaled Score: Varies with test. Generally ≥ 44 correct $\Rightarrow$ 800. For each question incorrect with raw score less than 44, -10 points

$$\begin{cases} R \geq 44 & S = 800 \\ R < 44 & S = 800 - 10(44 - R) \end{cases}$$

Scaled	Raw
800	44
700	35
600	25
500	12

- Strategies/Approaches
  - Read questions carefully
  - easy questions 1st (tend to be at beginning of test) Return later to hard questions
  - Pace yourself
  - Mark the correct # on the answer sheet
  - Back solve if don't know
  - ~~The~~ Read all choices

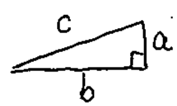
- 60% problems use calculator, but
  decide on solving strategy first
  Don't overuse calculator

Before the Test

- Know the content, topics on exam (preparation class)
- Know test format
- test directions
- practice under testing conditions - September
- Calculator - scientific or graphing
  - backup batteries / calculator

I. $\boxed{\text{The Real Number System}}$ (Algebra p. 2)

$\boxed{\text{Review}}$

- Pythagorean Thm  $c^2 = a^2 + b^2$
- $(a+b)^2 = a^2 + 2ab + b^2$
  $(a+b)(a-b) = a^2 - b^2$

10:20

Natural Numbers $\mathbb{N} = \{1, 2, 3, \dots\}$

Integers $\mathbb{Z} = \{\dots -2, -1, 0, 1, 2, \dots\}$

Rational Numbers $\mathbb{Q} = \{$#s that can be written as fractions$\}$

$\frac{1}{2}$, $0.\overline{3} = \frac{1}{3}$, $0.\overline{45} = \frac{5}{11}$

$= \{\frac{m}{n} : m \in \mathbb{Z}, n \in \mathbb{Z}, n \neq 0\}$

$= \{$all terminating or repeating decimals$\}$

Irrational Numbers $\mathbb{Q}^c = \{$numbers that cannot be put in fraction form$\}$

$\pi = 3.14159\dots$ $0.717717771\,7771\dots$ $= \{$all nonterminating nonrepeating decimals$\}$

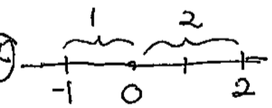
$e = 2.71828$

Real Numbers $\mathbb{R} = \{$all rational or irrational numbers$\}$
$= \mathbb{Q} \cup \mathbb{Q}^c$

$\boxed{\text{Absolute Values}}$ (Algebra p. 13)

Defn Absolute value of a real number a

$|a| \equiv \begin{cases} a & \text{if } a \geq 0 \\ -a & \text{if } a < 0 \end{cases}$ (distance from origin)

ex 

$|-1| = 1$ $|2| = 2$

Thms $|a| \geq 0$

$|ab| = |a||b|$

$\left|\frac{a}{b}\right| = \frac{|a|}{|b|}$

$|a-b| = |b-a|$, $|a| = |-a|$

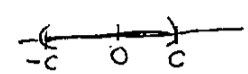
Triangle Inequality $|a+b| \leq |a| + |b|$

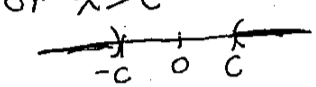
ex $|1 + (-2)| \leq |1| + |-2|$
$|-1| \leq 1 + 2$
$1 \leq 3$

ex Write w/o absolute value symbols

$\left|\frac{2x}{|x| + |x-2|}\right|$, given $0 < x < 2$

$= \frac{|2x|}{|x| + |x-2|} = \frac{2x}{x - (x-2)} = \frac{2x}{2} = x$

Note $|x| < c$, $c \geq 0 \iff -c < x < c$ 

$|x| > c$, $c \geq 0 \iff x < -c$ or $x > c$ 

ex Kaplan P.42 #10
Soln Set of $|\frac{2}{3}u - 5| > 8$

$\frac{2}{3}u - 5 < -8$ or $\frac{2}{3}u - 5 > 8$

$\frac{2}{3}u < -3$ or $\frac{2}{3}u > 13$

$u < -\frac{9}{?}$ or $u > \frac{39}{?}$

II. Exponents and Radicals

(Algebra P.17 §1.3, SAT §1.4 P.93 Part)

Rules

x, y, a, b are real numbers

Product	$x^a x^b = x^{a+b}$	$x^0 = 1 \quad (x \neq 0)$	$n \in \mathbb{N}$
Quotient	$\dfrac{x^a}{x^b} = x^{a-b}$	$x^{-a} = \dfrac{1}{x^a}$	$x^{\frac{1}{n}} = \sqrt[n]{x}$
Power	$(x^a)^b = x^{ab}$	$x^a y^a = (xy)^a$	

Where from? a, b $\begin{cases} \cdot \text{ Natural Numbers} \\ \cdot \text{ Rational Numbers} \\ \cdot \text{ Irrational ''} \end{cases}$

Natural Number Exponent

Defn $x \in \mathbb{R},\ n \in \mathbb{N}$

$$x^n \equiv \underbrace{x \cdot x \cdots x}_{n \text{ factors of } x}$$

Defn $x \neq 0,\ x \in \mathbb{R} \qquad x^0 \equiv 1$

Defn $x \neq 0,\ x \in \mathbb{R},\ n \in \mathbb{N}$

$$x^{-n} \equiv \frac{1}{x^n}$$

$\Rightarrow x^m x^n = x^{m+n}$

$(x^m)^n = x^{mn}$

$\dfrac{x^m}{x^n} = x^{m-n}$

Ex $\dfrac{5^{-2}}{7^{-1}} = \dfrac{7}{5^2} = \dfrac{7}{25}$

Undefined Expressions

$x \in \mathbb{R}$, ① $\dfrac{x}{0}$ ② 0^n, n is negative ③ 0^0

Why? undefined ① Say $x \neq 0$ suppose $\dfrac{x}{0} = c$ any $\mathbb{R}$

$x = 0 \cdot c = 0$

$x = 0$

$\rightarrow\leftarrow$

Say $x = 0$ $\frac{0}{0} = c \Rightarrow 0 = 0 \cdot c \Rightarrow c = 0$ But $\dfrac{x}{x} = 1$

② $0^{-2} = \dfrac{1}{0^2} = \dfrac{1}{0}$ undefined

③ 0^0 undefined

$x^0 \equiv 1 \quad | \quad 0^n = 0$

$0^0 = 1$

Ex $\dfrac{x^0 y^{-3}}{z-4}$ Need assume $x \neq 0,\ y \neq 0,\ z \neq 4$

Rational Exponent

Defn $n \in \mathbb{N}$ $\quad x^{1/n} \equiv \sqrt[n]{x}$ $\qquad$ meaningful $(x^{1/n})^n = x^1$

Note $\sqrt[n]{x} \geq 0$

$\left[\begin{array}{l} 2^4 = 16 \\ (-2)^4 = 16 \end{array}\right. \qquad \sqrt[4]{16} = 2$

Defn ~~n~~ $m, n \in \mathbb{N},\ x \in \mathbb{R}$

$x^{m/n} \equiv (x^{\frac{1}{n}})^m = (x^m)^{\frac{1}{n}}$

$(\sqrt[n]{x})^m = \sqrt[n]{x^m}$

can still show rules apply

Irrational Exponent

$x^{\pi} = x^{3.141\ldots} = x^{3 + \frac{1}{10} + \frac{4}{100} + \frac{1}{1000} + \cdots}$

$= x^3\, x^{\frac{1}{10}}\, x^{\frac{4}{100}}\, x^{\frac{1}{1000}} \cdots$

Simplifying Even Roots

Ex $\sqrt{x^2}$ $\quad x = 2 \quad \sqrt{2^2} = \sqrt{4} = 2$

$x = -2 \quad \sqrt{(-2)^2} = \sqrt{4} = 2$

$$\boxed{\sqrt{x^2} = \begin{cases} x & x \geq 0 \\ -x & x < 0 \end{cases} = |x|}$$

ex $\sqrt[3]{x^3} = x$ $\quad \sqrt[3]{-1} = -1$

Thm $\boxed{\begin{array}{l} n \text{ even natural number } \sqrt[n]{x^n} = |x| \\ n \text{ odd natural number } \sqrt[n]{x^n} = x \end{array}}$

ex Simplify $\sqrt[4]{32x^3y^4} = \sqrt[4]{2^4\, 2\, x^3 y^4}$

$= 2\sqrt[4]{2x^3} \quad 2|y|\sqrt[4]{2x^3}$

ex P.24
Rationalize the denominator

a) $\dfrac{2}{\sqrt{3}+\sqrt{2}}\left(\dfrac{\sqrt{3}-\sqrt{2}}{\sqrt{3}-\sqrt{2}}\right) = \dfrac{2\sqrt{3}-2\sqrt{2}}{3-2}$

b) $\dfrac{a+\sqrt{5}}{a-\sqrt{5}}\left(\dfrac{a+\sqrt{5}}{a+\sqrt{5}}\right) = \dfrac{a^2+2a\sqrt{5}+5}{a^2-5}$

ex P.94 $\dfrac{3^{n-2}\cdot 9^{2-n}}{3^{2-n}} = \dfrac{3^{n-2}\; 3^{2(2-n)}}{\cancel{3^{2-n}}} = 3^{n-2+2-n} = 3^0 = 1$

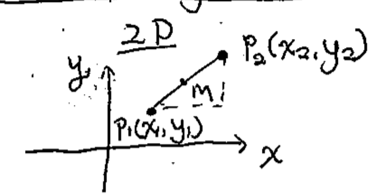
ex $(\sqrt[3]{2})(\sqrt[5]{4})(\sqrt[9]{8}) = 2^{\frac{1}{3}+\frac{2}{5}+\frac{3}{9}} \approx 2.1$

ex ($y^2 = x \Rightarrow y = \pm\sqrt{x}$ $\quad y^2=4,\ y=2$ or -2) $\quad$ ($y^3 = x \quad y = \sqrt[3]{x}$) ←

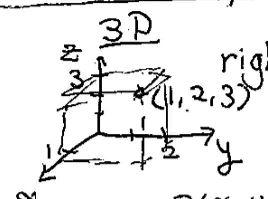
III Rectangular (Euclidean) Coordinate System

SAT P.134

2D



3D right-handed system



Midpoint $M =$
$\left(\dfrac{x_1+x_2}{2}, \dfrac{y_1+y_2}{2}\right)$

$P_1(x_1,y_1,z_1)$ $\quad P_2(x_2,y_2,z_2)$

$M = \left(\dfrac{x_1+x_2}{2}, \dfrac{y_1+y_2}{2}, \dfrac{z_1+z_2}{2}\right)$

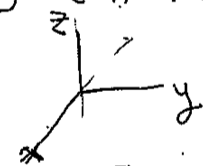
Distance by Pythag. Thm

$d(P_1,P_2) = d(P_2,P_1) \geq 0$
$= \sqrt{(x_2-x_1)^2+(y_2-y_1)^2}$

Distance (Pythag Thm twice)

$d(P_1,P_2) = \sqrt{(x_2-x_1)^2+(y_2-y_1)^2+(z_2-z_1)^2}$

ex $(-4,0,7)$ on which plane?



xz plane
$y=0$

10:50

(Polar & Trig.) $\quad \angle$ in degree or radians $\left[\begin{array}{l}360^\circ \\ 2\pi \text{ radians}\end{array}\right.$

IV Arcs & Angles

P.65

• The number π $\quad C = 2\pi$

Defn
One radian = measure of the central angle subtended by an arc of length r on a circle with radius r

Arc Length

length of arc subtended by θ on circle w/ radius r

$\boxed{s = r\theta}$

$\dfrac{s}{\theta} = \dfrac{r}{1}$

Radian Degree Conversion

Know $\quad 2\pi \quad \theta = \dfrac{s}{r} = \dfrac{2\pi}{r} \Rightarrow 360^\circ = 2\pi$

$\boxed{\dfrac{\pi \text{ radians}}{180^\circ}}$

P.65

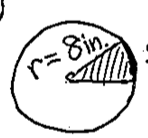
ex 1 $\quad 30^\circ\left(\dfrac{\pi}{180^\circ}\right) = \dfrac{\pi}{6}$ radians

$\dfrac{\pi}{4}\left(\dfrac{180^\circ}{\pi}\right) = 45^\circ$

Formulas $\quad$ θ in radians
arc length $s = r\theta$
area of sector $A = \frac{1}{2}r^2\theta$

$C = 2\pi r$
area circle $= \pi r^2$

ⓔⓧ SAT 1.66



arc length $s = 6\pi$ in.

area?

$s = r\theta$

$\text{area} = \frac{1}{2}r^2\theta = \frac{1}{2}r\,(r\theta)$

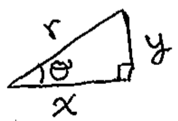
$= \frac{1}{2}(8\text{in})\,6\pi\text{ in.}$

$= 24\pi\text{ in.}^2$

V. Trigonometry & Unit Circle

SAT §1-3 P.61~65

• Right Triangles



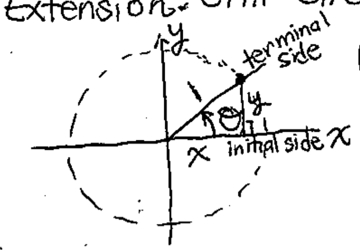
$0 < \theta < 90^\circ$

$\sin\theta = \frac{y}{r}$ $\qquad \csc\theta = \frac{r}{y} = \frac{1}{\sin\theta}$

$\cos\theta = \frac{x}{r}$ $\qquad \sec\theta = \frac{r}{x} = \frac{1}{\cos\theta}$

$\tan\theta = \frac{y}{x} = \frac{\sin\theta}{\cos\theta}$ $\qquad \cot\theta = \frac{x}{y} = \frac{1}{\tan\theta} = \frac{\cos\theta}{\sin\theta}$

• Extension - Unit Circle



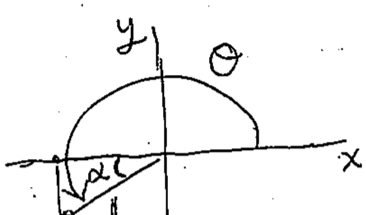
(counterclockwise $\theta \ge 0$
clockwise $\theta < 0$)

$\sin\theta = \frac{y}{1}$

$\cos\theta = \frac{x}{1}$

$\tan\theta = y/x$

ⓔⓧ. $\theta = 0 \begin{cases} y = 0 & \sin 0^\circ \\ x = 1 & \cos 0^\circ \end{cases}$

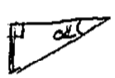


angle α = reference angle
(smallest between terminal side and x-axis)

$0^\circ \le \alpha \le 90^\circ$

$|\sec\theta_1| = |\sec\theta_2| = |\sec\theta_3| = |\sec\theta_4| = \sec\alpha$

ⓔⓧ $\underset{\sin\theta}{y} < 0$ (lengths same) $\quad \underset{\cos\theta}{x} < 0$



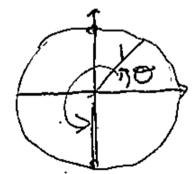
$\left.\begin{matrix}\sin\alpha \\ \cos\alpha\end{matrix}\right\} > 0$

$\boxed{\begin{aligned}&\text{Trig. fn. of } \theta \\ &= \pm\text{ Trig fn. of } \alpha\end{aligned}}$

All Students Take Calculus

II	I
$x<0, y>0$	$x>0, y>0$
only $\sin\theta$, $\csc\theta > 0$	All trig fns. > 0
III	**IV**
$x<0, y<0$	$x>0, y<0$
only $\tan\theta$, $\cot\theta > 0$	only $\cos\theta$, $\sec\theta > 0$

Know



$-1 \le \sin\theta \le 1$

$-1 \le \cos\theta \le 1$

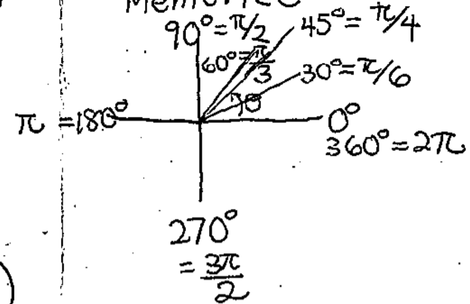
• θ in degrees or radians - Calculator mode

$\sin(\theta + k2\pi) = \sin\theta$

trig

~~θ 0° 30° 60°~~

Memorize

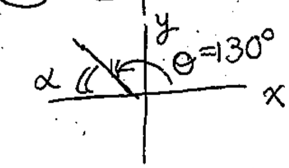


Memorize

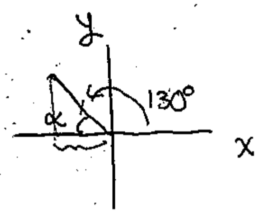
θ	0°	30° $\pi/6$	45° $\pi/4$	60° $\pi/3$	90° $\pi/2$
$\sin\theta$	0	$1/2$	$\sqrt{2}/2$	$\sqrt{3}/2$	1
$\cos\theta$	1	$\sqrt{3}/2$	$\sqrt{2}/2$	$1/2$	0
$\tan\theta$	0	$\sqrt{3}/3 = 1/\sqrt{3}$	1	$\sqrt{3}$	∞ (90° not in domain)

P.63

ex Express $\cos 130^\circ$ in terms of $\angle\alpha$



Soln.



$\cos 130^\circ$ negative

$\left[\begin{array}{l}\alpha = 180^\circ - 130^\circ = 50^\circ \\ \cos\alpha > 0\end{array}\right.$ $\Rightarrow$ $\cos 130^\circ = -\cos 50^\circ$

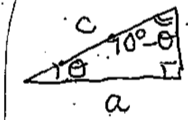
★ Cofunction Pairs $\left[\begin{array}{l}\sin \ \& \ \cos \\ \tan \ \& \ \cot \\ \sec \ \& \ \csc\end{array}\right.$

$\sin\alpha = \cos\beta \Longleftarrow_{\alpha,\beta}$ complementary angles $\alpha+\beta=90^\circ$

Cofunctions of complementary angles are equal

(P.430 Algebra)

Visualize



$\sin\theta = \frac{b}{c} = \cos(90^\circ-\theta)$

etc.

True for any θ

$\sin(90^\circ-\theta)=\cos\theta$ $\cos(90^\circ-\theta)=\sin\theta$

$\tan(90^\circ-\theta)=\cot\theta$ $\cot(90^\circ-\theta)=\tan\theta$

$\sec(90^\circ-\theta)=\csc\theta$ $\csc(90^\circ-\theta)=\sec\theta$

SAT P.64

ex If both angles are acute & $\sin(3x+20^\circ)=\cos(2x-40^\circ)$ Find x

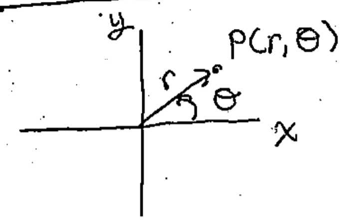
soln: $3x+20^\circ + 2x - 40^\circ = 90^\circ$

$5x - 20^\circ = 90^\circ$

$x = 22^\circ$

ex ~~x is an angle in quadrant III~~ 11:10 11:20

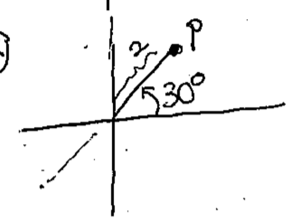
$\boxed{\text{VI. Polar Coordinates}}$ §2.1 P.128~129



$\left[\begin{array}{l}\text{counterclockwise } \theta \geq 0 \\ \text{clockwise } \theta < 0\end{array}\right.$

$\left[\begin{array}{l}r \geq 0 \text{ emanate outwards} \\ r < 0 \text{ go in other direction}\end{array}\right.$

ex

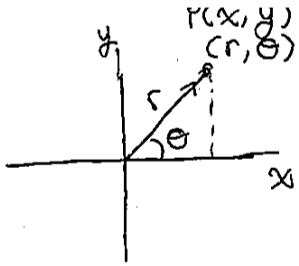


$(2, 30^\circ) = (2, 390^\circ) = (2, -330^\circ)$

$= (-2, 210^\circ) = (-2, -150^\circ)$

$(r, \theta) = (r, \theta + 2\pi n)$ $n \in \mathbb{Z}$

$= (-r, \theta + (\text{odd}\# \cdot \pi))$



Polar → Rectangular

$y = r\sin\theta$

$x = r\cos\theta$

Rectangular → Polar

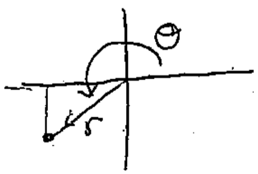
$r = \sqrt{x^2+y^2}$

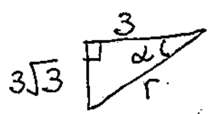
$\theta = \tan^{-1}\left(\frac{y}{x}\right)$

careful of this

Calculator: $-\frac{\pi}{2} < \theta < \frac{\pi}{2}$

(ex) Point P has rectangular coordinates $(-3, -3\sqrt{3})$. P in polar coordinates?



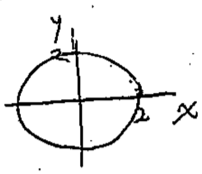


$$\left\{\begin{array}{l} r = \sqrt{3^2+(3\sqrt{3})^2} = \sqrt{36} = \boxed{6} \\ \alpha = \tan^{-1}\left(\frac{3\sqrt{3}}{3}\right) = \tan^{-1}(\sqrt{3}) = 60^\circ \\ \theta = 180^\circ + 60^\circ = \boxed{240^\circ} \\ \boxed{P = (6, 240^\circ)} \end{array}\right.$$

(Notice $\tan^{-1}\left(\frac{-3\sqrt{3}}{-3}\right) = 60^\circ$)

(ex) Describe the graphs

a) $r = 2$

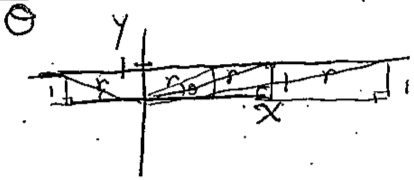


$\{(r,\theta) \mid r=2, \ \theta \text{ anything}\}$

b) $r = \frac{1}{\sin\theta}$

$r\sin\theta = 1$

$y = 1$



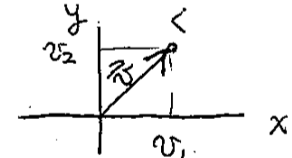
$\{(r,\theta) \mid \sin\theta = 1/r\}$

11:30

VII. Vectors §3.5 P.150–160

Defn: Vector is an ordered pair of real numbers

$\vec{v} = \langle v_1, v_2 \rangle$, $v_1, v_2 \in \mathbb{R}$



Note: • A vector is not a point (it's an arrow)

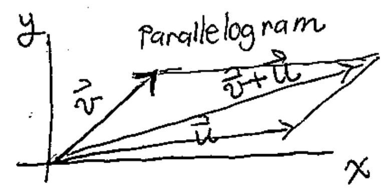
• A vector has both magnitude and direction

• $\boxed{\text{Norm}}$ $\|\vec{v}\|$ • $\boxed{\text{mag}}$ $|\vec{v}| = \boxed{\sqrt{v_1^2+v_2^2}}$

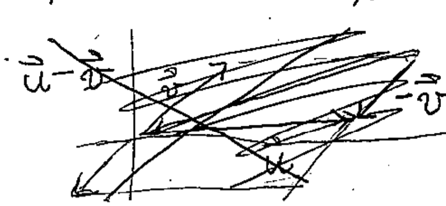
• Addition $\vec{v} = \langle v_1, v_2 \rangle$

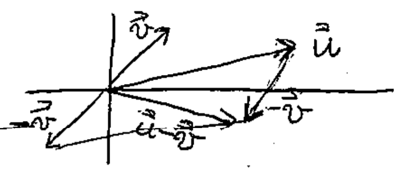
$\vec{u} = \langle u_1, u_2 \rangle$

$$\boxed{\begin{array}{l}\text{Resultant Vector} \\ \vec{v} + \vec{u} \equiv \langle v_1+u_1, v_2+u_2 \rangle\end{array}}$$

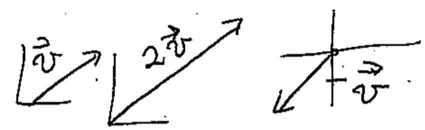


$\vec{u} - \vec{v}$?



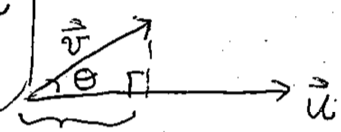


• $\boxed{\begin{array}{l} c \in \mathbb{R} \quad \text{Scalar mult.} \\ c\vec{v} \equiv \langle cv_1, cv_2 \rangle \end{array}}$



• Dot Product (inner product) = scalar

$$\boxed{\begin{array}{l} \vec{u} \cdot \vec{v} \equiv u_1v_1 + u_2v_2 = \vec{v} \cdot \vec{u} \\ \quad = \|\vec{u}\| \|\vec{v}\| \cos\theta \\ \text{in physics} \end{array}}$$



projection: $\|\vec{v}\|\cos\theta$

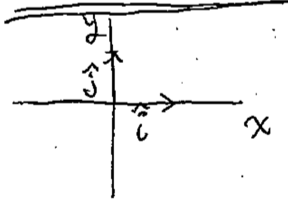
$\boxed{\vec{u} \perp \vec{v} \iff \vec{u} \cdot \vec{v} = 0}$

$\|\vec{u}\| \|\vec{v}\| \cos 90° = 0$

$\vec{v}$ ⌞ $\vec{u}$

Note: $\langle a,b \rangle \perp \langle b,-a \rangle$
$\langle a,b \rangle \perp \langle -b,a \rangle$

- Unit Vectors



$\hat{i} \equiv \langle 1,0 \rangle$

$\hat{j} \equiv \langle 0,1 \rangle$

- A unit vector $\vec{u} = \langle u_1, u_2 \rangle$ has magnitude 1 $\sqrt{u_1^2+u_2^2} = 1$

$\boxed{\vec{v} = \langle v_1, v_2 \rangle = v_1 \hat{i} + v_2 \hat{j}}$

check $v_1 \langle 1,0 \rangle + v_2 \langle 0,1 \rangle = \langle v_1, 0 \rangle + \langle 0, v_2 \rangle$
$= \langle v_1 + 0, 0 + v_2 \rangle$

ex $\vec{u} = \langle 6,-4 \rangle$, $\vec{v} = \langle 2,3 \rangle$

(a) $\vec{u}+\vec{v} = \langle 6+2, -4+3 \rangle = \langle 8,-1 \rangle$

b) $\|\vec{u}\| = \sqrt{6^2+16} = \sqrt{52} = 2\sqrt{13}$

c) $\vec{v}$ in terms of $\hat{i}, \hat{j}$

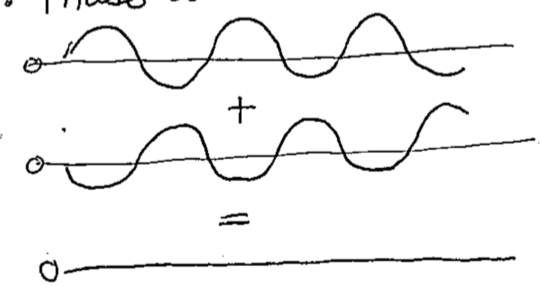
$\vec{v} = 2\hat{i} + 3\hat{j}$

d) $\vec{u} \perp \vec{v}$? $\vec{u} \cdot \vec{v} = 6 \cdot 2 + (-4)3$
$= 12 - 12 = 0$

yes

$\boxed{\text{VIII Complex Numbers}}$ §3.2 P.142 – P.1104

- App: Phase of waves, Convenient representation



Defn The Imaginary # $\boxed{i \equiv \sqrt{-1}}$

	Value
i	i
i^2	-1
i^3	$-i$
i^4	1
i^5	i

$-i^2 = -(-1)$

$\boxed{\begin{array}{l} n \in \mathbb{N} \\ i^n = i^{n \bmod 4} \\ \text{remainder of } n/4 \end{array}}$

ex $i^{58} = i^{4(14)+2} = i^2 = -1$

$\sqrt{-12} = \sqrt{i^2 2^2 3} = 2i\sqrt{3}$ *Calculator in $a+bi$ mode else NONREAL ANS.

Standard Form of a complex number

Defn. z complex number

$z = a+bi$ where $a, b \in \mathbb{R}$
a = real part
b = imaginary part

Defn Complex conjugate

$\bar{z} \equiv a - bi$

$\bar{\bar{z}} = z$

$\overline{z+w} = \bar{z} + \bar{w}$

$\overline{zw} = \bar{z}\,\bar{w}$

$\overline{\left(\frac{z}{w}\right)} = \frac{\bar{z}}{\bar{w}}$

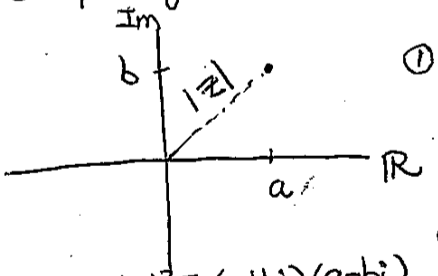
$z \in \mathbb{R}$ iff $z = \bar{z}$

Addition $z = a+bi$
$w = c+di$

$z+w = (a+c) + (b+d)i$

Multiplication $zw = (a+bi)(c+di) = ac + adi + bci + bdi^2$
$= (ac-bd) + (ad+bc)i$

• Graphing $z = a + bi$



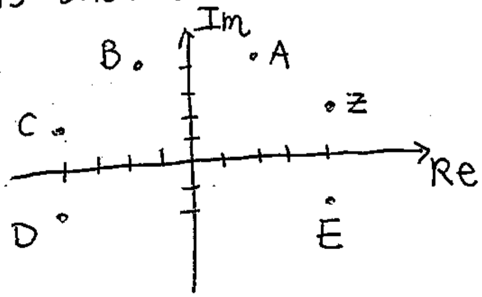
① modulus = distance from origin

$$\boxed{|z| \equiv \sqrt{a^2+b^2}}$$

② $|z|^2 = (a+bi)(a-bi)$
$= z\bar{z}$

③ $|zw| = |z||w|$
$\left|\frac{z}{w}\right| = \frac{|z|}{|w|}$ $w \neq 0$
$|z+w| \le |z| + |w|$ Triangle ineqlty

ex p.144
z is shown. Which point could be iz?



$z = a+bi = 4+2i$
$iz = i(4+2i) = 4i + 2i^2 = \cancel{4i-2}$
$= -2+4i$ (B)

ex Write $\frac{2-7i}{3+5i}$ in standard form

② $\boxed{a^2+b^2 = (a+bi)(a-bi)}$

$$\frac{2-7i}{3+5i} \cdot \frac{3-5i}{3-5i} = \frac{6-35+(-21-10)i}{9+25} = -\frac{29}{34} - \frac{31}{34}i$$