53. $\dfrac{x}{2} + \dfrac{y}{5} + \dfrac{z}{4} = 1$

$10x + 4y + 5z = 20$

54. $4x + 3y + 6z - 12 = 0$

55. $\dfrac{x}{3} + \dfrac{y}{5} = 1$

$5x + 3y = 15$

56. $3y + 4z - 12 = 0$

REVIEW EXERCISES

1. (a) $\overline{PQ} = \sqrt{(7-3)^2 + (-5-2)^2 + (-4-[-1])^2} = \sqrt{16+49+25} = 3\sqrt{10}$

(b) Midpoint of $\overline{QR}$: $\left(\dfrac{7+5}{2}, \dfrac{-5+6}{2}, \dfrac{4-3}{2}\right) = \left(6, \dfrac{1}{2}, \dfrac{1}{2}\right)$

(c) Let $X = (x, y, z)$. Then

$$(7,-5,4) = \left(\frac{3+x}{2}, \frac{2+y}{2}, \frac{-1+z}{2}\right) \Longrightarrow (x,y,z) = (11,-12,9).$$

(d) Midpoint of $\overline{PR}$: $(4,4,-2)$; radius of the sphere:

$$r = \frac{1}{2}|\overline{PR}| = \frac{1}{2}\sqrt{(5-3)^2 + (6-2)^2 + (-3+1)^2} = \sqrt{6}.$$

The equation of the sphere is: $(x-4)^2 + (y-4)^2 + (z+2)^2 = 6.$

2. (a) $\overline{PQ} = \sqrt{6^2 + 1^2 + (-7)^2} = \sqrt{36+1+49} = \sqrt{86}$

(b) Midpoint of $\overline{QR}$: $(\dfrac{-2+1}{2}, \dfrac{1-1}{2}, \dfrac{4-6}{2}) = (-\frac{1}{2}, 0, -1)$

(c) Let $X = (x, y, z)$. Then

$$(-2,1,4) = \left(\frac{4+x}{2}, \frac{2+y}{2}, \frac{-3+z}{2}\right) \Longrightarrow (x,y,z) = (-8,0,11).$$

(d) Midpoint of $\overline{PR}$: $\left(\frac{5}{2}, \frac{1}{2}, -\frac{9}{2}\right)$; radius of sphere:.

$$r = \frac{1}{2}|\overline{PR}| = \frac{1}{2}\sqrt{(4-1)^2 + (2+1)^2 + (-3+6)^2} = \tfrac{1}{2}\sqrt{27}.$$

The equation of the sphere is: $(x-\frac{5}{2})^2 + (y-\frac{1}{2})^2 + (z+\frac{9}{2})^2 = \frac{27}{4}.$

3. radius: $\sqrt{2^2 + (-3)^2 + 1^2} = \sqrt{14}$

equation: $(x-2)^2 + (y+3)^2 + (z-1)^2 = 14$

4. radius: $\frac{1}{2}\sqrt{4^2 + 6^2 + 4^2} = \sqrt{17}$

midpoint: $\left(\dfrac{-1+3}{2}, \dfrac{4-2}{2}, \dfrac{2+6}{2}\right) = (1,1,4)$

equation: $(x-1)^2 + (y-1)^2 + (z-4)^2 = 17$

5. By completing the square, the equation can be written as

$$(x+1)^2 + (y+2)^2 + (z-4)^2 = 4 = 2^2.$$

center: $(-1, -2, 4)$. radius: 2

6. By completing the square, the equation can be written as

$$(x-3)^2 + (y+5)^2 + (z-1)^2 = 33.$$

center: $(3, -5, 1)$ radius: $\sqrt{33}$

7. $\frac{3}{2}\,\mathbf{i} + \mathbf{j} - \frac{1}{2}\,\mathbf{k}$

8. $29\,\mathbf{i} + 2\,\mathbf{j} - 4\,\mathbf{k}$

9. $\mathbf{b} + \mathbf{c} = 3\,\mathbf{i} + 7\,\mathbf{j} + \mathbf{k}, \quad \mathbf{a} \cdot (\mathbf{b} + \mathbf{c}) = (3\,\mathbf{i} + 2\,\mathbf{j} - \mathbf{k}) \cdot (3\,\mathbf{i} + 7\,\mathbf{j} + \mathbf{k}) = 22$

10. $\mathbf{a} + \mathbf{b} = 8\,\mathbf{i} + 5\,\mathbf{j} - \mathbf{k}); \quad \|\mathbf{a} + \mathbf{b}\| = \sqrt{64 + 25 + 1} = 3\sqrt{10}$

11. $\|\mathbf{c}\|^2 = (-2)^2 + 4^2 + 1^2 = 21$

12. $\mathbf{b} \times \mathbf{b} = 0$. Therefore $\|\mathbf{b} \times \mathbf{b}\| = 0$

13. $2\mathbf{a} - \mathbf{b} = \mathbf{i} + \mathbf{j} - 2\,\mathbf{k}; \quad (2\mathbf{a} - \mathbf{b}) \cdot \mathbf{c} = (\mathbf{i} + \mathbf{j} - 2\,\mathbf{k}) \cdot (-2\,\mathbf{i} + 4\,\mathbf{j} + \mathbf{k}) = 0$

14. $\mathbf{b} + \mathbf{c} = 3\,\mathbf{i} + 7\,\mathbf{j} + \mathbf{k}; \quad \mathbf{a} \times (\mathbf{b} + \mathbf{c}) = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 3 & 2 & -1 \\ 3 & 7 & 1 \end{vmatrix} = 9\,\mathbf{i} - 6\,\mathbf{j} + 15\,\mathbf{k}$

15. $\|\mathbf{a}\| = \sqrt{14}; \quad \mathbf{u_a} = \frac{1}{\sqrt{14}}(3\,\mathbf{i} + 2\,\mathbf{j} - \mathbf{k})$

16. $\|\mathbf{c}\| = \sqrt{21}; \quad -\mathbf{u_c} = -\frac{1}{\sqrt{21}}(-2\,\mathbf{i} + 4\,\mathbf{j} + \mathbf{k})$

17. $\cos\theta = \frac{\mathbf{a} \cdot \mathbf{c}}{\|\mathbf{a}\|\|\mathbf{c}\|} = \frac{1}{\sqrt{14}\sqrt{21}} = \frac{1}{7\sqrt{6}}; \quad \theta \cong 1.51$ radians

18. $\cos\theta = \frac{\mathbf{b} \cdot \mathbf{c}}{\|\mathbf{b}\|\|\mathbf{c}\|} = \frac{2}{\sqrt{34}\sqrt{21}}; \quad \theta \cong 85.7^\circ$

19. $\|\mathbf{a}\| = \sqrt{14}; \quad \cos\alpha = \frac{3}{\sqrt{14}},\ \alpha \cong 0.64$ radians, $\cos\beta = \frac{2}{\sqrt{14}},\ \beta \cong 1.01,\ \cos\gamma = \frac{-1}{\sqrt{14}},\ \gamma \approx 1.84$

20. $\text{comp}_{\,\mathbf{a}}\mathbf{b} = \mathbf{b} \cdot \mathbf{u_a} = (5\,\mathbf{i} + 3\,\mathbf{j}) \cdot \frac{1}{\sqrt{14}}(3\,\mathbf{i} + 2\,\mathbf{j} - \mathbf{k}) = \frac{21}{\sqrt{14}}$

21. $\mathbf{b} \times \mathbf{c} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 5 & 3 & 0 \\ -2 & 4 & 1 \end{vmatrix} = 3\,\mathbf{i} - 5\,\mathbf{j} + 26\,\mathbf{k};$

$\text{comp}_{\,\mathbf{a}}(\mathbf{b} \times \mathbf{c}) = (\mathbf{b} \times \mathbf{c}) \cdot \mathbf{u_a} = (3\,\mathbf{i} - 5\,\mathbf{j} + 26\,\mathbf{k}) \cdot \frac{1}{\sqrt{14}}(3\,\mathbf{i} + 2\,\mathbf{j} - \mathbf{k}) = -\frac{27}{\sqrt{14}}$

22. $\mathbf{a}\times\mathbf{c} = 6\,\mathbf{i} - \mathbf{j} + 16\,\mathbf{k},\ \|\mathbf{a}\times\mathbf{c}\| = \sqrt{293};\quad \mathbf{u} = \pm\dfrac{6\,\mathbf{i} - \mathbf{j} + 16\,\mathbf{k}}{\sqrt{293}}$

23. $V = |(\mathbf{a}\times\mathbf{b})\cdot\mathbf{c}|;\quad (\mathbf{a}\times\mathbf{b})\cdot\mathbf{c} = \begin{vmatrix} 3 & 2 & -1 \\ 5 & 3 & 0 \\ -2 & 4 & 1 \end{vmatrix} = -27;\quad V = |-27| = 27$

24. $A = \dfrac{1}{2}\|\mathbf{a}\times\mathbf{b}\|;\quad \mathbf{a}\times\mathbf{b} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 3 & 2 & -1 \\ 5 & 3 & 0 \end{vmatrix} = 3\,\mathbf{i} - 5\,\mathbf{j} - \mathbf{k};\quad A = \dfrac{1}{2}\sqrt{35}$

25. (a) Direction vector: $\overrightarrow{QR} = (6, -3, 3)$; scalar parametric equations for the line:

$$x = 1 + 6t,\quad y = 1 - 3t,\quad z = 1 + 3t.$$

(b) Normal vector: $\overrightarrow{PR} = (3, -3, 2)$; equation of the plane:

$$3(x-1) + (-3)(y-1) + 2(z-1) = 0.$$

(c) A normal vector for the plane is: $\overrightarrow{QR}\times\overrightarrow{PR} = (3, -3, -9)$ or $\mathbf{N} = (1, -1, -3)$;
an equation for the plane: $(x-1) - (y-1) - 3(z-1) = 0$

26. (a) Direction vector: $\overrightarrow{PQ} = (4, -7, 5)$; scalar parametric equations for the line:

$$x = 5 + 4t,\quad y = 6 - 7t,\quad z = -3 + 5t.$$

(b) Scalar parametric equations for the line l through P and Q are:

$$x = 3 + 4t,\quad y = 2 - 7t,\quad z = -1 + 5t.$$

For any point S on l, the vector $\overrightarrow{RS}$ is $(4t-2, -7t-4, 5t+2)$. We want $\overrightarrow{RS}\perp\overrightarrow{PQ}$:

$$(4t-2, -7t-4, 5t+2)\cdot(4, -7, 5) = 0 \Longrightarrow t = -\frac{1}{3}.$$

Therefore $\left(-\frac{10}{3}, -\frac{5}{3}, \frac{1}{3}\right)$ is a direction vector for the line that passes through R perpendicular to PQ. Scalar parametric equations for this line are:

$$x = 5 - \frac{10}{3}t,\quad y = 6 - \frac{5}{3}t,\quad z = -3 + \frac{1}{3}t.$$

(c) This plane is determined by the points P, Q, R. A normal vector for the plane is:

$$\overrightarrow{PQ}\times\overrightarrow{PR} = -6\,\mathbf{i} + 18\,\mathbf{j} + 30\,\mathbf{k}\quad\text{or}\quad \mathbf{i} - 3\,\mathbf{j} - 5\,\mathbf{k}.$$

An equation for the plane is: $(x-3) - 3(y-2) - 5(z+1) = 0$ or $x - 3y - 5z = 2$.

27. Solve, if possible, the system of equations: $t = 1 - u,\quad -t = 1 + 3u,\quad -6 + 2t = 2u$. In this case, the solution is $t = 2, u = -1$. The lines intersect at the point $(2, -2, -2)$.

28. Solve, if possible, the system of equations: $1 - 2t = 3 + 2u,\quad 3 + 3t = 1 - u,\quad 5t = 6 + 3u$. In this case there is no solution; the lines are skew.

29. The lines l_1 and l_2 written in scalar parametric form are:

$$l_1: x = 1 + 2t, \quad y = -2 - t, \quad z = 3 + 4t; \quad l_2: \ x = -2 + u, \quad y = 3 + 3u, \quad z = u.$$

Solve, if possible, the system of equations: $1 + 2t = -2 + u$, $-2 - t = 3 + 3u$, $3 + 4t = u$. In this case there is no solution; the lines are skew.

30. Scalar parametric equations for the line l are: $x = -1 + t$, $y = -2 + t$, $-1 + t$ and a direction vector for l is $\mathbf{d} = (1, 1, 1)$. The point $Q\ (-1 + t, -2 + t, -1 + t)$ is a point on l and $\overrightarrow{PQ} = (-4 + t, -3 + t, 1 + t)$ is the vector from P to Q. We want $\mathbf{d} \cdot \overrightarrow{PQ} = 0$:

$$(-4 + t, -3 + t, 1 + t) \cdot (1, 1, 1) = 0 \Longrightarrow 3t = 6 \quad \text{and} \quad t = 2.$$

Therefore $Q = (1, 0, 1)$.

31. (a) No. $\overrightarrow{PQ} = (4, -7, 5)$, $\overrightarrow{PR} = (2, -3, 2)$; the vectors are not parallel; the points are not collinear.
(b) $\overrightarrow{PQ} = (4, -7, 5)$, $\overrightarrow{PR} = (2, -3, 2)$, $\overrightarrow{PS} = (-2, 0, 1)$

$$(\overrightarrow{PQ} \times \overrightarrow{PR}) \cdot \overrightarrow{PS} = \begin{vmatrix} 4 & -7 & 5 \\ 2 & -3 & 2 \\ -2 & 0 & 1 \end{vmatrix} = 0.$$

The points are coplanar.

32. The scalar parametric equations for the line are:

$$x = -2 + 3t, \quad y = 1 + 2t, \quad z = -6 + t.$$

Substituting x, y, z into the equation for the plane gives

$$2(-2 + 3t) + (1 + 2t) - 3(-6 + t) + 6 = 0 \Longrightarrow t = -\frac{21}{5}.$$

Therefore the line intersects the plane at the point $\left(-\frac{73}{5}, -\frac{37}{5}, -\frac{51}{5}\right)$.

33. $\overrightarrow{PQ} \times \overrightarrow{PR} = -10\,\mathbf{i} + 5\,\mathbf{k}$ is a normal vector for the plane; so is $\mathbf{N} = 2\,\mathbf{i} - \mathbf{k}$.
An equation for the plane is: $2(x - 1) - (z - 1) = 0$ or $2x - z = 1$

34. $\mathbf{N} = 2\,\mathbf{i} + 3\,\mathbf{j} - 4\,\mathbf{k}$ is a normal vector for the plane. An equation for the plane is:

$$2(x - 2) + 3(y - 1) - 4(z + 3) = 0 \quad \text{or} \quad 2x + 3y - 4z = 19.$$

35. $\mathbf{N} = 3\,\mathbf{i} + 2\,\mathbf{j} - 1\,\mathbf{k}$ is a normal vector for the plane. An equation for the plane is:

$$3(x - 1) + 2(y + 2) - (z + 1) = 0 \quad \text{or} \quad 3x + 2y - z = 0.$$

36. The point $Q\,(2, -1, 0)$ is in the plane since it is on the line. $\mathbf{d} = 2\,\mathbf{i} + 3\,\mathbf{j} - 2\,\mathbf{k}$ is a direction vector for the line. The vector $\overrightarrow{PQ} \times \mathbf{d}$ is a normal vector for the plane.

$$\overrightarrow{PQ} \times \mathbf{d} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ -1 & 0 & -2 \\ 2 & 3 & -2 \end{vmatrix} = 6\,\mathbf{i} - 6\,\mathbf{j} - 3\,\mathbf{k}.$$

Take $\mathbf{N} = 2\,\mathbf{i} - 2\,\mathbf{j} - \mathbf{k}$ as a normal vector for the plane. An equation for the plane is:

$$2(x-3) - 2(y+1) - (z-2) = 0 \quad \text{or} \quad 2x - 2y - z = 6.$$

37. Let P be the plane that satisfies the conditions. A direction vector for the given line is $\mathbf{d} = (3, 2, 4)$; a normal vector for the given plane is $\mathbf{N} = (2, 1, -3)$. The cross product $\mathbf{d} \times \mathbf{N}$ is a normal vector for P.

$$\mathbf{d} \times \mathbf{N} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 3 & 2 & 4 \\ 2 & 1 & -3 \end{vmatrix} = -10\,\mathbf{i} + 17\,\mathbf{j} - \mathbf{k}.$$

The point $Q\,(-1, 1, 2)$ is on the plane. An equation for P is:

$$-10(x+1) + 17(y-1) - (z-2) = 0 \quad \text{or} \quad 10x - 17y + z + 25 = 0.$$

38. Let P be the plane that satisfies the given conditions. The vectors $\mathbf{N}_1 = 3\,\mathbf{i} + \mathbf{j} - \mathbf{k}$ and $\mathbf{N}_2 = 2\,\mathbf{i} + \mathbf{j} + 4\,\mathbf{k}$ are normal vectors for the given planes. The vector $\mathbf{d} = \mathbf{N}_1 \times \mathbf{N}_2 = 5\,\mathbf{i} - 14\,\mathbf{j} + \mathbf{k}$ is a direction vector for the line of intersection. Solving the equations $3x + y - z = 2,\ 2x + y + 4z = 1$ simultaneously, we find that $Q\,(1, -1, 0)$ lies on the line of intersection (set $z = 0$ and solve for x and y). Now, the vector

$$\mathbf{N} = \overrightarrow{PQ} \times \mathbf{d} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ -1 & -2 & 3 \\ 5 & -14 & 1 \end{vmatrix} = 40\,\mathbf{i} + 16\,\mathbf{j} + 24\,\mathbf{k} \quad \text{or} \quad 5\,\mathbf{i} + 2\,\mathbf{j} + 3\,\mathbf{k}$$

is a normal vector for P. An equation for P is: $5(x-2) + 2(y-1) + 3(z+3) = 0$.

39. The line l which passes through Q and R has direction vector $\mathbf{d} = \overrightarrow{QR} = (2, 1, -2)$. By (13.5.6), the distance from P to l is given by

$$d(P, l) = \frac{\|\,\overrightarrow{QP} \times \mathbf{d}\|}{\|\mathbf{d}\|} = \frac{\|(2, 4, -5) \times (2, 1, -2)\|}{3} = \frac{9}{3} = 3.$$

40. By (13.6.5), the distance from P to the plane is given by

$$d = \frac{|1(2) - 2(1) + 2(-1) + 5|}{\sqrt{1+4+4}} = \frac{3}{3} = 1.$$

41. The normals are: $\mathbf{N}_1 = (2, 1, 1),\ \mathbf{N}_2 = (2, 2, -1)$. The cosine of the angle between the planes is:

$$\cos\theta = \frac{|\mathbf{N}_1 \cdot \mathbf{N}_2|}{\|\mathbf{N}_1\|\|\mathbf{N}_2\|} = \frac{5}{\sqrt{54}} \quad \text{and} \quad \theta \cong 0.822 \text{ radians.}$$

42. The normals are: $\mathbf{N}_1 = (2, -3, 1),\ \mathbf{N}_2 = (1, 4, -5)$. The cosine of the angle between the planes is:

$$\cos\theta = \frac{|\mathbf{N}_1 \cdot \mathbf{N}_2|}{\|\mathbf{N}_1\|\|\mathbf{N}_2\|} = \frac{15}{7\sqrt{12}} \quad \text{and} \quad \theta \cong 0.904 \text{ radians.}$$

43. The normal vectors to the two planes are: $\mathbf{N}_1 = 3\,\mathbf{i} + 5\,\mathbf{j} + 2\,\mathbf{k},\ \mathbf{N}_2 = \mathbf{i} + 2\,\mathbf{j} - \mathbf{k}$. A direction vector for the line of intersection is:

$$\mathbf{N}_1 \times \mathbf{N}_2 = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 3 & 5 & 2 \\ 1 & 2 & -1 \end{vmatrix} = -9\,\mathbf{i} + 5\,\mathbf{j} + \mathbf{k}.$$

A solution of the pair of equations $3x + 5y + 2z - 4 = 0 \quad x + 2y - z - 2 = 0$ is $x = -2,\ y = 2,\ z = 0$ (set $z = 0$ and solve for x and y). Scalar parametric equations for the line of intersection are:

$$x = -2 - 9t, \quad y = 2 + 5t, \quad z = t.$$

44. The normal vectors to the two planes are: $\mathbf{N}_1 = \mathbf{i} - 2\,\mathbf{j} + 2\,\mathbf{k},\ \mathbf{N}_2 = 3\,\mathbf{i} - \mathbf{j} - \mathbf{k}$. A direction vector for the line of intersection is:

$$\mathbf{N}_1 \times \mathbf{N}_2 = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 1 & -2 & 2 \\ 3 & -1 & -1 \end{vmatrix} = 4\,\mathbf{i} + 7\,\mathbf{j} + 5\,\mathbf{k}.$$

A solution of the pair of equations $x - 2y + 2z = 1 \quad 3x - y - z = 2$ is $x = 0,\ y = -5/4,\ z = -3/4$ (set $x = 0$ and solve for y and z). Scalar parametric equations for the line of intersection are:

$$x = 4t, \quad y = -\tfrac{5}{4} + 7t, \quad z = -\tfrac{3}{4} + 5t.$$

45. $\mathbf{a} \times \mathbf{b} = -5\,\mathbf{i} + 11\,\mathbf{j} + 7\,\mathbf{k}$ is perpendicular to both $\mathbf{a}$ and $\mathbf{b}$; $\|\mathbf{a} \times \mathbf{b}\| = \sqrt{195}$. The vectors are:

$$\pm\frac{4}{\sqrt{195}}(-5\,\mathbf{i} + 11\,\mathbf{j} + 7\,\mathbf{k}).$$

46. Since $\mathbf{a} \times (\mathbf{b} \times \mathbf{c}) = (\mathbf{a} \cdot \mathbf{c})\mathbf{b} - (\mathbf{a} \cdot \mathbf{b})\mathbf{c}$ (13.4.11),

$$(\mathbf{a} \times \mathbf{b}) \times (\mathbf{c} \times \mathbf{d}) = [(\mathbf{a} \times \mathbf{b}) \cdot \mathbf{d}]\mathbf{c} - [(\mathbf{a} \times \mathbf{b}) \cdot \mathbf{c}]\mathbf{d}.$$

47. $(\|\mathbf{b}\|\mathbf{a} - \|\mathbf{a}\|\mathbf{b}) \cdot (\|\mathbf{b}\|\mathbf{a} + \|\mathbf{a}\|\mathbf{b}) = \|\mathbf{a}\|^2\|\mathbf{b}\|^2 + \|\mathbf{a}\|\|\mathbf{b}\|\mathbf{a} \cdot \mathbf{b} - \|\mathbf{a}\|\|\mathbf{b}\|\mathbf{a} \cdot \mathbf{b} - \|\mathbf{a}\|^2\|\mathbf{b}\|^2 = 0.$
Therefore, $(\|\mathbf{b}\|\mathbf{a} - \|\mathbf{a}\|\mathbf{b}) \perp (\|\mathbf{b}\|\mathbf{a} + \|\mathbf{a}\|\mathbf{b})$

48. $\|\mathbf{a} + \mathbf{b}\|^2 - \|\mathbf{a} - \mathbf{b}\|^2 = (\mathbf{a} + \mathbf{b}) \cdot (\mathbf{a} + \mathbf{b}) - (\mathbf{a} - \mathbf{b}) \cdot (\mathbf{a} - \mathbf{b}) = \|\mathbf{a}\|^2 + \|\mathbf{b}\|^2 + 2\mathbf{a} \cdot \mathbf{b} - \|\mathbf{a}\|^2 - \|\mathbf{b}\|^2 + 2\mathbf{a} \cdot \mathbf{b} = 4\mathbf{a} \cdot \mathbf{b}$

49. Let $\mathbf{a}$ and $\mathbf{b}$ be adjacent sides of a parallelogram. Then the diagonals of the parallelogram are $\mathbf{a} + \mathbf{b}$ and $\mathbf{a} - \mathbf{b}$. By Exercise 48, the diagonals have equal length iff $\mathbf{a} \perp \mathbf{b}$, which means that the parallelogram is a rectangle.

50. Let $A,\ B,\ C,\ D$ be the vertices of the quadrilateral, and let E, F, G, H be the midpoints of $\overline{AB},\ \overline{BC},\ \overline{CD},\ \overline{DA}$, respectively. Then, $\overline{EF} \parallel \overline{AC} \parallel \overline{GH}$ and $\overline{FG} \parallel \overline{BD} \parallel \overline{EH}$. Therefore $EFGH$ is a parallelogram.

51. Let $A,\ B,\ C$ be the vertices of a triangle. Without loss of generality, assume that $A\,(0,0),\ B\,(x_1,y_1),\ C\,(x_2,0)$. Let D and E be the midpoints of $\overline{AB}$ and $\overline{BC}$, respectively. Then $D\left(\dfrac{x_1}{2},\dfrac{y_1}{2}\right)$ and $E\left(\dfrac{x_1+x_2}{2},\dfrac{y_1}{2}\right)$. Now

$$\overrightarrow{DE}=\left(\frac{x_2}{2},0\right), \quad \text{and} \quad \overrightarrow{AC}=(x_2,0).$$

Therefore $\overrightarrow{DE}\|\overrightarrow{AC}$ and $\|\overrightarrow{DE}\|=\frac{1}{2}\|\overrightarrow{AC}\|$.

52. (a) If $A\neq 0$, then $(-\frac{C}{A},0)$ and $(-\frac{B+C}{A},1)$ are two points on l. Therefore $(\frac{B}{A},-1)$ or $(B,-A)$ are directional vectors for l. Thus

$$\mathbf{r}(t)=(-C/A)\,\mathbf{i}+(B\,\mathbf{i}-A\,\mathbf{j})t=(-C/A+Bt)\,\mathbf{i}+(-At)\,\mathbf{j}$$

is the parametrization of the line.

(b) $(A\,\mathbf{i}+B\,\mathbf{j})\cdot(B\,\mathbf{i}-A\,\mathbf{j})=0$

(c) $P=(-C/A,0)$ is a point on l and $\overrightarrow{OP}=-C/A\,\mathbf{i}$. By (13.5.6)

$$\frac{\|\overrightarrow{OP}\times(B\,\mathbf{i}-A\,\mathbf{j})\|}{\|(B\,\mathbf{i}-A\,\mathbf{j})\|}=\frac{|C|}{\sqrt{A^2+B^2}}.$$