1. Integrate $\mathbf{h}(x, y) = x^2y\,\mathbf{i} - xy\,\mathbf{j}$ over the indicated path:

(a) the line segment from (0,0) to (1,1).

(b) $\mathbf{r}(u) = u^2\,\mathbf{i} + u^3\,\mathbf{j},\ 0 \le u \le 1.$

2. Integrate $\mathbf{h}(x, y) = x^3\,\mathbf{i} + y^3\,\mathbf{j}$ over the indicated path:

(a) $\mathbf{r}(u) = \cos u\,\mathbf{i} + \sin u\,\mathbf{j}, \quad 0 \le u \le \pi/2.$

(b) $\mathbf{r}(u) = \cos^3 u\,\mathbf{i} + \sin^3 u\,\mathbf{j}, \quad 0 \le u \le \pi/2.$

3. Integrate $\mathbf{h}(x, y) = (2xy^2 + x)\,\mathbf{i} + (2x^2y - 1)\,\mathbf{j}$ over the indicated path:

(a) the line segment from $(-1, 2)$ to (2,4).

(b) the polygonal path from $(-1, 2)$ to (0,0) to (2,4).

(c) the line segment from $(-1, 2)$ to (0,0), followed by the parabolic path $y = x^2$ from (0,0) to (2,4).

4. Integrate $\mathbf{h}(x, y) = \dfrac{-y}{x^2 + y^2}\,\mathbf{i} + \dfrac{x}{x^2 + y^2}\,\mathbf{j}$ over the indicated path:

(a) the arc of the semicircle $y = \sqrt{4 - x^2}$ from (2,0) to $(-\sqrt{2}, \sqrt{2})$.

(b) the line segment from (2, 0) to $(-\sqrt{2}, \sqrt{2})$.

(c) the polygonal path that connects $(2, 0), (2, \sqrt{2}), (-\sqrt{2}, \sqrt{2})$ in that order.

5. Integrate $\mathbf{h}(x, y, z) = \sin y\,\mathbf{i} + xe^{xy}\,\mathbf{j} + \sin z\,\mathbf{k}$ over the curve

$$\mathbf{r}(u) = u^2\,\mathbf{i} + u\,\mathbf{j} + u^3\,\mathbf{k},\ 0 \le u \le 3.$$

6. Integrate $\mathbf{h}(x, y, z) = x^2\,\mathbf{i} + xy\,\mathbf{j} + z^2\,\mathbf{k}$ over the curve

$$\mathbf{r}(u) = \cos u\,\mathbf{i} + \sin u\,\mathbf{j} + u^2\,\mathbf{k},\ 0 \le u \le \pi/2.$$

Exercises 11–13. Verify that **h** is a gradient. Then evaluate the line integral of **h** over the indicated curve C in two ways: (a) by carrying out the integration; (b) by applying the fundamental theorem for line integrals.

11. $\mathbf{h}(x, y) = (ye^{xy} + 2x)\,\mathbf{i} + (xe^{xy} - 2y)\,\mathbf{j}$;
$C: \mathbf{r}(u) = u\,\mathbf{i} + u^2\,\mathbf{j},\ 0 \le u \le 2.$

12. $\mathbf{h}(x, y) = (2xy^2 + 2y)\,\mathbf{i} + (2x^2y + 2x)\,\mathbf{j}$;
$C:\ \mathbf{r}(u) = 3u\,\mathbf{i} + (1 + 4u)\,\mathbf{j},\ 0 \le u \le 1.$

13. $\mathbf{h}(x, y, z) = 4x^3y^3z^2\,\mathbf{i} + 3x^4y^2z^2\,\mathbf{j} + 2x^4y^3z\,\mathbf{k}.$
$C:\ \mathbf{r}(u) = u\,\mathbf{i} + u^2\,\mathbf{j} + u^3\,\mathbf{k},\ 0 \le u \le 1.$

14. Evaluate $\int_C y^2 dx + (x^2 - xy)\,dy$ where C is the path given from (0, 0) to (2, 8):

(a) the straight-line path.

(b) the polygonal path (0,0) to (2, 0) to (2, 8).

(c) the cubic path $y = x^3$.

15. Evaluate $\int_C 2xy^{1/2}\,dx + yx^{1/2}\,dy$ where C is the path given from (1, 0) to (0, 1):

(a) the straight-line path.

(b) the polygonal path (1, 0) (1, 1), (0, 1).

(c) the quarter-circle $y = \sqrt{1 - x^2}$.

16. Evaluate $\int_C z\,dx + x\,dy + y\,dz$ where C is the circular helix $\mathbf{r}(u) = a\cos u\,\mathbf{i} + a\sin u\,\mathbf{j} + u\,\mathbf{k}$, from $u = 0$ to $u = 2\pi$.

17. Evaluate $\int_C ye^{xy}dx + \cos x\,dy + (xy/z)\,dz$ where C is the twisted cubic $\mathbf{r}(u) = u\,\mathbf{i} + u^2\,\mathbf{j} + u^3\,\mathbf{k}$, from $u = 0$ to $u = 2$.

Exercises 19–20. Evaluate the line integral (a) directly; (b) by applying Green's theorem.

19. $\oint_C xy^2\,dx - x^2y\,dy$ where C is the boundary of the region that lies between the curves $y = x^2$ and $y^2 = x$.

20. $\oint_C (x^2 + y^2)\,dx + (x^2 - y^2)\,dy$ where C is the unit circle $x^2 + y^2 = 1$.

Exercises 21–26. Evaluate the line integral using Green's theorem.

21. $\oint_C (x - 2y^2)\,dx + 2xy\,dy$ where C is the rectangle with vertices $(0, 0)$, $(2, 0)$, $(2, 1)$, $(0, 1)$.

22. $\oint_C xy\,dx + (\frac{1}{2}x^2 + xy)\,dy$ where C is the upper half of the ellipse $x^2 + 4y^2 = 1$ together with the interval $[-1, 1]$.

23. $\oint_C \ln(x^2 + y^2)\,dx + \ln(x^2 + y^2)\,dy$ where C is the boundary of the upper half of the annular region bounded by the circles $x^2 + y^2 = 1$ and $x^2 + y^2 = 4$.

24. $\oint_C (1/y)\,dx + (1/x)\,dy$ where C is the boundary of the region enclosed by the x-axis, the line $x = 4$, and the curve $y = \sqrt{x}$.

25. $\oint_C y^2 dx$ where C is the cardioid $r = 1 + \sin\theta$, $\theta \in [0, 2\pi]$.

26. $\oint_C e^y \cos x\,dx - e^y \sin x\,dy$ where C is the rectangle with vertices $(0, 0)$, $(\pi/2, 0)$, $(\pi/2, 1)$, $(0, 1)$.

Exercises 27–28. Use Green's theorem to find the area of the region bounded by the curves.

27. $y = 4 - x^2$ and $y = 0$.

28. $xy = 3$ and $x + y = 4$.