Exercises 1–10. Evaluate.

1. $\displaystyle\int_0^1 \int_y^{\sqrt{y}} xy^2\,dx\,dy.$

2. $\displaystyle\int_0^1 \int_{-y}^{y} e^{x+y}\,dx\,dy.$

3. $\displaystyle\int_0^1 \int_x^{3x} 2ye^{x^3}\,dy\,dx.$

4. $\displaystyle\int_1^2 \int_0^{\ln x} xe^y\,dy\,dx.$

5. $\displaystyle\int_0^{\pi/4} \int_0^{2\sin\theta} r\cos\theta\,dr\,d\theta.$

6. $\displaystyle\int_{-1}^2 \int_0^4 \int_0^1 xyz\,dx\,dy\,dz.$

7. $\displaystyle\int_0^2 \int_0^{2-3x} \int_0^{x+y} x\,dz\,dy\,dx.$

8. $\displaystyle\int_0^{\pi/2} \int_z^{\pi/2} \int_0^{\sin z} 3x^2 \sin y\,dx\,dy\,dz.$

9. $\displaystyle\int_{-\pi/2}^0 \int_0^{2\sin\theta} \int_0^{r^2} r^2\cos\theta\,dz\,dr\,d\theta.$

10. $\displaystyle\int_{-\pi/6}^{\pi/2} \int_0^{\pi/2} \int_0^1 \rho^3 \sin\varphi\cos\varphi\,d\rho\,d\theta\,d\varphi.$

Exercises 11–14. Sketch the region that gives rise to the repeated integral, change the order of integration, and then evaluate.

11. $\displaystyle\int_0^1 \int_y^1 e^{x^2}\,dx\,dy.$

12. $\displaystyle\int_0^2 \int_{\frac{1}{2}x}^1 \cos y^2\,dy\,dx.$

13. $\displaystyle\int_0^1 \int_0^{\sqrt{1-x^2}} \frac{1}{\sqrt{1-y^2}}\,dy\,dx.$

14. $\displaystyle\int_0^1 \int_0^{1-x} y\cos(x+y)\,dy\,dx.$

Exercises 15–22. Evaluate.

15. $\iint_{\Omega} xy\,dx\,dy$; $\Omega: 0 \le x^2 + y^2 \le 1, x, y \ge 0$.

16. $\iint_{\Omega} (x - y)\,dx\,dy$; Ω the region between the curves $y^2 = 3x$ and $y^2 = 4 - x$.

17. $\iint_{\Omega} (x^2 - xy)\,dx\,dy$; Ω the region between the curves $y = x$ and $y = 3x - x^2$.

18. $\iint_{\Omega} x(x - 1)e^{xy}\,dx\,dy$; Ω the triangular region in the first quadrant bounded by $x = 0$, $y = 0$, and $x + y = 2$.

19. $\iiint_{T} xyz\,dx\,dy\,dz$; T the solid bounded by the cylinder $z = 4 - y^2$ and the planes $x = 0, z = 0, y = x$.

20. $\iiint_{T} z\,dx\,dy\,dz$; T the solid bounded by the planes $x = 0, y = 0, z = 0, y + z = 1, x + z = 1$.

21. $\iiint_{T} xy\,dx\,dy\,dz$; T the solid in the first octant bounded by the coordinate planes and the hemisphere $z = \sqrt{4 - x^2 - y^2}$.

22. $\iiint_{T} (x^2 + 2z)\,dx\,dy\,dz$; T the solid bounded by the planes $z = 0$ and $y + z = 4$, and the cylinder $y = x^2$.

Exercises 23–24. Use polar coordinates to evaluate the integral.

23. $\int_0^2 \int_0^{\sqrt{4-y^2}} e^{\sqrt{x^2+y^2}}\,dx\,dy$.

24. $\int_{-1}^1 \int_0^{\sqrt{1-x^2}} \arctan(y/x)\,dy\,dx$.

25. Find the volume of the solid bounded by the paraboloid $z = 9 - x^2 - y^2$ and the xy-plane.

26. Find the volume of the solid bounded above by the paraboloid $z = 2 - x^2 - y^2$ and below by the region between the curves $y = x^2$ and $x = y^2$ in the xy-plane.

27. Find the volume of the solid in the first octant bounded by $z = x^2 + y^2$, $x + y = 1$, and the coordinate planes.

28. Find the volume of the solid in the first octant bounded by the cylinder $x^2 + y^2 = 9$ and the planes $z = y$ and $z = 0$.

Exercises 35–42. Use triple integrals to find the volume of the solid. Use rectangular, cylindrical, or spherical coordinates, whichever seems appropriate.

35. The solid bounded above by the plane $2x + 2y - z + 1 = 0$, on the sides by the planes $y = x$, $x = 2$, $y = 0$, and below by $z = 0$.

36. The solid bounded above by the paraboloid $z = 4x^2 + 4y^2$, below by the plane $z = -1$, and on the sides by the cylinders $y = x^2$ and $y = x$.

37. The solid bounded above by the elliptic paraboloid $z = 12 - x^2 - 2y^2$ and below by the elliptic paraboloid $z = 2x^2 + y^2$.

38. The solid in the first octant inside the cylinder $y^2 + z^2 = 1$ and bounded by the plane $2x + y + z = 2$.

39. The solid bounded above by the sphere $x^2 + y^2 + z^2 = 4$ and below by the plane $z = 1$.

40. The solid in the first octant bounded by the cylinder $x^2 + z = 16$, the coordinate planes, and the plane $3x + 4y = 12$.

41. The solid that lies outside the cone $z = \sqrt{x^2 + y^2}$ and inside the hemisphere $z = \sqrt{1 - x^2 - y^2}$.

42. The solid that lies above the cone $z = \sqrt{x^2 + y^2}$ and below the hemisphere $z = \sqrt{1 - x^2 - y^2}$.

43. A homogeneous solid in the first octant is bounded by the cylinders $x^2 + z^2 = 1$ and $y^2 + z^2 = 1$.