Exercises 53–56. Find the differential df.

53. $f(x, y) = 3x^3 - 5x^2y^2 + 2x - y$.

54. $f(x, y) = y \tan x^2 - 2xy^2 + 3$.

55. $f(x, y, z) = \dfrac{xyz}{x + y + z}$.

56. $f(x, y, z) = e^{yz} - \ln(y^2 + xz)$.

Exercises 57–58. Use differentials to find the approximate value.

57. $e^{0.02}\sqrt{15.2 + (1.01)^3}$.

58. $\sqrt[3]{64.5}\cos^2(28°)$.

59. A silo is in the shape of a right circular cylinder 22 feet high and 10 feet in diameter. The top and lateral surface are to be given a coat of paint 0.01 inches thick. Estimate by a differential the amount of paint required. Express your answer in gallons. (There are 231 cubic inches in a gallon.)

Exercises 60–63. Determine whether or not the vector function is a gradient. If so, find all the functions with that gradient.

60. $(6x^2y^2 - 8xy + 2x)\,\mathbf{i} + (4x^3y - 4x^2 - 8)\,\mathbf{j}$.

61. $(2xy + 3 - y \sin x)\,\mathbf{i} + (x^2 + 2y + 1 + \cos x)\,\mathbf{j}$.

62. $(xy^2 + 2y^2)\,\mathbf{i} + (2y^3 - x^2y + 2x)\,\mathbf{j}$.

63. $(e^y \sin z + 2x)\,\mathbf{i} + (xe^y \sin z - y^2)\,\mathbf{j} + xe^y \cos z\,\mathbf{k}$.