Exercises 24–26. Find du/dt.

24. $u(x, y) = \arctan xy$; $x = \tan t$, $y = e^{2t}$.

25. $u(x, y) = 3xy^2 - x^2$; $x = t^2 + 2t$, $y = 3t$.

26. $u(x, y, z) = \dfrac{1}{\sqrt{x^2 + y^2 + z^2}}$; $x = \cos t$, $y = \sin t$, $z = t$.

27. A triangle has sides x and y, and included angle θ. Given that x and y increase at the rate of 2 inches per second but the area of the triangle is kept constant, at what rate is θ changing when $x = 4$ inches, $y = 5$ inches, and $\theta = \pi/3$ radians?

28. View the trunk of a tree as a right circular cylinder. Suppose that a certain tree grows in such a manner that the radius increases at the rate of 4 centimeters per year and the height increases at the rate of 1.5 meters per year. At what rate is the volume of the trunk changing when the radius is 12 centimeters and the height is 10 meters?

29. Assume that $u = u(x, y)$ is differentiable, and set $x = s + t$, $y = s - t$. Show that

$$\left(\frac{\partial u}{\partial x}\right)^2 - \left(\frac{\partial u}{\partial y}\right)^2 = \frac{\partial u}{\partial s}\frac{\partial u}{\partial t}.$$

30. Assume that $u = u(x, y)$ has continuous second partials. Show that if $x = e^s \cos t$ and $y = e^s \sin t$, then

$$\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = e^{-2s}\left[\frac{\partial^2 u}{\partial s^2} + \frac{\partial^2 u}{\partial t^2}\right].$$

Exercises 31–32. Find a normal vector and a tangent vector at the point P. Write scalar parametric equations for the tangent line and the normal line.

31. $x^3 - 3x^2y + y^2 = 5$; $P(1, -1)$.

32. $\cos \pi xy = -\frac{1}{2}$; $P(\frac{1}{3}, 2)$.

Exercises 33–36. Write an equation for the tangent plane and scalar parametric equations for the normal line at the point P.

33. $z = x^{1/2} + y^{1/2}$; $P(1, 1, 2)$.

34. $x^2 + y^2 + z^2 = 9$; $P(1, 2, -2)$.

35. $z^3 + xyz - 2 = 0$; $P(1, 1, 1)$.

36. $z = e^{3x} \sin 3y$; $P(0, \pi/6, 1)$.

37. Show that the hyperboloids $x^2 + 2y^2 - 4z^2 = 8$ and $4x^2 - y^2 + 2z^2 = 14$ are mutually perpendicular at the point $(2, 2, 1)$.

38. Show that every line normal to the sphere $x^2 + y^2 + z^2 = a^2$ passes through the origin.

Exercises 39–44. Find the stationary points and the local extreme values.

39. $f(x, y) = x^2y - 2xy + 2y^2 - 15y - 2.$

40. $f(x, y) = 3x^2 - 3xy^2 + y^3 + 3y^2.$

41. $f(x, y) = x^3 + y^3 - 18xy.$

42. $f(x, y) = x^3 + y^2 - 6x^2 + y - 4.$

43. $f(x, y) = (x - y)(1 - xy).$

44. $f(x, y) = xy^2e^{-(x^2+y^2)/2}.$

Exercises 45–48. Find the absolute extreme values taken on by f on the set indicated.

45. $f(x, y) = x^2 + y^2 - 2x + 2y + 2;\qquad x^2 + y^2 \leq 4.$

46. $f(x, y) = 2x^2 - 4x + y^2 - 4y + 3;$ the closed triangular region bounded by the lines $x = 0$, $y = 2$, $y = 2x$.

47. $f(x, y) = 4x^2 - xy + y^2 + y;\qquad x^2 + \frac{1}{4}y^2 \leq 1.$

48. $f(x, y) = x^4 + 2y^3;\qquad x^2 + y^2 \leq 1.$

49. Find the point of the plane $3x + 2y - z = 5$ that is closest to the point $P(1, -2, 3)$. What is the distance from the point P to the plane?

50. Maximize $3x - 2y + z$ on the sphere $x^2 + y^2 + z^2 = 14$.

51. Find the maximum and minimum values of $f(x, y, z) = x + y - z$ on the ellipsoid

$$\frac{x^2}{4} + \frac{y^2}{4} + z^2 = 1.$$

52. Closed rectangular boxes 16 cubic feet in volume are to be constructed from three types of metal. The cost of the metal for the bottom of the box is \$0.50 per square foot, for the sides of the box \$0.25 per square foot, for the top \$0.10 per square foot. Find the dimensions that minimize cost of material.