Exercises 1–6. Find the gradient.

1. $f(x, y) = 2x^2 - 4xy + y^3$.
2. $f(x, y) = \dfrac{xy}{x^2 + y^2}$.
3. $f(x, y) = e^{xy} \tan 2x$.
4. $f(x, y, z) = \ln \sqrt{x^2 + y^2 + z^2}$.
5. $f(x, y, z) = x^2 e^{-yz} \sec z$.
6. $f(x, y, z) = e^{-3z}(\sin xy - \cos y)$.

Exercises 7–10. Find the directional derivative at the point P in the direction indicated.

7. $f(x, y) = x^2 - 2xy$ at $P(1, -2)$ in the direction of $\mathbf{i} + 2\,\mathbf{j}$.
8. $f(x, y) = xe^{xy}$ at $P(2, 0)$ in the direction of $\mathbf{i} + \sqrt{3}\,\mathbf{j}$.
9. $f(x, y, z) = xy^2 + 2yz + 3zx^2$ at $P(1, -2, 3)$ in the direction of $\frac{1}{3}\mathbf{i} - \frac{2}{3}\mathbf{j} + \frac{2}{3}\mathbf{k}$.
10. $f(x, y, z) = \ln(x^2 + y^2 + z^2)$ at $P(1, 2, 3)$ in the direction of $\mathbf{i} - \mathbf{j} + \mathbf{k}$.
11. Find the directional derivative of $f(x, y) = 3x^2 - 2xy^2 + 1$ at $(3, -2)$ toward the origin.
12. Find the directional derivative of $f(x, y, z) = xy^2z - 3xyz$ at $(1, -1, 2)$ in the direction of increasing t along the path $\mathbf{r}(t) = t\,\mathbf{i} + \cos \pi t\,\mathbf{j} + 2e^{t-1}\,\mathbf{k}$.
13. Find the directional derivatives of $f(x, y, z) = \sqrt{x^2 + y^2 + z^2}$ at $(3, -1, 4)$ in the directions parallel to the line
$$\frac{x-3}{4} = \frac{y+1}{-3} = z - 4.$$
14. Find the directional derivative of $f(x, y) = e^{2x}(\cos y - \sin y)$ at the point $(\frac{1}{2}, -\frac{1}{2}\pi)$ in the direction in which f increases most rapidly.
15. Find the directional derivative of $f(x, y, z) = \sin xyz$ at the point $(\frac{1}{2}, \frac{1}{3}, \pi)$ in the direction in which f decreases most rapidly.

16. The intensity of light in a neighborhood of the point (4, 3) is given by the function

$$I(x, y) = 10 - x^2 - 3y^2.$$

Find the path of a light-seeking particle that originates at the center of the neighborhood.

17. The temperature in a neighborhood of the origin is given by the function

$$T(x, y) = 100 + e^{-x} \cos y.$$

Find the path of a heat-fleeing particle that originates at the origin.

18. Determine the path of steepest descent along the surface $z = 4x^2 + y^2$ from each of the following points: (a) $(1, 1, 5)$; (b) $(1, -2, 8)$.

Exercises 19–20. Find the unit vector in the direction in which f increases most rapidly at P and give the rate of the change of f in that direction.

19. $f(x, y) = e^x \arctan y; \quad P(0, 1).$

20. $f(x, y, z) = \dfrac{x - z}{y + z}; \quad P(-1, 1, 3).$

Exercises 21–23. Find the rate of change of f with respect to t along the curve.

21. $f(x, y) = 2x^2 - 3y^3; \quad \mathbf{r}(t) = \sqrt{t}\,\mathbf{i} + e^{2t}\,\mathbf{j}.$

22. $f(x, y) = \sin x + \cos xy; \quad \mathbf{r}(t) = t^2\,\mathbf{i} + \mathbf{j}.$

23. $f(x, y, z) = \dfrac{x}{y} - \dfrac{z}{x}; \quad \mathbf{r}(t) = \sin t\,\mathbf{i} + \cos t\,\mathbf{j} + \tan t\,\mathbf{k}.$