

Exercises 1–4. Find the domain and range of the function.

1. $f(x, y) = \frac{1}{\sqrt{y - x^2}}$. 2. $f(x, y) = e^{-(x^2+y^2)}$.

3. $f(x, y, z) = \sqrt{z - x^2 - y^2}$.

4. $f(x, y, z) = \ln(x + 2y + z)$.

5. Express as a function of two variables x and y .

(a) The volume of a circular cone of radius x and height y .

(b) The volume of a box of length x height y given that the box is twice as long as it is wide.

(c) The angle between the vectors $\mathbf{i} + 2\mathbf{j}$ and $x\mathbf{i} + y\mathbf{j}$.

6. A rectangular box is inscribed in the ellipsoid $(x^2/a^2) + (y^2/b^2) + (z^2/c^2) = 1$. The sides of the box are parallel to the coordinate planes and the vertices of the box are on the ellipsoid. Express the volume of the box as a function of x and y .

Exercises 7–12. Identify the surface and find the traces. Then sketch the surface.

7. $4x^2 + 9y^2 + 36z^2 = 36$. 8. $4z^2 - x^2 - y^2 = 4$.

9. $z = y^2 - x^2$. 10. $4x^2 + 9z^2 = y$.

11. $x^2 = y^2 + z^2$. 12. $z^2 = 9x^2 + 4y^2 - 36$.

Exercises 13–16. Sketch the cylinder.

13. $x^2 + z^2 = 4$. 14. $y - x^2 = 1$.

15. $4y^2 + 9z^2 - 36 = 0$. 16. $(y - 1)^2 + (z - 1)^2 = 1$.

Exercises 17–20. Identify the level curves $f(x, y) = c$ and sketch the curves corresponding to the indicated values of c .

17. $f(x, y) = 2x^2 + 3y^2$; $c = 0, 6, 12$.

18. $f(x, y) = \sqrt{x^2 + y^2 - 4}$; $c = 0, 1, 2, \sqrt{5}$.

19. $f(x, y) = \frac{x}{y^2}$; $c = -4, -1, 1, 4$.

20. $f(x, y) = e^{x^2+y^2}$; $c = e, e^4, e^9$.

Exercises 21–22. Identify the c -level surface and sketch it.

21. $f(x, y, z) = 2x + y + 3z; \quad c = 6.$

22. $f(x, y, z) = x^2 + y^2 + 4z^2; \quad c = 16.$

23. Set $f(x, y) = (y^2 + 1)e^x$. Find an equation for the level curve that passes through the point indicated

(a) $P(0, 0).$ (b) $P(\ln 2, 1).$ (c) $P(1, -1).$

24. Set $f(x, y, z) = x^2 \cos yz$. Find an equation for the level surface that passes through the point indicated.

(a) $P(2, 0, 1).$ (b) $P(1, \pi, -1).$ (c) $P(4, \pi, \frac{1}{2}).$

Exercises 25–26. Find $f_x(x, y)$ and $f_y(x, y)$ by forming the appropriate difference quotient and taking the limit as h tends to 0.

25. $f(x, y) = x^2 + 2xy.$

26. $f(x, y) = y^2 \cos 2x.$

Exercises 27–36. Calculate the first-order partial derivatives.

27. $f(x, y) = x^2y - 2xy^3.$

28. $g(x, y) = \frac{x}{\sqrt{x^2 + y^2}}.$

29. $z = x^2 \sin xy^2.$

30. $f(x, y) = e^{xy} \ln(y/x).$

31. $h(x, y) = e^{-x} \cos(2x - y).$

32. $u = y^2 \sec x + x^2 \tan y.$

33. $f(x, y, z) = \frac{2xy}{x + y + z}.$

34. $w = x \arctan(y - z).$

35. $g(x, y, z) = \ln \sqrt{x^2 + y^2 + z^2}.$

36. $h(u, v, w) = e^{uv} \sin uw.$

Exercises 37–42. Calculate the second-order partial derivatives.

37. $f(x, y) = x^3y^2 - 4xy^3 + 2x - y.$

38. $g(x, y) = x^2 \ln(y - x).$

39. $g(x, y) = xy \sin(xy).$

40. $f(x, y) = x^2 e^{x/y}.$

41. $f(x, y, z) = x^2 e^{2y} \cos(2z + 1).$

42. $g(x, y, z) = 2x^2yz^3 + e^{xyz}.$

Exercises 43–44. The surface $z = 2x^2 + 3xy$ intersects the plane in a curve C . Give parametric equations for the line tangent to C at the point indicated.

43. Plane $y = 2$; $P(1, 2, 8)$. **44.** Plane $x = 2$; $P(2, -1, 2)$.

45. The surface $z = \sqrt{20 - 2x^2 - 3y^2}$ is the top half of an ellipsoid centered at the origin.

(a) The surface intersects the plane $x = 2$ in a curve C_1 . Write scalar parametric equations for the line tangent to C_1 at the point $(2, 1, 3)$.

(b) The surface intersects the plane $y = 1$ in a curve C_2 . Write scalar parametric equations for the line tangent to C_2 at the point $(2, 1, 3)$.

(c) The lines found in (a) and (b) determine a plane. Write an equation in (x, y, z) for this plane.

Exercises 46–50. Specify the interior and the boundary of the set. State whether the set is open, closed, or neither. Then sketch the set.

46. $\{(x, y) : 0 \leq x \leq 3, 2 < y \leq 5\}$.

47. $\{(x, y) : 0 < x^2 + y^2 < 4\}$.

48. $\{(x, y) : x + y \geq 4\}$.

49. $\{(x, y, z) : y^2 + z^2 \leq 4, 0 \leq x \leq 2, y \geq 0, z \geq 0\}$.

50. $\{(x, y, z) : 0 \leq x^2 + y^2 \leq z < 4\}$.

51. Let g be a twice differentiable function of one variable and set $f(x, y) = g(xy)$.

(a) Show that

$$x \frac{\partial f}{\partial x} - y \frac{\partial f}{\partial y} = 0.$$

(b) Show that

$$x^2 \frac{\partial^2 f}{\partial x^2} - y^2 \frac{\partial^2 f}{\partial y^2} = 0.$$

52. Show that $f(x, y) = \arctan(y/x)$ satisfies Laplace's equation $f_{xx} + f_{yy} = 0$.
53. Is there a function f of x and y with everywhere continuous second partials such that

$$\frac{\partial f}{\partial x} = xe^{xy} \quad \text{and} \quad \frac{\partial f}{\partial y} = ye^{xy}?$$

54. Set $f(x, y) = \frac{2x^2y}{x^4 + y^2}$. Evaluate the limit of $f(x, y)$ as (x, y) approaches the origin along
- (a) the x -axis; (b) the y -axis;
(c) the line $y = mx$; (d) the parabola $y = ax^2$.

Does $\lim_{(x,y) \rightarrow (0,0)} f(x, y)$ exist?