Exercises 1–4. Calculate the first and second derivatives.

1. $\mathbf{f}(t) = 3t^2\,\mathbf{i} - 5t^3\,\mathbf{j}$.

2. $\mathbf{f}(t) = e^{2t}\,\mathbf{i} + \ln(t^2+1)\,\mathbf{j}$.

3. $\mathbf{f}(t) = e^t \cos t\,\mathbf{i} - \cos 2t\,\mathbf{j} + 3\,\mathbf{k}$.

4. $\mathbf{f}(t) = \sinh t\,\mathbf{i} - t^2 e^{-t}\,\mathbf{j} + \cosh t\,\mathbf{k}$.

Exercises 5–6. Evaluate.

5. $\displaystyle\int_0^2 [2t\,\mathbf{i} + (t^2 - 1)\,\mathbf{j}]\,dt$.

6. $\displaystyle\int_0^{\pi} [\sin 2t\,\mathbf{i} + 2\cos t\,\mathbf{j} + \sqrt{t}\,\mathbf{k}]\,dt$.

Exercises 7–10. Sketch the curve traced out by the tip of the radius vector and indicate the direction in which the curve is traversed as t increases.

7. $\mathbf{r}(t) = 2t^2\,\mathbf{i} - t\,\mathbf{j}, \quad t \geq 0$.

8. $\mathbf{r}(t) = e^{-t}\,\mathbf{i} + 2e^{2t}\,\mathbf{j}, \quad t$ real.

9. $\mathbf{r}(t) = t\,\mathbf{i} + 3\sin t\,\mathbf{j} + 4\cos t\,\mathbf{k}, \quad 0 \leq t \leq 2\pi$.

10. $\mathbf{r}(t) = t\,\mathbf{i} + t\,\mathbf{j} + \sin t\,\mathbf{k}, \quad t \geq 0$.

11. Define a vector function $\mathbf{r}$ on the interval $[0, 2\pi]$ that traces out the ellipse $16x^2 + 4y^2 = 64$ in the manner indicated.

(a) Once in the counterclockwise direction starting at the point (0, 4).

(b) Twice in the clockwise direction starting at the point (−2, 0).

12. Define a vector function $\mathbf{r}$ that traces out the directed line segment from $(1, 1, -2)$ to $(3, 5, 4)$.

13. Find $\mathbf{f}(t)$ given that $\mathbf{f}'(t) = t^2\,\mathbf{i} + (e^{2t} + 1)\,\mathbf{j} + \sqrt{1 + 2t}\,\mathbf{k}$ and $\mathbf{f}(0) = \mathbf{i} - 3\,\mathbf{j} + 3\,\mathbf{k}$.

14. Find $\mathbf{f}(t)$ given that $\mathbf{f}'(t) = -\mathbf{f}(t)$ for all t and $\mathbf{f}(0) = \mathbf{i} + 2\mathbf{k}$.

Exercises 15–18. Calculate $\mathbf{f}'(t)$.

15. $\mathbf{f}(t) = 3(2t\,\mathbf{i} + t^4\,\mathbf{j} - \mathbf{k}) + 4(t^2\,\mathbf{i} + \mathbf{j} - 3t\,\mathbf{k})$.

16. $\mathbf{f}(t) = [(e^{2t}\,\mathbf{i} + e^{-2t}\,\mathbf{j} + t\,\mathbf{k}) \cdot (e^{-t}\,\mathbf{i} + e^{2t}\,\mathbf{j})]\,\mathbf{j}$.

17. $\mathbf{f}(t) = (t\,\mathbf{i} + t^2\,\mathbf{j} - (1/t)\,\mathbf{k}) \times ([1/t]\,\mathbf{i} + t^3\,\mathbf{j} + 2t\,\mathbf{k})$.

18. $\mathbf{f}(t) = [(t^3\,\mathbf{i} + t^2\,\mathbf{j} + t\,\mathbf{k}) \cdot (\cos t\,\mathbf{i} + \sin t\,\mathbf{j} + 3\cos t\,\mathbf{k})]\,\mathbf{k}$.

19. An object moves along a curve C in such a manner that the tangent vector $\mathbf{r}'(t)$ is always $2\mathbf{r}(t)$. Find parametric equations for C given that $\mathbf{r}(0) = (1, 2, 1)$. Sketch the curve.

20. Set $\mathbf{F}(t) = e^{2t}\,\mathbf{i} + e^{-2t}\,\mathbf{j}$. Show that $\mathbf{F}(t)$ and $\mathbf{F}''(t)$ are parallel for all t. Is there a value of t for which $\mathbf{F}$ and $\mathbf{F}'$ have the same direction?

Exercises 21–22. Find the tangent vector $\mathbf{r}'(t)$ at the indicated point and parametrize the tangent line at that point.

21. $\mathbf{r}(t) = (t^2 + 2t + 1)\,\mathbf{i} + (3t + 1)\,\mathbf{j} + (t^3 + t + 1)\,\mathbf{k}$ at $P(1, 1, 1)$.

22. $\mathbf{r}(t) = \sin 2t\,\mathbf{i} + \cos 2t\,\mathbf{j} + t\,\mathbf{k}$ at $t = \pi/3$.

23. Show that the curves

$$\mathbf{r}_1(t) = 2t\,\mathbf{i} + t^2\,\mathbf{j} + t\,\mathbf{k}, \quad \mathbf{r}_2(u) = (1-u)\mathbf{i} + (2-u^2)\,\mathbf{j} + u^2\,\mathbf{k}$$

intersect at the point (2, 1, 1) and find the angle of intersection. Express your answer in radians.

24. Find the points, if any, on the curve $\mathbf{r}(t) = t^2\,\mathbf{i} + (1-t^2)\,\mathbf{j} - t^2\,\mathbf{k}$ at which $\mathbf{r}(t)$ and the line tangent to the curve meet at right angles.

Exercises 25–26. Sketch the curve showing the direction of the curve and displaying both $\mathbf{r}(t)$ and $\mathbf{r}'(t)$ at the value of t indicated.

25. $\mathbf{r}(t) = t\,\mathbf{i} + e^{2t}\,\mathbf{j}; \qquad t = 0.$

26. $\mathbf{r}(t) = 2\sin t\,\mathbf{i} - 3\cos t\,\mathbf{j}; \qquad t = \pi/6.$

27. Find the unit tangent and principal normal for the elliptical helix

$$\mathbf{r}(t) = (\cos t + t\sin t)\,\mathbf{i} + (\sin t - t\cos t)\,\mathbf{j} + \tfrac{1}{2}\sqrt{3}\,t^2\,\mathbf{k}.$$

28. Show that along the circular helix $\mathbf{r}(t) = a\cos t\,\mathbf{i} + a\sin t\,\mathbf{j} + bt\,\mathbf{k}$. the angle between $\mathbf{k}$ and the tangent vector $\mathbf{r}'(t)$ remains constant.

Exercises 29–30. Find the unit tangent, the principal normal, and write an equation in x, y, z for the osculating plane at

the point on the curve that corresponds to the indicated value of t.

29. $\mathbf{r}(t) = 2t\,\mathbf{i} + \ln t\,\mathbf{j} - t^2\,\mathbf{k}; \qquad t = 1.$

30. $\mathbf{r}(t) = \cos t\,\mathbf{i} + \cos t\,\mathbf{j} - \sqrt{2}\sin t\,\mathbf{k}; \qquad t = \pi/4.$

Exercises 31–34. Find the length of the curve.

31. $\mathbf{r}(t) = 2t\,\mathbf{i} + \frac{2}{3}t^{3/2}\,\mathbf{j}$ from $t = 0$ to $t = 5$.

32. $\mathbf{r}(t) = e^{t}\mathbf{i} + e^{-t}\mathbf{j} - t\sqrt{2}\,\mathbf{k}$ from $t = 0$ to $t = \ln 3$.

33. $\mathbf{r}(t) = \sinh t\,\mathbf{i} + \cosh t\,\mathbf{j} + t\,\mathbf{k}$ from $t = 0$ to $t = 1$.

34. $\mathbf{r}(t) = \cos t\,\mathbf{i} + \sin t\,\mathbf{j} + \cosh t\,\mathbf{k}$ from $t = 0$ to $t = \ln 2$.