

1. Throughout this exercise let

$$P(3, 2, -1), \quad Q(7, -5, 4), \quad R(5, 6, -3).$$

- (a) Find the length of the line segment  $\overline{PQ}$ .
- (b) Find the midpoint of the line segment  $\overline{QR}$ .
- (c) Given that  $Q$  is the midpoint of the line segment  $\overline{PX}$ , what are the coordinates of  $X$ ?
- (d) Find an equation for the sphere which passes through  $P$  and is centered at the midpoint of  $\overline{PR}$ .

2. Exercise 1 for  $P(4, 2, -3)$ ,  $Q(-2, 1, 4)$ ,  $R(1, -1, -6)$ .

**Exercises 3–4.** Write an equation for the sphere that satisfies the given conditions.

3. Centered at  $(2, -3, 1)$ , passes through the origin.
4. The line segment that joins  $(-1, 4, 2)$  to  $(3, -2, 6)$  is a diameter.

**Exercises 5–6.** Show that the equation represents a sphere; find the center and the radius.

5.  $x^2 + y^2 + z^2 + 2x + 4y - 8z + 17 = 0.$

6.  $x^2 + y^2 + z^2 - 6x + 10y - 2z + 2 = 0.$

**Exercises 7–24.** Set  $\mathbf{a} = 3\mathbf{i} + 2\mathbf{j} - \mathbf{k}$ ,  $\mathbf{b} = 5\mathbf{i} + 3\mathbf{j}$ ,  $\mathbf{c} = -2\mathbf{i} + 4\mathbf{j} + \mathbf{k}$ . Determine the following.

7.  $\frac{1}{2}\mathbf{a}.$
8.  $\mathbf{a} + 4\mathbf{b} - 3\mathbf{c}.$
9.  $\mathbf{a} \cdot (\mathbf{b} + \mathbf{c}).$
10.  $\|\mathbf{a} + \mathbf{b}\|.$
11.  $\|\mathbf{c}\|^2.$
12.  $\|\mathbf{b} \times \mathbf{b}\|.$
13.  $(2\mathbf{a} - \mathbf{b}) \cdot \mathbf{c}.$
14.  $\mathbf{a} \times (\mathbf{b} + \mathbf{c}).$
15. The unit vector in the direction of  $\mathbf{a}.$
16. The unit vector in the direction opposite to the direction of  $\mathbf{c}.$
17. The angle between  $\mathbf{a}$  and  $\mathbf{c}.$  Express your answer in radians rounded to the nearest hundredth of a radian.

20.  $\text{comp}_a \mathbf{b}$ .

21.  $\text{comp}_a (\mathbf{b} \times \mathbf{c})$ .

22. The unit vectors perpendicular to  $\mathbf{a}$  and  $\mathbf{c}$ .

23. The volume of the parallelepiped determined by  $\mathbf{a}$ ,  $\mathbf{b}$ ,  $\mathbf{c}$ .

24. The area of the triangle determined by  $\mathbf{a}$ ,  $\mathbf{b}$ ,  $\mathbf{a} - \mathbf{b}$ .

25. The points  $P(1, 1, 1)$ ,  $Q(-2, 1, 0)$ ,  $R(4, -2, 3)$  are given.

(a) Find scalar parametric equations for the line that passes through  $P$  and is parallel to  $\overrightarrow{QR}$ .

(b) Find an equation for the plane through  $P$  that is perpendicular to  $\overrightarrow{PR}$ .

(c) Find an equation for the plane that passes through  $P$ ,  $Q$ ,  $R$ .

26. The points  $P(3, 2, -1)$ ,  $Q(7, -5, 4)$ ,  $R(5, 6, -3)$  are given.

(a) Find scalar parametric equations for the line that passes through  $R$  and is parallel to the line determined by  $P$  and  $Q$ .

(b) Find scalar parametric equations for the line that passes through  $R$  and is perpendicular to the line determined by  $P$  and  $Q$ .

(c) The lines in parts (a) and (b) determine a plane. Find an equation in rectangular coordinates for this plane.

**Exercises 27–29.** Determine whether the lines  $l_1$  and  $l_2$  are parallel, skew, or intersecting. If they intersect, find the point of intersection.

27.  $l_1 : x = t, y = -t, z = -6 + 2t$ ;  $l_2 : x = 1 - u,$   
 $y = 1 + 3u, z = 2u.$

28.  $l_1 : \mathbf{r}(t) = (1 - 2t)\mathbf{i} + (3 + 3t)\mathbf{j} + 5t\mathbf{k}$ ;

$l_2 : \mathbf{R}(u) = (3 + 2u)\mathbf{i} + (1 - u)\mathbf{j} + (6 + 3u)\mathbf{k}.$

29.  $l_1 : \frac{x-1}{2} = \frac{y+2}{-1}, \frac{z-3}{4}$ ;  $l_2 : x+2 = \frac{y-3}{3} = z.$

30. The point  $P(3, 1, -2)$  and the line  $l : x + 1 = y + 2 = z + 1$  are given. Find The point  $Q$  on  $l$  for which  $\overrightarrow{PQ} \perp l$ .
31. (a) Are the points  $P(3, 2, -1)$ ,  $Q(7, -5, 4)$ ,  $R(5, -1, 1)$  collinear?  
 (b) Are the points  $P(3, 2, -1)$ ,  $Q(7, -5, 4)$ ,  $R(5, -1, 1)$ ,  $S(1, 2, 0)$  coplanar?
32. Where does the line
- $$\frac{x + 2}{3} = \frac{y - 1}{2} = \frac{z + 6}{1}$$
- intersect the plane  $2x + y - 3z + 6 = 0$ ?

**Exercises 33–38.** Write an equation for the plane that satisfies the given conditions.

33. Contains the points  $P(1, -2, 1)$ ,  $Q(2, 0, 3)$ ,  $R(0, 1, -1)$ .
34. Contains the point  $P(2, 1, -3)$  and is perpendicular to the line

$$\frac{x + 1}{2} = \frac{y - 1}{3} = -\frac{z}{4}$$

35. Contains the point  $P(1, -2, -1)$  and is parallel to the plane  $3x + 2y - z = 4$ .
36. Contains the point  $P(3, -1, 2)$  and the line  $x = 2 + 2t$ ,  $y = -1 + 3t$ ,  $z = -2t$ .
37. Contains the line  $x = -1 + 3t$ ,  $y = 1 + 2t$ ,  $z = 2 + 4t$  and is perpendicular to the plane  $2x + y - 3z + 4 = 0$ .
38. Contains the point  $P(2, 1, -3)$  and the line formed by intersecting the planes

$$3x + y - z = 2, \quad 2x + y + 4z = 1.$$

39. Find the distance from the point  $P(4, 6, -4)$  to the line which passes through  $Q(2, 2, 1)$  and  $R(4, 3, -1)$ .
40. Find the distance from the point  $P(2, 1, -1)$  to the plane  $x - 2y + 2z + 5 = 0$ .

**Exercises 41–42.** Find the angle between the planes.

41.  $2x + y + z + 3 = 0$ ,  $4z + 4y - 2z - 9 = 0$ .
42.  $2x - 3y + z + 2 = 0$ ,  $x + 4y - 5z - 6 = 0$ .

**Exercises 43–44.** Find a set of scalar parametric equations for the line formed by intersecting the planes.

43.  $3x + 5y + 2z - 4 = 0, \quad x + 2y - z - 2 = 0.$

44.  $x - 2y + 2z = 1, \quad 3x - y - z = 2.$

45. Set  $\mathbf{a} = 2\mathbf{i} - \mathbf{j} + 3\mathbf{k}$  and  $\mathbf{b} = 3\mathbf{i} + 2\mathbf{j} - \mathbf{k}$ . Find all vectors of length 4 that are perpendicular to both  $\mathbf{a}$  and  $\mathbf{b}$ .

46. Show that, for all vectors  $\mathbf{a}, \mathbf{b}, \mathbf{c}, \mathbf{d}$ ,

$$(\mathbf{a} \times \mathbf{b}) \times (\mathbf{c} \times \mathbf{d}) = [(\mathbf{a} \times \mathbf{b}) \cdot \mathbf{d}]\mathbf{c} - [(\mathbf{a} \times \mathbf{b}) \cdot \mathbf{c}]\mathbf{d}.$$

47. Show that, for all vectors  $\mathbf{a}$  and  $\mathbf{b}$ ,

$$(\|\mathbf{b}\|\mathbf{a} - \|\mathbf{a}\|\mathbf{b}) \perp (\|\mathbf{b}\|\mathbf{a} + \|\mathbf{a}\|\mathbf{b}).$$

48. Verify that, for all vectors  $\mathbf{a}$  and  $\mathbf{b}$ ,

$$\|\mathbf{a} + \mathbf{b}\|^2 - \|\mathbf{a} - \mathbf{b}\|^2 = 4\mathbf{a} \cdot \mathbf{b}.$$

49. Use Exercise 46 to prove that the diagonals of a parallelogram have equal length iff the parallelogram is a rectangle.

50. Show that the midpoints of the sides of a quadrilateral are the four vertices of a parallelogram.

51. Use vector methods to show that the line segment that joins the midpoints of two sides of a triangle is parallel to the third side. Show that the length of this line segment is one-half the length of the third side.

52. The setting is the  $xy$ -plane.

(a) Verify that the line  $l : Ax + By + C = 0$  can be parametrized by setting

$$\mathbf{r}(t) = (-C/A)\mathbf{i} + (B\mathbf{i} - A\mathbf{j})t$$

provided that  $A \neq 0$ .

(b) Show that  $A\mathbf{i} + B\mathbf{j}$  is normal to  $l$ .

(c) Show that

$$d(O, l) = \frac{|C|}{\sqrt{A^2 + B^2}}$$

by vector methods.