REVIEW EXERCISES

1. domain $\{(x,y) : y > x^2\}$, range $(0,\infty)$

2. domain $\{(x,y) : x, y \in R\}$, range $(0,\infty)$

3. domain $\{(x,y,x) : z \geq x^2 + y^2\}$, dange $[0,+\infty)$

4. domain $\{(x,y,z) : x + 2y + z > 0\}$, range R

5. (a) $f(x,y) = \dfrac{1}{3}\pi x^2 y;$

(b) $f(x,y) = \dfrac{1}{2}yx^2;$

(c) $\theta = \arccos \dfrac{x+2y}{\sqrt{5}\sqrt{x^2+y^2}}$

6. Assume one of the vertices is $(x,y,z), x > 0, y > 0, z > 0.$

$$V = 8cxy\sqrt{1 - \frac{x^2}{a^2} - \frac{y^2}{b^2}}$$

7. ellipsoid
$xy-$trace: ellipse $4x^2 + 9y^2 = 36$
$xz-$trace: ellipse $4x^2 + 36z^2 = 36$
$yz-$trace: ellipse $9y^2 + 36z^2 = 36$

8. hyperboloid of two sheets
$xy-$trace: none
$xz-$trace: hyperbola $4z^2 - x^2 = 4$
$yz-$trace: hyperbola $4z^2 - y^2 = 4$

9. hyperbolic paraboloid
$xy-$trace: lines $x = \pm y$
$xz-$trace: parabola $z = -x^2$
$yz-$trace: parabola $z = y^2$

10. elliptic paraboloid
$xy-$trace: parabola $4x^2 = y$
$xz-$trace: $(0,0)$
$yz-$trace: parabola $9z^2 = y$

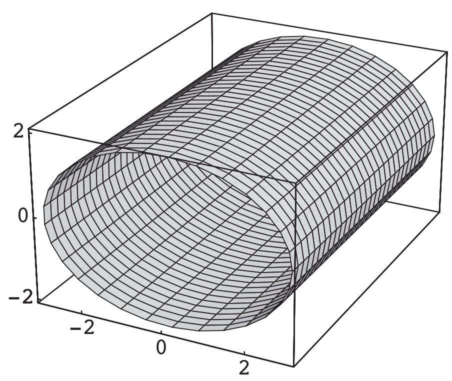
11. cone
$xy-$trace: lines $x = \pm y$
$xz-$trace: lines $x = \pm z$
$yz-$trace: $(0,0)$

12. hyperboloid of one sheet
$xy-$trace: ellipse $9x^2 + 4y^2 = 36$
$xz-$trace: hyperbola $z^2 = 9x^2 - 36$
$yz-$trace: hyperbola $z^2 = 4y^2 - 36$

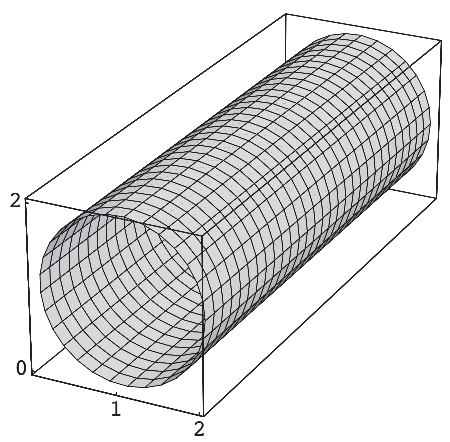
13.

14.

15.



16.



17. $c = 0, \Longrightarrow 0 = 2x^2 + 3y^2 \Longrightarrow (0,0)$
$c = 6, \Longrightarrow 6 = 2x^2 + 3y^2$, ellipse
$c = 12, \Longrightarrow 6 = 2x^2 + 3y^2$, ellipse

18. $c = 0, \Longrightarrow 0 = x^2 + y^2 - 4$, circle
$c = 1, \Longrightarrow 5 = x^2 + y^2$, circle
$c = 2, \Longrightarrow 8 = x^2 + y^2$, circle
$c = \sqrt{5}, \Longrightarrow 9 = x^2 + y^2$, circle

19. $c = -4, \Longrightarrow x = -4y^2$, parabola
$c = -1, \Longrightarrow x = -y^2$, parabola
$c = 1, \Longrightarrow x = y^2$, parabola
$c = 4, \Longrightarrow x = 4y^2$, parabola
the origin is omitted

20. $c = 1, \Longrightarrow x^2 + y^2 = 1$ circle
$c = 4, \Longrightarrow x^2 + y^2 = 4$ circle
$c = 9, \Longrightarrow x^2 + y^2 = 9$ circle

21. $c = 6,\ 2x + y + 3z = 6$, plane

22. $c = 16, x^2 + y^2 + 4z^2 = 16$, ellipsoid

23. (a) $f(0,0) = 1$, level curve: $f(x,y) = 1$
(b) $f(\ln 2, 1) = 4$, level curve: $f(x,y) = 4$
(c) $f(1,-1) = 2e$, level curve: $f(x,y) = 2e$

24. (a) $f(2,0,1) = 4$, level surface: $f(x,y,z) = 4$
(b) $f(1,\pi,-1) = -1$, level surface:
$f(x,y,z) = -1$
(c) $f(4,\pi,1/2) = 0$, level surface: $f(x,y,z) = 0$

25.
$$f_x = \lim_{h\to 0}\frac{f(x+h,y) - f(x,y)}{h} = \lim_{h\to 0}\frac{(x+h)^2 + 2(x+h)y - x^2 - 2xy}{h}$$
$$= \lim_{h\to 0}(2x + h + 2y) = 2x + 2y$$
$$f_y = \lim_{h\to 0}\frac{f(x,y+h) - f(x,y)}{h} = \lim_{h\to 0}\frac{x^2 + 2x(y+h) - x^2 - 2xy}{h} = 2x$$

26.
$$f_x = \lim_{h\to 0}\frac{f(x+h,y) - f(x,y)}{h} = \lim_{h\to 0}\frac{y^2\cos 2(x+h) - y^2\cos 2x}{h}$$
$$= y^2\lim_{h\to 0}\frac{\cos 2(x+h) - \cos 2x}{h}$$
$$= y^2\cos' 2x = -2y^2\sin 2x$$

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$$f_y = \lim_{h\to 0} \frac{f(x, y+h) - f(x, y)}{h} = \lim_{h\to 0} \frac{(y+h)^2 \cos 2x - y^2 \cos 2x}{h}$$
$$= \cos 2x \lim_{h\to 0} \frac{(y+h)^2 - y^2}{h} = 2y \cos 2x$$

27. $f_x = 2xy - 2y^3; \qquad f_y = x^2 - 6xy^2$

28. $g_x = (x^2 + y^2)^{-1/2} - x^2(x^2 + y^2)^{-3/2}; \qquad g_y = -xy(x^2 + y^2)^{-3/2}$

29. $\dfrac{\partial z}{\partial x} = 2x \sin(xy^2) + x^2 y^2 \cos(xy^2); \qquad \dfrac{\partial z}{\partial y} = 2x^3 y \cos(xy^2).$

30. $f_x = ye^{xy} \ln(y/x) - \dfrac{1}{x} e^{xy} \qquad f_y = xe^{xy} \ln(y/x) + \frac{e^{xy}}{y}$

31. $h_x = -e^{-x} \cos(2x - y) - 2e^{-x} \sin(2x - y) \qquad h_y = e^{-x} \sin(2x - y)$

32. $u_x = y^2 \sec x \tan x + 2x \tan y \qquad u_y = 2y \sec x + x^2 \sec^2 y$

33. $f_x = \dfrac{2y^2 + 2yz}{(x + y + z)^2}; \qquad f_y = \dfrac{2x^2 + 2xz}{(x + y + z)^2}; \qquad f_z = \dfrac{-2xy}{(x + y + z)^2}$

34. $w_x = \arctan(y - z) \qquad w_y = \dfrac{x}{1 + (y - z)^2} \qquad w_z = \dfrac{-x}{1 + (y - z)^2}$

35. $\dfrac{\partial g}{\partial x} = \dfrac{x}{x^2 + y^2 + z^2}; \qquad \dfrac{\partial g}{\partial y} = \dfrac{y}{x^2 + y^2 + z^2}; \qquad \dfrac{\partial g}{\partial z} = \dfrac{z}{x^2 + y^2 + z^2}.$

36. $h_u = ve^{uv} \sin uw + we^{uv} \cos uw; \qquad h_v = ue^{uv} \sin uw; \qquad h_w = ue^{uv} \cos uw$

37. $f_x = 3x^2 y^2 - 4y^3 + 2, \qquad f_y = 2x^3 y - 12xy^2 - 1;$

$f_{xx} = 6xy^2, \qquad f_{yy} = 2x^3 - 24xy, \qquad f_{yx} = f_{xy} = 6x^2 y - 12y^2$

38. $g_x = 2x \ln(y - x) - \dfrac{x^2}{y - x}, \qquad g_{xx} = 2 \ln(y - x) - \dfrac{4x}{y - x} - \dfrac{x^2}{(y - x)^2};$

$g_y = \dfrac{x^2}{y - x}, \quad g_{yy} = -\dfrac{x^2}{(y - x)^2}, \qquad g_{xy} = g_{yx} = \dfrac{2x}{y - x} + \dfrac{x^2}{(y - x)^2}$

39. $g_x = y \sin xy + xy^2 \cos xy, \quad g_{xx} = 2y^2 \cos xy - xy^3 \sin xy;$

$g_y = x \sin xy + x^2 y \cos xy, \qquad g_{yy} = 2x^2 \cos xy - yx^3 \sin xy,$

$g_{xy} = g_{yx} = \sin xy + 3xy \cos xy - x^2 y^2 \sin xy$

40. $f_x = 2xe^{x/y} + \dfrac{x^2}{y} e^{x/y}, \qquad f_{xx} = 2e^{x/y} + \dfrac{4x}{y} e^{x/y} + \dfrac{x^2}{y^2} e^{x/y};$

$f_y = -\dfrac{x^3}{y^2} e^{x/y}, \qquad f_{yy} = \dfrac{2x^3}{y^3} e^{x/y} + \dfrac{x^4}{y^4} e^{x/y}, \qquad f_{xy} = f_{yx} = -\dfrac{3x^2}{y^2} e^{x/y} - \dfrac{x^3}{y^3} e^{x/y}$

41. $f_x = 2xe^{2y}\cos(2z+1),\ f_y = 2x^2e^{2y}\cos(2z+1),\ f_z = -2x^2e^{2y}\sin(2z+1);$

$f_{xx} = 2e^{2y}\cos(2z+1),\ f_{yy} = 4x^2e^{2y}\cos(2z+1),\ f_{zz} = -4x^2e^{2y}\cos(2z+1);$

$f_{xy} = f_{yx} = 4xe^{2y}\cos(2z+1),\ f_{xz} = f_{zx} = -4xe^{2y}\sin(2z+1),\ f_{yz} = f_{zy} = -4x^2e^{2y}\sin(2z+1)$

42. $g_x = 4xyz^3 + yze^{xyz}, \qquad g_y = 2x^2z^3 + xze^{xyz}, \qquad g_z = 6x^2yz^2 + xye^{xyz};$

$g_{xx} = 4yz^3 + y^2z^2e^{xyz}, \qquad g_{yy} = x^2z^2e^{xyz}, \qquad g_{zz} = 12x^2yz + x^2y^2e^{xyz};$

$g_{xy} = g_{yx} = 4xz^3 + ze^{xyz} + xyz^2e^{xyz}, \qquad g_{xz} = g_{zx} = 12xyz^2 + ye^{xyz} + xy^2ze^{xyz}$

$g_{yz} = g_{zy} = 6x^2z^2 + xe^{xyz} + x^2yze^{xyz}$

43. $\dfrac{\partial z}{\partial x} = 4x + 6|_{(1,2,8)} = 10$

$x = 1 + t, \quad y = 2, \quad z = 8 + 10t$

44. $\dfrac{\partial z}{\partial y} = 3x|_{(2,-1,2)} = 6$

$x = 2, \quad y = -1 + t, \quad z = 2 + 6t$

45. (a) $z_y(2,1) = \dfrac{-6y}{2\sqrt{20 - 2x^2 - 3y^2}}(2,1) = -1$; the equation for l_1 is:

$x = 2; \quad y = 1 - t; \quad z = 3 + t$

(b) $z_x(2,1) = \dfrac{-4x}{2\sqrt{20 - 2x^2 - 3y^2}}(2,1) = -\dfrac{4}{3}$; the equation for l_2 is:

$x = 2 - \dfrac{3}{4}t; \quad y = 1; \quad z = 3 + t$

(c) The normal vector for this plane is: $-\mathbf{i} - \dfrac{3}{4}\mathbf{j} - \dfrac{3}{4}\mathbf{k}$ or $4\,\mathbf{i} + 3\,\mathbf{j} + 3\,\mathbf{k}$;
an equation for the plane is: $4(x-2) + 3(y-1) + 3(z-3) = 0.$

46. Neither.
interior: $\{(x,y) : 0 < x < 3, 2 < y < 5\}$
boundary: $\{(x,y) : x = 0 \text{ or } x = 3,\ 2 \le y \le 5\} \cup \{(x,y) : y = 2 \text{ or } y = 5,\ 0 \le x \le 3\}$

47. Open.
interior: $\{(x,y) : 0 < x^2 + y^2 < 4\}$
boundary: $\{(0,0)\} \cup \{(x,y) : x^2 + y^2 = 4\}$

48. Closed.
interior: $\{(x,y) : x + y > 4\}$
boundary: $\{(x,y) : x + y = 4\}$

49. Closed.
interior: $\{(x,y,z) : 0 < x < 2,\ 0 < y,\ 0 < z,\ y^2 + z^2 < 4\}$
boundary: the quarter disks $x = 0,\ y^2 + z^2 \le 4; \quad x = 2,\ y^2 + z^2 \le 4;$
the squares $z = 0,\ 0 \le x, y \le 2; \quad y = 0, 0 \le x,\ z \le 2;$ and
the cylindrical surface $y^2 + z^2 = 4,\ 0 \le x \le 2,\ y,\ z \ge 0$

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50. Neither.

interior: $\{(x, y, z) : 0 < x^2 + y^2 < z < 4\}$

boundary: the cone $z = x^2 + y^2$ and the disk $x^2 + y^2 \leq 4,\ z = 4$

51. (a) $f_x = yg'(xy), \quad f_y = xg'(xy); \qquad xf_x - yf_y = xyg' - xyg' = 0$

(b) $f_{xx} = y^2 g''(xy), \quad f_{yy} = x^2 g''(xy); \qquad x^2 f_{xx} - y^2 f_{yy} = x^2y^2g'' - x^2y^2g'' = 0$

52. $f_x = -\dfrac{y}{x^2 + y^2}, \quad f_y = \dfrac{x}{x^2 + y^2}; \qquad f_{xx} = \dfrac{2xy}{(x^2 + y^2)^2}, \quad f_{yy} = \dfrac{-2xy}{(x^2 + y^2)^2}$

$f_{xx} + f_{yy} = 0$

53. No. $\dfrac{\partial^2 f}{\partial y \partial x} = x^2 e^{xy} \neq y^2 e^{xy} = \dfrac{\partial^2 f}{\partial x \partial y}$

54. (a) $\lim_{x \to 0} f(x, 0) = \lim_{x \to 0} 0 = 0$

(b) $\lim_{y \to 0} f(0, y) = \lim_{y \to 0} 0 = 0$

(c) $\lim_{x \to 0} \dfrac{2x^2 mx}{x^4 + m^2 x^2} = \lim_{x \to 0} \dfrac{2mx}{m^2 + x^2} = 0$

(d) $\lim_{x \to 0} \dfrac{2x^2 ax^2}{x^4 + a^2 x^4} = \dfrac{2a}{1 + a^2}$

$\lim_{(x,y) \to (0,0)} f(x, y)$ does not exist.