2. $\dfrac{dE}{dt} = \dfrac{1}{2}m\dfrac{d}{dt}(v^2) - m\rho\dfrac{d}{dt}(r^{-1})$

$\dfrac{d}{dt}(v^2) = \dfrac{d}{dt}(\mathbf{v}\cdot\mathbf{v}) = 2(\mathbf{a}\cdot\mathbf{v})$

$\dfrac{d}{dt}(r^{-1}) = -\dfrac{1}{r^2}\dfrac{dr}{dt} = -\dfrac{1}{r^3}\left(r\dfrac{dr}{dt}\right) = -\dfrac{1}{r^3}(\mathbf{r}\cdot\mathbf{v})$ (using 13.2.3)

$\dfrac{dE}{dt} = m(\mathbf{a}\cdot\mathbf{v}) + \dfrac{m\rho}{r^3}(\mathbf{r}\cdot\mathbf{v}) = (\mathbf{a}\cdot m\mathbf{v}) + \left(\dfrac{\rho\mathbf{r}}{r^3}\cdot m\mathbf{v}\right) = \left(a + \dfrac{\rho\mathbf{r}}{r^3}\right)\cdot m\mathbf{v} = 0$

3.
$$\begin{aligned}\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2 &= \left[\frac{d}{dt}(r\cos\theta)\right]^2 + \left[\frac{d}{dt}(r\sin\theta)\right]^2\\ &= \left[r(-\sin\theta)\frac{d\theta}{dt} + \frac{dr}{dt}\cos\theta\right]^2 + \left[r\cos\theta\frac{d\theta}{dt} + \frac{dr}{dt}\sin\theta\right]^2\\ &= r^2\sin^2\theta\left(\frac{d\theta}{dt}\right)^2 + \left(\frac{dr}{dt}\right)^2\cos^2\theta + r^2\cos^2\theta\left(\frac{d\theta}{dt}\right)^2 + \left(\frac{dr}{dt}\right)^2\sin^2\theta\\ &= \left(\frac{dr}{dt}\right)^2 + r^2\left(\frac{d\theta}{dt}\right)^2\end{aligned}$$

4. Using $\dot{r} = \frac{dr}{d\theta}\dot{\theta}$ and $\dot{\theta} = \frac{L}{mr^2}$
we have
$$\begin{aligned}E + \frac{m\rho}{r} &= \frac{1}{2}m(\dot{r}^2 + r^2\dot{\theta}^2) = \frac{1}{2}\left[\left(\frac{dr}{d\theta}\right)^2\dot{\theta}^2 + r^2\dot{\theta}^2\right]\\ &= \frac{1}{2}m\left[\left(\frac{dr}{d\theta}\right)^2\frac{L^2}{m^2r^4} + \frac{L^2}{m^2r^2}\right] = \frac{L^2}{2m}\left[\frac{1}{r^4}\left(\frac{dr}{d\theta}\right)^2 + \frac{1}{r^2}\right]\end{aligned}$$
and therefore
$$E = \frac{L^2}{2m}\left[\frac{1}{r^2} + \frac{1}{r^4}\left(\frac{dr}{d\theta}\right)^2\right] - \frac{m\rho}{r}$$

5. Substitute
$$r = \frac{a}{1+e\cos\theta},\quad \left(\frac{dr}{d\theta}\right)^2 = \left[\frac{-a}{(1+e\cos\theta)^2}\cdot(-e\sin\theta)\right]^2 = \frac{(ae\sin\theta)^2}{(1+e\cos\theta)^4}$$
into the right side of the equation and you will see that, with a and e^2 as given, the expression reduces to E.

REVIEW EXERCISES

1. $\mathbf{f}'(t) = 6t\,\mathbf{i} - 15t^2\,\mathbf{j}, \qquad \mathbf{f}''(t) = 6\,\mathbf{i} - 30t\,\mathbf{j}$

2. $\mathbf{f}'(t) = 2e^{2t}\,\mathbf{i} + \dfrac{2t}{1+t^2}\,\mathbf{j}, \qquad \mathbf{f}''(t) = 4e^{2t}\,\mathbf{i} + \dfrac{2-2t^2}{(1+t^2)^2}\,\mathbf{j}$

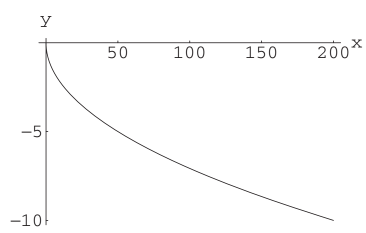
3. $\mathbf{f}'(t) = (e^t\cos t - e^t\sin t)\,\mathbf{i} + 2\sin 2t\,\mathbf{j}, \qquad \mathbf{f}''(t) = -2e^t\sin t\,\mathbf{i} + 4\cos 2t\,\mathbf{j}$

4. $\mathbf{f}'(t) = \cosh t\,\mathbf{i} - (2t - t^2)e^{-t}\,\mathbf{j} + \sinh t\mathbf{k}, \qquad \mathbf{f}''(t) = \sinh t\,\mathbf{i} + (t^2 - 4t + 2)e^{-t}\,\mathbf{j} + \cosh t\,\mathbf{k}$

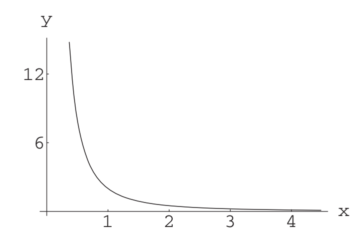
5. $\displaystyle\int_0^2 \left[2t\,\mathbf{i} + (t^2 - 1)\,\mathbf{j}\right]\,dt = \left[t^2\,\mathbf{i} + \left(\frac{1}{3}t^3 - t\right)\,\mathbf{j}\right]_0^2 = 4\,\mathbf{i} + \frac{2}{3}\,\mathbf{j}$

6. $\displaystyle\int_0^\pi [\sin 2t\,\mathbf{i} + 2\cos t\,\mathbf{j} + \sqrt{t}\,\mathbf{k}]\,dt = \left[-\frac{1}{2}\cos 2t\,\mathbf{i} + 2\sin t\,\mathbf{j} + \frac{2}{3}t^{3/2}\,\mathbf{k}\right]_0^\pi = \frac{2}{3}\pi^{3/2}\,\mathbf{k}$

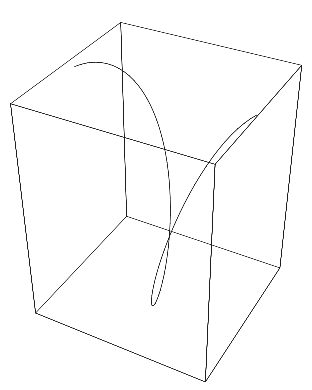
7.



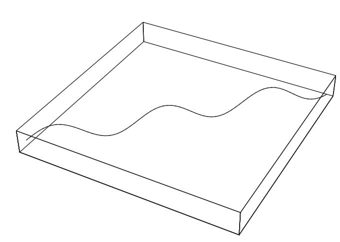
8.



9.



10.



11. (a) $\mathbf{r}(t) = 2\cos\left(t + \frac{\pi}{2}\right)\,\mathbf{i} + 4\sin\left(t + \frac{\pi}{2}\right)\,\mathbf{j}$ (b) $\mathbf{r}(t) = -2\cos 2t\,\mathbf{i} + 4\sin 2t\,\mathbf{j}$

12. direction vector: $\mathbf{d} = (2, 4, 6)$; $\mathbf{r}(t) = (1 + 2t)\,\mathbf{i} + (1 + 4t)\,\mathbf{j} + (-2 + 6t)\,\mathbf{k}, \quad 0 \le t \le 1$

13. $\mathbf{f}\,(t) = \frac{1}{3}t^3\,\mathbf{i} + (\frac{1}{2}e^{2t} + t)\,\mathbf{j} + \frac{1}{3}(2t+1)^{3/2}\mathbf{k} + \mathbf{C}.$

$\mathbf{f}\,(0) = \mathbf{i} - 3\,\mathbf{j} + 3\mathbf{k} \Longrightarrow \mathbf{C} = \mathbf{i} - \frac{7}{2}\,\mathbf{j} + \frac{8}{3}\mathbf{k};\ \ \mathbf{f}\,(t) = \left(\frac{1}{3}t^3 + 1\right)\mathbf{i} + \left(\frac{1}{2}e^{2t} + t - \frac{7}{2}\right)\mathbf{j} + \left(\frac{1}{3}(2t+1)^{3/2} + \frac{8}{3}\right)\mathbf{k}$

14. $\mathbf{f}'(t) = -\mathbf{f}\,(t) \Longrightarrow \mathbf{f}\,(t) = \mathbf{f}_0 e^{-t}$

$\mathbf{f}\,(0) = \mathbf{i} + 2\mathbf{k} \Longrightarrow \mathbf{f}_0 = \mathbf{i} + 2\mathbf{k}$ and so $\mathbf{f}\,(t) = e^{-t}\,\mathbf{i} + 2e^{-t}\mathbf{k}$

15. $\mathbf{f}'(t) = (6\,\mathbf{i} + 12t^3\,\mathbf{j}) + (8t\,\mathbf{i} - 12\,\mathbf{k}) = (6 + 8t)\,\mathbf{i} + 12t^3\,\mathbf{j} - 12\mathbf{k}$

16. Note: f is not a vector function. $f(t) = e^t + 1 \Longrightarrow f'(t) = e^t$

17. $\mathbf{f}(t) = (t^2 + 2t^3)\,\mathbf{i} - \left(2t^2 + \dfrac{1}{t^2}\right)\mathbf{j} + (t^4 - t)\,\mathbf{k}, \quad \mathbf{f}'(t) = (2t + 6t^2)\,\mathbf{i} - \left(4t - \dfrac{2}{t^3}\right)\mathbf{j} + (4t^3 - 1)\,\mathbf{k}$

18. Note: f is not a vector function. $f(t) = t^3 \cos t + t^2 \sin t + 3t \cos t = (t^3 + 3t)\cos t + t^2 \sin t$
$f'(t) = -(t^3 + 3t)\sin t + (3t^2 + 3)\cos t + 2t \sin t + t^2 \cos t = (4t^2 + 3)\cos t - (t^3 + t)\sin t$

19. $\mathbf{r}'(t) = 2\mathbf{r}(t) \Longrightarrow \mathbf{r}(t) = \mathbf{r}_0 e^{2t}$
$\mathbf{r}(0) = (1, 2, 1) \Longrightarrow \mathbf{r}_0 = (1, 2, 1)$ and $\mathbf{r}(t) = (e^{2t}, 2e^{2t}, e^{2t})$

20. $\mathbf{F}(t) = e^{2t}\,\mathbf{i} + e^{-2t}\,\mathbf{j}, \quad \mathbf{F}'(t) = 2e^{2t}\,\mathbf{i} - 2e^{-2t}\,\mathbf{j}, \quad \mathbf{F}''(t) = 4e^{2t}\,\mathbf{i} + 4e^{-2t}\,\mathbf{j}$
Since $\mathbf{F}''(t) = 4\mathbf{F}(t)$ for all t, $\mathbf{F}$ and $\mathbf{F}''$ are parallel.
$\mathbf{F}$ and $\mathbf{F}'$ will have the same direction for some value of t iff there is a number $k > 0$ such that $e^{2t}\,\mathbf{i} + e^{-2t}\,\mathbf{j} = k(2e^{2t}\,\mathbf{i} - 2e^{-2t}\,\mathbf{j})$. No such value of k exists.

21. The tip of $\mathbf{r}(t)$ is $P(1, 1, 1)$ when $t = 0$.
$\mathbf{r}'(t) = (2t + 2)\,\mathbf{i} + 3\,\mathbf{j} + (3t^2 + 1)\,\mathbf{k}, \quad \mathbf{r}'(0) = 2\,\mathbf{i} + 3\,\mathbf{j} + \mathbf{k}$
Scalar parametric equations for the tangent line are: $x = 1 + 2t, \quad y = 1 + 3t, \quad z = 1 + t.$

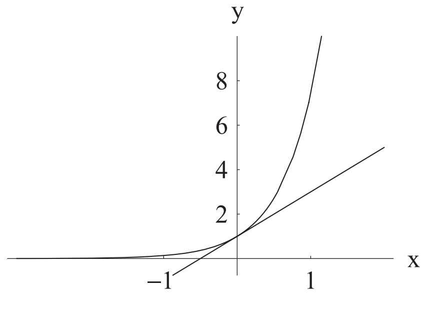
22. $\mathbf{r}(\pi/3) = (\sqrt{3}/2, -1/2, \pi/3)$
$\mathbf{r}'(t) = (2\cos 2t, -2\sin 2t, 1); \quad \mathbf{r}'(\pi/3) = (-1, -\sqrt{3}, 1)$
Scalar parametric equations for the tangent line are: $x = \frac{\sqrt{3}}{2} - t, \quad y = -\frac{1}{2} - \sqrt{3}\,t, \quad z = \frac{\pi}{3} + t$

23. $\mathbf{r}_1(t) = (2, 1, 1)$ at $t = 1$; $\mathbf{r}_2(u) = (2, 1, 1)$ at $u = -1$. Therefore the curves intersect at the point $(2, 1, 1)$.
$\mathbf{r}_1'(t) = (2\,\mathbf{i} + 2t\,\mathbf{j} + \mathbf{k}, \quad \mathbf{r}_1(1) = 2\,\mathbf{i} + 2\,\mathbf{j} + \mathbf{k}; \quad \mathbf{r}_2(u) = -\mathbf{i} - 2u\,\mathbf{j} + 2u\,\mathbf{k}, \quad \mathbf{r}_2'(-1) = -\mathbf{i} + 2\,\mathbf{j} - 2\,\mathbf{k}.$
Since $\mathbf{r}_1'(1) \cdot \mathbf{r}_2'(-1) = 0$, the angle of intersection is $\pi/2$ radians

24. $\mathbf{r}(t) = 2t\,\mathbf{i} + (1 - t^2)\,\mathbf{j} - t^2\,\mathbf{k}, \quad \mathbf{r}'(t) = 2t\,\mathbf{i} - 2t\,\mathbf{j} - 2t\,\mathbf{k}.$
$(2t\,\mathbf{i} + (1 - t^2)\,\mathbf{j} - t^2\,\mathbf{k}) \cdot (2t\,\mathbf{i} - 2t\; wj - 2t\,\mathbf{k}) = 4t^3; \quad 4t^3 = 0 \Longrightarrow t = 0$
The curve and the tangent line meet at right angles at the point where $t = 0$; $(0, 1, 0)$.

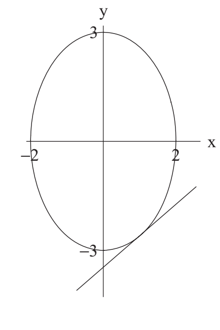
25. $\mathbf{r}(t) = t\,\mathbf{i} + e^{2t}\,\mathbf{j}, \quad \mathbf{r}(0) = \mathbf{j};$

$\mathbf{r}'(t) = \mathbf{i} + 2e^{2t}\,\mathbf{j}, \quad \mathbf{r}'(0) = \mathbf{i} + 2\,\mathbf{j}$



26. $\mathbf{r}(t) = 2\sin t\,\mathbf{i} - 3\cos t\,\mathbf{j}, \quad \mathbf{r}(\pi/6) = \mathbf{i} - \frac{3}{2}\sqrt{3}\,\mathbf{j}$

$\mathbf{r}'(t) = 2\cos t\,\mathbf{i} + 3\sin t\,\mathbf{j} \quad \mathbf{r}'(\pi/6) = \sqrt{3}\,\mathbf{i} + \frac{3}{2}\,\mathbf{j}$



27. $\mathbf{r}'(t) = t\cos t\,\mathbf{i} + t\sin t\,\mathbf{j} + \sqrt{3}t\mathbf{k}; \quad ||\mathbf{r}'|| = \dfrac{ds}{dt} = 2t$

unit tangent vector: $\mathbf{T} = \dfrac{\mathbf{r}'(t)}{||\mathbf{r}'(t)||} = \dfrac{1}{2}(\cos t\,\mathbf{i} + \sin t\,\mathbf{j} + \sqrt{3}\,\mathbf{k}).$

$\mathbf{T}'(t) = -\dfrac{1}{2}\sin t\,\mathbf{i} + \dfrac{1}{2}\cos t\,\mathbf{j}; \quad ||\mathbf{T}'(t)|| = \dfrac{1}{2}.$

principal normal vector: $\mathbf{N} = \dfrac{\mathbf{T}'(t)}{||\mathbf{T}'(t)||} = -\sin t\,\mathbf{i} + \cos t\,\mathbf{j}$

28. $\mathbf{r}'(t) = (-a\sin t\,\mathbf{i} + a\cos t\,\mathbf{j} + b\,\mathbf{k}.$

The cosine of the angle θ between $\mathbf{r}'(t)$ and $\mathbf{k}$ is: $\dfrac{\mathbf{r}'\cdot\mathbf{k}}{||\mathbf{r}'||\,||\mathbf{k}||} = \dfrac{|b|}{\sqrt{a^2+b^2}} = \text{constant}.$

Therefore θ is a constant.

29. $\mathbf{r}'(t) = 2\,\mathbf{i} + \dfrac{1}{t}\,\mathbf{j} - 2t\mathbf{k}; \quad ||\mathbf{r}'(t)|| = \dfrac{1+2t^2}{t}.$

$\mathbf{T}(t) = \dfrac{\mathbf{r}'(t)}{||\mathbf{r}'(t)||} = \dfrac{2t}{2t^2+1}\,\mathbf{i} + \dfrac{1}{2t^2+1}\,\mathbf{j} - \dfrac{2t^2}{2t^2+1}\mathbf{k}; \quad \mathbf{T}(1) = \dfrac{2}{3}\,\mathbf{i} + \dfrac{1}{3}\,\mathbf{j} - \tfrac{2}{3}\,\mathbf{k}$

$\mathbf{T}'(t) = \dfrac{2-4t^2}{(2t^2+1)^2}\,\mathbf{i} - \dfrac{4t}{(2t^2+1)^2}\,\mathbf{j} - \dfrac{4t}{(2t^2+1)^2}\mathbf{k}; \quad \mathbf{T}'(1) = -\tfrac{2}{9}\,\mathbf{i} - \tfrac{4}{9}\,\mathbf{j} - \tfrac{4}{9}\,\mathbf{k}, \quad ||\mathbf{T}'(1)|| = \tfrac{2}{3}$

$\mathbf{N}(1) = \frac{\mathbf{T}'(1)}{||\mathbf{T}'(1)||} = -\tfrac{1}{3}\,\mathbf{i} - \tfrac{2}{3}\,\mathbf{j} - \tfrac{2}{3}\,\mathbf{k}$

A normal vector for the osculating plane is: $(2\,\mathbf{i} + \mathbf{j} - 2\,\mathbf{k}) \times (\mathbf{i} + 2\,\mathbf{j} + 2\,\mathbf{k}) = 6\,\mathbf{i} - 6\,\mathbf{j} + 3\,\mathbf{k}.$

Since $\mathbf{r}(1) = 2\,\mathbf{i} - \mathbf{k}$, an equation for the osculating plane is

$$6(x-2) - 6y + 3(z+1) = 0 \quad \text{or} \quad 2x - 2y + z = 3.$$

30. $\mathbf{r}'(t) = -\sin t\,\mathbf{i} - \sin t\,\mathbf{j} - \sqrt{2}\cos t\,\mathbf{k}; \quad ||\mathbf{r}'(t)|| = \sqrt{2}.$

$\mathbf{T}(t) = \dfrac{\mathbf{r}'(t)}{||\mathbf{r}'(t)||} = \dfrac{1}{\sqrt{2}}(-\sin t\,\mathbf{i} - \sin t\,\mathbf{j} - \sqrt{2}\cos t\,\mathbf{k}); \quad \mathbf{T}(\pi/4) = -\dfrac{1}{2}\,\mathbf{i} - \dfrac{1}{2}\,\mathbf{j} - \dfrac{1}{\sqrt{2}}\,\mathbf{k}$

$\mathbf{T}'(t) = \dfrac{1}{\sqrt{2}}(-\cos t\,\mathbf{i} - \cos t\,\mathbf{j} + \sqrt{2}\sin t\,\mathbf{k}); \quad \mathbf{T}'(\pi/4) = -\dfrac{1}{2}\,\mathbf{i} - \dfrac{1}{2}\,\mathbf{j} + \dfrac{1}{\sqrt{2}}\,\mathbf{k}, \quad ||\mathbf{T}'(\pi/4)|| = 1$

$\mathbf{N}(\pi/4) = \dfrac{\mathbf{T}'(\pi/4)}{||\mathbf{T}'(\pi/4)||} = -\dfrac{1}{2}\,\mathbf{i} - \dfrac{1}{2}\,\mathbf{j} + \dfrac{1}{\sqrt{2}}\,\mathbf{k}$

A normal vector for the osculating plane is: $(\mathbf{i} + \mathbf{j} + \sqrt{2}\,\mathbf{k}) \times (\mathbf{i} + \mathbf{j} - \sqrt{2}\,\mathbf{k}) = -2\sqrt{2}\,\mathbf{i} + 2\sqrt{2}\,\mathbf{j}.$

Since $\mathbf{r}(\pi/4) = \dfrac{\sqrt{2}}{2}(\mathbf{i} + \mathbf{j} - 2\,\mathbf{k}$, an equation for the osculating plane is

$$-2\sqrt{2}\left(x - \frac{\sqrt{2}}{2}\right) + 2\sqrt{2}\left(y - \frac{\sqrt{2}}{2}\right) = 0 \quad \text{or} \quad x - y = 0.$$

31. $\mathbf{r}'(t) = 2\,\mathbf{i} + t^{1/2}\,\mathbf{j}; \quad L = \displaystyle\int_0^5 ||\mathbf{r}'(t)||dt = \int_0^5 \sqrt{4+t}\,dt = \frac{38}{3}$

32. $\mathbf{r}'(t) = e^t\,\mathbf{i} - e^{-t}\,\mathbf{j} - \sqrt{2}\mathbf{k}; \quad ||\mathbf{r}'(t)|| = e^t + e^{-t}; \quad L = \displaystyle\int_0^{\ln 3} (e^t + e^{-t})dt = \Big[e^t - e^{-t}\Big]_0^{\ln 3} = \frac{8}{3}$

33. $\mathbf{r}'(t) = \cosh t\,\mathbf{i} + \sinh t\,\mathbf{j} + \mathbf{k};\quad ||\mathbf{r}'(t)|| = \sqrt{\cosh^2 t + \sinh^2 t + 1} = \sqrt{2}\,\cosh t;$

$L = \int_0^1 \sqrt{2}\,\cosh t\,dt = \left[\sqrt{2}\,\sinh t\right]_0^1 = \sqrt{2}\sinh 1.$

34. $\mathbf{r}'(t) = -\sin t\,\mathbf{i} - \cos t\,\mathbf{j} + \sinh t\,\mathbf{k};\quad ||\mathbf{r}'|| = \sqrt{1+\sinh^2 t} = \cosh t;$

$\int_0^{\ln 2} \cosh t\,dt = \left[\sinh t\right]_0^{\ln 2} = \frac{3}{4}$

35. (a) $\mathbf{r}'(t) = -\sin t\,\mathbf{i} + \cos t\,\mathbf{j} + t^{1/2}\,\mathbf{k};\quad ||\mathbf{r}'(t)|| = \sqrt{1+t}.$

$s = \int_0^t ||\mathbf{r}'(u)||\,du = \int_0^t \sqrt{1+t}\,dt = \left[\frac{2}{3}(1+u)^{3/2}\right]_0^t = \frac{2}{3}(1+t)^{3/2} - \frac{2}{3}$

(b) $t = \left(\frac{3}{2}s+1\right)^{2/3} - 1 = \phi(s);\quad \mathbf{R}(s) = \cos\,\phi(s)\,\mathbf{i} + \sin\,\phi(s)\,\mathbf{j} + \frac{2}{3}[\phi(s)]^{3/2}\,\mathbf{k}$

(c) $\mathbf{R}'(s) = \left[-\sin\,\phi(s)\,\mathbf{i} + \cos\,\phi(s)\,\mathbf{j} + \phi(s)^{1/2}\,\mathbf{k}\right]\phi'(s)$

$||\mathbf{R}'(s)|| = \phi'(s)\sqrt{1+\phi(s)} = \frac{2}{3}\left[\frac{3}{2}s+1\right]^{-1/3}\left(\frac{3}{2}\right)\sqrt{\left(\frac{3}{2}s+1\right)^{2/3}} = 1$

36. velocity: $\mathbf{v} = \mathbf{r}'(t) = -\sin t\,\mathbf{i} + \cos t\,\mathbf{j} + 2\sin 2t\,\mathbf{k}$ speed: $s = ||\mathbf{v}|| = \sqrt{1+4\sin^2 2t}$
acceleration: $\mathbf{a} = \mathbf{r}''(t) = -\cos t\,\mathbf{i} - \sin t\,\mathbf{j} + 4\cos 2t\,\mathbf{k};\quad ||\mathbf{a}|| = \sqrt{1+16\cos^2 2t}$

37. $\mathbf{r}''(t) = -\cos t\,\mathbf{i} - \sin t\,\mathbf{j}$ and $\mathbf{r}'(0) = \mathbf{k} \Longrightarrow \mathbf{r}'(t) = -\sin t\,\mathbf{i} + (\cos t - 1)\,\mathbf{j} + \mathbf{k}.$
Thus: velocity $\mathbf{v} = -\sin t\,\mathbf{i} + (\cos t - 1)\,\mathbf{j} + \mathbf{k}$ and speed $||\mathbf{v}|| = \sqrt{3 - 2\cos t}.$
$\mathbf{r}'(t) = -\sin t\,\mathbf{i} + (\cos t - 1)\,\mathbf{j} + \mathbf{k}$ and $\mathbf{r}(0) = \mathbf{i} \Longrightarrow \mathbf{r}(t) = \cos t\,\mathbf{i} + (\sin t - t)\,\mathbf{j} + t\,\mathbf{k}.$

38. The acceleration vector remains perpendicular to the path means:

$$\mathbf{r}'' \cdot \mathbf{r}' = 0.$$

$\mathbf{r}'(t) = f'(t)\,\mathbf{i} + 2f(t)f'(t)\,\mathbf{j},\quad \mathbf{r}''(t) = f''(t)\,\mathbf{i} + [2f'(t)^2 + 2f(t)f''(t)]\,\mathbf{j}$
$\mathbf{r}'' \cdot \mathbf{r}' = 0 \Longrightarrow f'(t)f''(t) + 4f(t)[f'(t)]^3 + 4f^2(t)f'(t)f''(t) = 0$

39. $y' = \frac{3}{2}x^{1/2},\quad y'' = \frac{3}{4}x^{-1/2};$

$\kappa = \frac{|y''|}{[1+(y')^2]^{3/2}} = \frac{\frac{3}{4}x^{-1/2}}{[1+\frac{9}{4}x]^{3/2}} = \frac{6}{\sqrt{x}(4+9x)^{3/2}}$

40. $y' = -2\sin 2x,\quad y'' = -4\cos 2x;\qquad \kappa = \frac{4|\cos 2x|}{[1+4\sin^2 2x]^{3/2}}$

41. $x(t) = 2e^{-t},\ y(t) = e^{-2t} \Longrightarrow x'(t) = -2e^{-t},\ y'(t) = -2e^{-2t} \Longrightarrow x''(t) = 2e^{-t},\ y''(t) = 4e^{-2t}$

$\kappa = \frac{|(-2e^{-t})(4e^{-2t}) - (-2e^{-2t})(2e^{-t})|}{[4e^{-2t} + 4e^{-4t}]^{3/2}} = \frac{1}{2(1+e^{-2t})^{3/2}} = \frac{e^{3t}}{2(e^{2t}+1)^{3/2}}$