SECTION 3.5

28. $\dfrac{dy}{dx} = \dfrac{dy}{ds}\,\dfrac{ds}{dt}\,\dfrac{dt}{dx} = \dfrac{2}{(1-s)^2}\left(1+\dfrac{1}{t^2}\right)\dfrac{1}{2\sqrt{x}}$

At $x = 2$, we have $t = \sqrt{2}$ and $s = \sqrt{2}/2$. Thus $\dfrac{dy}{dx} = \dfrac{2}{(1-\sqrt{2}/2)^2}\left(1+\dfrac{1}{2}\right)\dfrac{1}{2\sqrt{2}} = \dfrac{3}{3\sqrt{2}-4}$.

37. $(g \circ f \circ h)'(2) = g'(f(h(2)))\ f'(h(2))h'(2) = g'(1)f'(0)h'(2) = (0)(2)(2) = 0$

42. $f'(x) = \dfrac{1}{2\sqrt{x^2+1}}(2x) = \dfrac{x}{\sqrt{x^2+1}}$

$$f''(x) = \frac{\sqrt{x^2+1}(1) - x\dfrac{x}{\sqrt{x^2+1}}}{\left(\sqrt{x^2+1}\right)^2} = \frac{1}{(x^2+1)^{3/2}}$$

61. $T'(x) = 2f(x)\cdot f'(x) + 2g(x)\cdot g'(x) = 2f(x)\cdot g(x) - 2g(x)\cdot f(x) = 0$

64. Suppose $P(x) = (x-a)^3 q(x)$, where $q(a) \neq 0$. Then

$$P'(x) = 3(x-a)^2 q(x) + (x-a)^3 q'(x)$$

$$P''(x) = 6(x-a)q(x) + 6(x-a)^2 q'(x) + (x-a)^3 q''(x)$$

$$P'''(x) = 6q(x) + 18(x-a)q'(x) + 9(x-a)^2 q''(x) + (x-a)^3 q'''(x)$$

and it follows that $P(a) = P'(a) = P''(a) = 0$, $P'''(a) \neq 0$.

Now suppose that $P(a) = P'(a) = P''(a) = 0$ and $P'''(a) \neq 0$.

$$P(a) = 0 \quad \Longrightarrow \quad P(x) = (x-a)g(x) \quad \text{for some polynomial } g.$$

Then $P'(x) = g(x) + (x-a)g'(x)$ and

$$P'(a) = 0 \quad \Longrightarrow \quad g(a) = 0 \text{ and so } g(x) = (x-a)h(x) \text{ for some polynomial } h.$$

Therefore, $P(x) = (x-a)^2 h(x)$. Now $P''(x) = 2h(x) + 4(x-a)h'(x) + (x-a)^2 h''(x)$ and

$$P''(a) = 0 \quad \Longrightarrow \quad h(a) = 0 \text{ and so } h(x) = (x-a)q(x) \text{ for some polynomial } q.$$

Therefore, $P(x) = (x-a)^3 q(x)$. Finally, $P'''(a) \neq 0$ implies $q(a) \neq 0$.

SECTION 3.6

23. $y = \sin^2 x + \cos^2 x = 1$ so $\dfrac{dy}{dx} = \dfrac{d^2 y}{dx^2} = 0$

27. $$\frac{d}{dt}\left[t^2\frac{d^2}{dt^2}(t\cos 3t)\right] = \frac{d}{dt}\left[t^2\frac{d}{dt}(\cos 3t - 3t\sin 3t)\right]$$
$$= \frac{d}{dt}[t^2(-3\sin 3t - 3\sin 3t - 9t\cos 3t)]$$
$$= \frac{d}{dt}[-6t^2\sin 3t - 9t^3\cos 3t]$$
$$= (-18t^2\cos 3t - 12t\sin 3t) + (27t^3\sin 3t - 27t^2\cos 3t)$$
$$= (27t^3 - 12t)\sin 3t - 45t^2\cos 3t$$

55. $$\frac{d^n}{dx^n}(\cos x) = \begin{cases} (-1)^{(n+1)/2}\sin x, & n \text{ odd} \\ (-1)^{n/2}\cos x, & n \text{ even} \end{cases}$$

68. (a) f must be continuous at 0:

$$\lim_{x\to 0^+} f(x) = \lim_{x\to 0^+}\cos x = 1, \quad \lim_{x\to 0^-} f(x) = \lim_{x\to 0^-}(ax+b) = b;\ \text{thus } b = 1$$

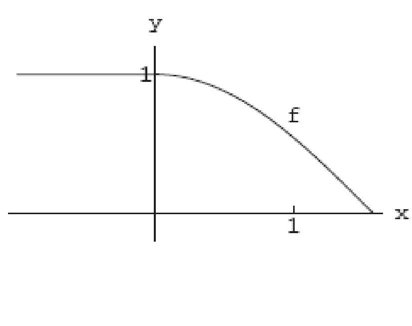
Differentiable at 0 :

$$\lim_{h\to 0^+}\frac{f(h)-f(0)}{h} = \lim_{h\to 0^+}\frac{\cos h - 1}{h} = 0,$$

$$\lim_{h\to 0^-}\frac{f(h)-f(0)}{h} = \lim_{h\to 0^+}\frac{ah+1-1}{h} = a$$

Therefore, f is differentiable at 0 if $a = 0$ and $b = 1$.

(b)



SECTION 3.7

16.

$$\sin^2 x + \cos^2 y = 1$$
$$2\,\sin x\,\cos x - 2\,\cos y\,\sin y\frac{dy}{dx} = 0$$
$$\sin 2x - \sin 2y\,\frac{dy}{dx} = 0$$
$$\frac{dy}{dx} = \frac{\sin 2x}{\sin 2y}$$
$$\frac{d^2y}{dx^2} = \frac{\sin 2y(\cos 2x)2 - \sin 2x(\cos 2y)2\dfrac{dy}{dx}}{\sin^2 2y}$$

Substituting $\dfrac{dy}{dx} = \dfrac{\sin 2x}{\sin 2y}$ and using the double angle formulas, we find that

$$\frac{d^2y}{dx^2} = \frac{8\left[\cos^2 x\cos^2 y(\sin^2 y - \sin^2 x) - \sin^2 x\sin^2 y(\cos^2 y - \cos^2 x)\right]}{\sin^3 2y} = 0$$

since $\sin^2 x = \sin^2 y$ and $\cos^2 x = \cos^2 y$ from the original equation.

20. $x = \sin^2 y \qquad 1 = 2\,\sin y\,\cos y\dfrac{dy}{dx} = \sin 2y\dfrac{dy}{dx}.$

At $(1/2, \pi/4)$, we get $\dfrac{dy}{dx} = 1$. Differentiating again,

$$0 = 2\,\cos 2y\left(\frac{dy}{dx}\right)^2 + \sin 2y\frac{d^2y}{dx^2}.$$

At $(1/2, \pi/4)$, we get $\dfrac{d^2y}{dx^2} = 0.$

55. Differentiate the equation $(x^2+y^2)^2 = x^2 - y^2$ implicitly with respect to x :

$$2(x^2+y^2)\left(2x+2y\,\frac{dy}{dx}\right) = 2x - 2y\,\frac{dy}{dx}$$

Now set $dy/dx = 0$. This gives

$$2x(x^2+y^2) = x$$
$$x^2+y^2 = \frac{1}{2} \quad (x \neq 0)$$

Substituting this result into the original equation, we get

$$x^2 - y^2 = \frac{1}{4}$$

Now

$$\begin{array}{l} x^2+y^2 = 1/2 \\ x^2-y^2 = 1/4 \end{array} \Rightarrow\ x = \pm\,\frac{\sqrt{6}}{4}, \quad y = \pm\,\frac{\sqrt{2}}{4}$$

Thus, the points on the curve at which the tangent line is horizontal are:

$$(\sqrt{6}/4, \sqrt{2}/4), \quad (\sqrt{6}/4, -\sqrt{2}/4), \quad (-\sqrt{6}/4, \sqrt{2}/4), \quad (-\sqrt{6}/4, -\sqrt{2}/4).$$

58. The circle has equation $x^2+(y-a)^2 = 1$ and $2x+2(y-a)\dfrac{dy}{dx} = 0$. Thus $\dfrac{dy}{dx} = -\dfrac{x}{y-a}$.

A tangent line to the parabola has slope $\dfrac{dy}{dx} = 4x$. Now

$$4x = -\frac{x}{y-a} \quad\Longrightarrow\quad x(4y-4a+1) = 0 \quad\Longrightarrow\quad 4y-4a+1 = 0 \text{ since } x \neq 0$$

It follows that

$$y = a - \frac{1}{4} \quad\Longrightarrow\quad x = \pm\frac{\sqrt{15}}{4} \quad\Longrightarrow\quad y = \frac{15}{8}$$

Points of intersection: $\left(\pm\,\dfrac{\sqrt{15}}{4}, \dfrac{15}{8}\right)$