SECTION 3.3

20. $$\frac{d}{du}\left[u^2(1-u^2)(1-u^3)\right] = \frac{d}{du}[u^2 - u^4 - u^5 + u^7] = 2u - 4u^3 - 5u^4 + 7u^6$$

22. $$\frac{d}{dx}\left(\frac{x^3+x^2+x-1}{x^3-x^2+x+1}\right) = \frac{(x^3-x^2+x+1)(3x^2+2x+1)-(x^3+x^2+x-1)(3x^2-2x+1)}{(x^3-x^2+x+1)^2}$$
$$= \frac{-2x^4+8x^2+2}{(x^3-x^2+x+1^2)}$$

40. $$\frac{d^2}{dx^2}\left[(x^2-3x)\frac{d}{dx}(x+x^{-1})\right] = \frac{d^2}{dx^2}\left[(x^2-3x)(1-x^{-2})\right]$$
$$= \frac{d^2}{dx^2}[x^2-3x-1+3x^{-1}]$$
$$= \frac{d}{dx}(2x-3-3x^{-2}) = 2+6x^{-3}$$

54. Let $g(x) = \begin{cases} x^3 & x \geq 0 \\ 0 & x < 0 \end{cases}$

(a) $g'_+(0) = \lim\limits_{h \to 0^+} \dfrac{g(0+h) - g(0)}{h} = \lim\limits_{h \to 0^+} \dfrac{h^3 - 0}{h} = 0,$

$g'_-(0) = \lim\limits_{h \to 0^-} \dfrac{g(0+h) - g(0)}{h} = \lim\limits_{h \to 0^-} \dfrac{0}{h} = 0$

Therefore, g is differentiable at 0 and $g'(0) = 0$.

$g'(x) = \begin{cases} 3x^2 & x \geq 0 \\ 0 & x < 0 \end{cases}$

$g''_+(0) = \lim\limits_{h \to 0^+} \dfrac{g'(0+h) - g,(0)}{h} = \lim\limits_{h \to 0^+} \dfrac{3h^2 - 0}{h} = 0,$

$g''_-(0) = \lim\limits_{h \to 0^-} \dfrac{g'(0+h) - g'(0)}{h} = \lim\limits_{h \to 0^-} \dfrac{0}{h} = 0$

Therefore, g' is differentiable at 0 and $g''(0) = 0$.