SECTION 3.1

31. $f'(-1)$ does not exist

$$\lim_{h\to 0^-}\frac{f(-1+h)-f(-1)}{h}=\lim_{h\to 0^-}\frac{h-0}{h}=1$$

$$\lim_{h\to 0^+}\frac{f(-1+h)-f(-1)}{h}=\lim_{h\to 0^+}\frac{h^2-0}{h}=0$$

40. (a) $f'(x)=\begin{cases}2(x+1), & x<0\\ 2(x-1), & x>0.\end{cases}$

(b) $\displaystyle\lim_{h\to 0^-}\frac{f(0+h)-f(0)}{h}=\lim_{h\to 0^-}\frac{(h+1)^2-1}{h}=\lim_{h\to 0^-}(h+2)=2,$

$$\lim_{h\to 0^+}\frac{f(0+h)-f(0)}{h}=\lim_{h\to 0^+}\frac{(h-1)^2-1}{h}=\lim_{h\to 0^+}(h-2)=-2.$$

42. Continuity at $x=2 \implies 4B+2A=2$; differentiability at $x=2 \implies 4B+A=4$; $A=-2,\ B=\frac{3}{2}$

59. (a) Since $|\sin(1/x)|\le 1$ it follows that

$$-x\le f(x)\le x \qquad \text{and} \qquad -x^2\le g(x)\le x^2$$

Thus $\displaystyle\lim_{x\to 0} f(x)=f(0)=0$ and $\displaystyle\lim_{x\to 0} g(x)=g(0)=0$, which implies that f and g are continuous at 0.

(b) $\displaystyle\lim_{h\to 0}\frac{h\sin(1/h)-0}{h}=\lim_{h\to 0}\sin(1/h)$ does not exist.

(c) $\displaystyle\lim_{h\to 0}\frac{h^2\sin(1/h)-0}{h}=\lim_{h\to 0}h\sin(1/h)=0$. Thus g is differentiable at 0 and $g'(0)=0$.

SECTION 3.2

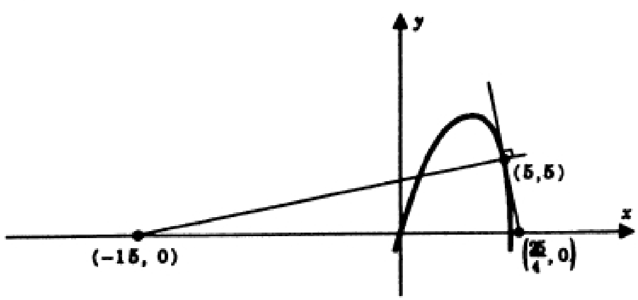
28. $f'(x) = 6xh(x) + 3x^2h'(x) - 5; \quad f'(0) = -5$

29. $f'(x) = h'(x) + \dfrac{h'(x)}{[h(x)]^2}, \quad f'(0) = h'(0) + \dfrac{h'(0)}{[h(0)]^2} = 2 + \dfrac{2}{3^2} = \dfrac{20}{9}$

30. $f'(x) = h'(x) + \dfrac{h(x) - xh'(x)}{h^2(x)}; \quad f'(0) = h'(0) + \dfrac{h(0)}{h^2(0)} = 2 + \dfrac{1}{3} = \dfrac{7}{3}$

50. First we need, $\lim\limits_{x\to -1^-} f(x) = \lim\limits_{x\to -1^+} f(x), \implies A + B = -B - A + 4$ or $A + B = 2.$
Next we need, $\lim\limits_{x\to -1^-} f'(x) = \lim\limits_{x\to -1^+} f'(x) \implies -2A = 5B + A$ or $3A + 5B = 0.$
Solving these equations gives $A = 5,\ B = -3.$

51.



slope of tangent at $(5,5)$ is $f'(5) = -4$
tangent $y - 5 = -4(x - 5)$ intersects
x-axis at $(\frac{25}{4}, 0)$
normal $y - 5 = \frac{1}{4}(x - 5)$ intersects
x-axis at $(-15, 0)$
area of triangle is
$\frac{1}{2}(5)(15 + \frac{25}{4}) = \frac{425}{8}$

54. First, $f(1) = 0$ and $f'(1) = 3 \implies A + B + C + D = 0$ and $3A + 2B + C = 3$
Next, $f(2) = 9$ and $f'(2) = 18 \implies 8A + 4B + 2C + D = 9$ and $12A + 4B + C = 18$
Solving these equations gives $A = 3,\ B = -6,\ C = 6,\ D = -3.$