SECTION 7.7

48. $$\int_4^6 \frac{dx}{(x-3)\sqrt{x^2-6x+8}} = \int_4^6 \frac{dx}{(x-3)\sqrt{(x-3)^2-1}} = [\operatorname{arcsec}(x-3)]_4^6 = \operatorname{arcsec} 3$$

49. $$\int_{-3}^{-2} \frac{dx}{\sqrt{4-(x+3)^2}} = \left[\arcsin\left(\frac{x+3}{2}\right)\right]_{-3}^{-2} = \frac{\pi}{6}$$

50. $$\int_{\ln 2}^{\ln 3} \frac{e^{-x}}{\sqrt{1-e^{-2x}}}\,dx = \left[-\arcsin e^{-x}\right]_{\ln 2}^{\ln 3} = \arcsin\left(\frac{1}{2}\right) - \arcsin\left(\frac{1}{3}\right) = \frac{\pi}{6} - \arcsin\left(\frac{1}{3}\right)$$

53. $$\left\{\begin{array}{c} u = x^2 \\ du = 2x\,dx \end{array}\right\}; \quad \int \frac{x}{\sqrt{1-x^4}}\,dx = \frac{1}{2}\int \frac{du}{\sqrt{1-u^2}} = \frac{1}{2}\arcsin u + C = \frac{1}{2}\arcsin x^2 + C$$

54. $$\int \frac{\sec^2 x}{\sqrt{9-\tan^2 x}}\,dx = \int \frac{du}{\sqrt{9-u^2}} = \arcsin\left(\frac{u}{3}\right) + C = \arcsin\left(\frac{\tan x}{3}\right) + C$$

60. $$\int \frac{\arctan x}{1+x^2}\,dx = \int u\,du = \frac{u^2}{2} + C = \frac{1}{2}(\arctan x)^2 + C$$

61. $$\left\{\begin{array}{c} u = \ln x \\ du = \frac{1}{x}\,dx \end{array}\right\}; \quad \int \frac{dx}{x\sqrt{1-(\ln x)^2}} = \int \frac{du}{\sqrt{1-u^2}} = \arcsin u + C = \arcsin(\ln x) + C$$

SECTION 8.2

9. $$\int x \ln\sqrt{x}\;dx = \frac{1}{2}\int x\ln x\,dx = \frac{1}{2}\left[\frac{1}{2}x^2\ln x - \frac{1}{2}\int x\,dx\right]$$

$$\boxed{\begin{array}{ll} u = \ln x, & dv = x\,dx \\ du = \dfrac{dx}{x}, & v = \dfrac{1}{2}x^2 \end{array}} \qquad = \tfrac{1}{4}x^2\ln x - \tfrac{1}{8}x^2 + C$$

$$\int_1^{e^2} x\,\ln\sqrt{x}\,dx = \left[\tfrac{1}{4}x^2\ln x - \tfrac{1}{8}x^2\right]_1^{e^2} = \tfrac{3}{8}e^4 + \tfrac{1}{8}$$

10. $$\left\{\begin{array}{c|c} u = x+1 & u(0)=1 \\ du = dx & u(3)=4 \end{array}\right\} \quad \int_0^3 x\sqrt{x+1}\,dx = \int_1^4 (u-1)\sqrt{u}\,du = \left[\frac{2}{5}u^{5/2} - \frac{2}{3}u^{3/2}\right]_1^4 = \frac{116}{15}$$

28. $$\int e^{3x}\cos 2x\,dx = \frac{1}{2}e^{3x}\sin 2x - \int \frac{3}{2}e^{3x}\sin 2x\,dx$$

$$\boxed{\begin{array}{ll} u = e^{3x} & dv = \cos 2x\,dx \\ du = 3e^{3x}\,dx & v = \dfrac{1}{2}\sin 2x \end{array}} \quad = \frac{1}{2}e^{3x}\sin 2x + \frac{3}{4}e^{3x}\cos 2x - \int \frac{9}{4}e^{3x}\cos 2x\,dx$$

$$\Longrightarrow \frac{13}{4}\int e^{3x}\cos 2x\,dx = \frac{1}{2}e^{3x}\sin 2x + \frac{3}{4}e^{3x}\cos 2x$$

$$\Longrightarrow \int e^{3x}\cos 2x\,dx = \frac{2}{13}e^{3x}\sin 2x + \frac{3}{13}e^{3x}\cos 2x + C$$

29.

$$\{t = x^2, \quad dt = 2x\,dx\}; \qquad \int x^3 \sin x^2\,dx = \frac{1}{2}\int t \sin t\,dt$$

$$\boxed{\begin{array}{ll} u = t, & dv = \sin t\,dt \\ du = dt, & v = -\cos t \end{array}}$$

$$= \frac{1}{2}\left[-t\cos t + \int \cos t\,dt\right]$$

$$= \tfrac{1}{2}\left(-t\cos t + \sin t\right) + C$$

$$= -\tfrac{1}{2}x^2\cos x^2 + \tfrac{1}{2}\sin x^2 + C$$

30.

$$\int x^3 \sin x\,dx = -x^3\cos x + \int 3x^2\cos x\,dx$$

$$\boxed{\begin{array}{ll} u = x^3 & dv = \sin x\,dx \\ du = 3x^2\,dx & v = -\cos x \end{array}}$$

$$= -x^3\cos x + 3x^2\sin x - \int 6x\sin x\,dx$$

$$= -x^3\cos x + 3x^2\sin x + 6x\cos x - \int 6\cos x\,dx$$

$$= -x^3\cos x + 3x^2\sin x + 6x\cos x - 6\sin x + C$$

31.

$$\left\{\begin{array}{l|l} u = 2x & x = 0 \implies u = 0 \\ du = 2\,dx & x = 1/4 \implies u = 1/2 \end{array}\right\};$$

$$\int_0^{1/4} \arcsin 2x\,dx = \frac{1}{2}\int_0^{1/2} \arcsin u\,du$$

$$= \frac{1}{2}\left[u\arcsin u + \sqrt{1-u^2}\right]_0^{1/2} \qquad \text{[by(8.2.5)]}$$

$$= \frac{1}{2}\left[\frac{\pi}{12} + \frac{\sqrt{3}}{2} - 1\right] = \frac{\pi}{24} + \frac{\sqrt{3}-2}{4}$$

32.

$$\left\{\begin{array}{l} u = \arcsin 2x \\ du = \dfrac{2}{\sqrt{1-4x^2}}\,dx \end{array}\right\} \qquad \int \frac{\arcsin 2x}{\sqrt{1-4x^2}}\,dx = \frac{1}{2}\int u\,du = \frac{1}{4}u^2 + C = \frac{1}{4}\left(\arcsin 2x\right)^2 + C$$

38.

$$\boxed{\begin{array}{ll} u = \cos(\ln x) & dv = dx \\ du = -\dfrac{\sin(\ln x)}{x}\,dx & v = x \end{array}} \quad \int \cos(\ln x)\,dx = x\cos(\ln x) + \int \sin(\ln x)\,dx$$

$$\boxed{\begin{array}{ll} u = \sin(\ln x) & dv = dx \\ du = \dfrac{\cos(\ln x)}{x}\,dx & v = x \end{array}} = x\cos(\ln x) + x\sin(\ln x) - \int \cos(\ln x)\,dx$$

$$\Longrightarrow \int \cos(\ln x)\,dx = \frac{1}{2}x\left[\cos(\ln x) + \sin(\ln x)\right] + C$$

39.

$$\int \sin(\ln x)\,dx = x\ \sin(\ln x) - \int \cos(\ln x)\,dx$$

$$\boxed{\begin{array}{ll} u = \sin(\ln x), & dv = dx \\ du = \cos(\ln x)\dfrac{1}{x}\,dx, & v = x \end{array}} \quad = x\ \sin(\ln x) - x\cos(\ln x) - \int \sin(\ln x)\,dx$$

$$\boxed{\begin{array}{ll} u = \cos(\ln x), & dv = dx \\ du = -\ \sin(\ln x)\dfrac{1}{x}\,dx, & v = x \end{array}}$$

Adding $\int \sin(\ln x)\,dx$ to both sides, we get

$$2\int \sin(\ln x)\,dx = x\ \sin(\ln x) - x\cos(\ln x)$$

so that

$$\int \sin(\ln x)\,dx = \tfrac{1}{2}\left[x\ \sin(\ln x) - x\cos(\ln x)\right] + C$$

40.

$$\boxed{\begin{array}{ll} u = (\ln x)^2 & dv = x^2 dx \\ du = 2\dfrac{\ln x}{x}\,dx & v = \dfrac{x^3}{3} \end{array}} \quad \int_1^{2e} x^2(\ln x)^2\,dx = \left[\frac{x^3}{3}(\ln x)^2\right]_1^{2e} - \int_1^{2e} \frac{2x^2}{3}\ln x\,dx$$

$$\boxed{\begin{array}{ll} u = \ln x & dv = \dfrac{2x^2}{3}\,dx \\ du = \dfrac{1}{x}\,dx & v = \dfrac{2x^3}{9} \end{array}} \quad = \left[\frac{x^3}{3}(\ln x)^2 - \frac{2x^3}{9}\ln x\right]_1^{2e} + \int_1^{2e} \frac{2x^2}{9}\,dx$$

$$= \left[\frac{x^3}{3}(\ln x)^2 - \frac{2x^3}{9}\ln x + \frac{2x^3}{27}\right]_1^{2e}$$

$$= \frac{8e^3}{3}\left[(\ln 2e)^2 - \frac{2}{3}\ln 2e + \frac{2}{9}\right] - \frac{2}{27}$$

SECTION 8.3

3. $\displaystyle\int_0^{\pi/6} \sin^2 3x\,dx = \int_0^{\pi/6} \frac{1-\cos 6x}{2}\,dx = \left[\frac{1}{2}x - \frac{1}{12}\sin 6x\right]_0^{\pi/6} = \frac{\pi}{12}$

4. $\displaystyle\int \cos^3 x\,dx = \int (1-\sin^2 x)\cos x\,dx = \sin x - \frac{1}{3}\sin^3 x + C$

8.
$$\begin{aligned}\int \sin^2 x\cos^4 x\,dx &= \int (\sin x\cos x)^2\cos^2 x\,dx = \int \frac{1}{4}\sin^2 2x(\frac{1}{2}+\frac{1}{2}\cos 2x)\,dx\\ &= \frac{1}{8}\int \sin^2 2x\,dx + \frac{1}{8}\int \sin^2 2x\cos 2x\,dx\\ &= \frac{1}{8}\left(\frac{1}{2}x - \frac{1}{8}\sin 4x\right) + \frac{1}{48}\sin^3 2x + C\end{aligned}$$

20.

$$\int_0^{\pi/2} \cos^4 x\,dx = \left[\frac{1}{4}\cos^3 x \sin x\right]_0^{\pi/2} + \frac{3}{4}\int_0^{\pi/2} \cos^2 x\,dx$$

$$= \left[\frac{1}{4}\cos^3 x \sin x + \frac{3}{8}x + \frac{3}{16}\sin 2x\right]_0^{\pi/2} = \frac{3\pi}{16}$$

21.

$$\int \sin^6 x\,dx = \int \left(\frac{1-\cos 2x}{2}\right)^3\,dx$$

$$= \frac{1}{8}\int (1 - 3\cos 2x + 3\cos^2 2x - \cos^3 2x)\,dx$$

$$= \frac{1}{8}\int \left[1 - 3\cos 2x + 3\left(\frac{1+\cos 4x}{2}\right) - \cos 2x\,(1-\sin^2 2x)\right]\,dx$$

$$= \frac{1}{8}\int \left(\frac{5}{2} - 4\cos 2x + \frac{3}{2}\cos 4x + \sin^2 2x \cos 2x\right)\,dx$$

$$= \tfrac{5}{16}x - \tfrac{1}{4}\sin 2x + \tfrac{3}{64}\sin 4x + \tfrac{1}{48}\sin^3 2x + C$$

22.

$$\int \cos^5 x \sin^5 x\,dx = \int \cos^5 x \sin^4 x \sin x\,dx = \int \cos^5 x(1-\cos^2 x)^2 \sin x\,dx$$

$$= \int \cos^5 x(1 - 2\cos^2 x + \cos^4 x)\sin x\,dx$$

$$= -\frac{\cos^6 x}{6} + \frac{1}{4}\cos^8 x - \frac{1}{10}\cos^{10} x + C$$

Equivalently, $\int \cos^5 x \sin^4 x \sin x\,dx$ gives $-\tfrac{1}{6}\cos^x + \tfrac{1}{4}\cos^8 x - \tfrac{1}{10}\cos^{10} x + C.$

34.

$$\int_0^{1/2} \cos \pi x \cos \frac{\pi}{2} x \, dx = \frac{1}{2} \int_0^{1/2} \left[\cos\left(\pi x - \frac{\pi}{2} x\right) + \cos\left(\pi x + \frac{\pi}{2} x\right) \right] \, dx$$
$$= \frac{1}{2} \int_0^{1/2} \left(\cos \frac{\pi}{2} x + \cos \frac{3\pi}{2} x \right) \, dx = \frac{1}{2} \left[\frac{2}{\pi} \sin \frac{\pi}{2} x + \frac{2}{3\pi} \sin \frac{3\pi}{2} x \right]_0^{1/2}$$
$$= \frac{2\sqrt{2}}{3\pi}$$

35.

$$\int_0^{\pi/4} \cos 4x \sin 2x \, dx = \int_0^{\pi/4} \frac{1}{2} (\sin 6x - \sin 2x) \, dx$$
$$= \left[-\tfrac{1}{12} \cos 6x + \tfrac{1}{4} \cos 2x \right]_0^{\pi/4} = -\tfrac{1}{6}$$

36.

$$\int (\sin 3x - \sin x)^2 \, dx = \int (\sin^2 3x - 2 \sin 3x \sin x + \sin^2 x) \, dx$$
$$= \int (\frac{1}{2} - \frac{1}{2} \cos 6x - \cos 2x + \cos 4x + \frac{1}{2} - \frac{1}{2} \cos 2x) \, dx$$
$$= x - \frac{1}{12} \sin 6x + \frac{1}{4} \sin 4x - \frac{3}{4} \sin 2x + C$$

37.

$$\int \tan^4 x \sec^4 x \, dx = \int \tan^4 x \, (\tan^2 x + 1) \sec^2 x \, dx$$
$$= \int (\tan^6 x + \tan^4 x) \sec^2 x \, dx$$
$$= \frac{1}{7} \tan^7 x + \frac{1}{5} \tan^5 x + C$$

SECTION 8.4

10. $\left\{\begin{array}{c} u = a^2 + x^2 \\ dx = 2x\,dx \end{array}\right\}$; $\quad \displaystyle\int \frac{x}{a^2+x^2}\,dx = \frac{1}{2}\int \frac{du}{u} = \frac{1}{2}\ln|u| + C$

$$= \frac{1}{2}\ln(a^2+x^2) + C$$

11. $\left\{\begin{array}{c} u = 4 - x^2 \\ du = -2x\,dx \end{array}\right\}$; $\quad \displaystyle\int x\sqrt{4-x^2}\,dx = -\frac{1}{2}\int u^{1/2}\,du = -\frac{1}{3}u^{3/2} + C$

$$= -\frac{1}{3}(4-x^2)^{3/2} + C$$

12. $\left\{\begin{array}{c} x = 4\sin u \\ dx = 4\cos u\,du \end{array}\right\}$; $\quad \displaystyle\int \frac{x^3}{\sqrt{16-x^2}}\,dx = \frac{256\sin^3 u\cos u}{\sqrt{16-16\sin^2 u}}\,du = 64\int \sin^3 u\,du$

$$= 64\int (1-\cos^2 u)\sin u\,du = 64\left(-\cos u + \frac{\cos^3 u}{3}\right) + C$$

$$= \frac{1}{3}\left(\sqrt{16-x^2}\right)^3 - 16\sqrt{16-x^2} + C$$

$$\int_0^2 \frac{x^3}{\sqrt{16-x^2}}\,dx = \left[\frac{1}{3}(16-x^2)^{3/2} + 64\sqrt{16-x^2}\right]_0^2 = \frac{128}{3} - 12\sqrt{12}$$

32. $\left\{ \begin{aligned} x+2 &= 3\tan u \\ dx &= 3\sec^2 u\, du \end{aligned} \right\};$

$$\begin{aligned} \int \frac{x+2}{\sqrt{x^2+4x+13}}\,dx &= \int \frac{x+2}{\sqrt{(x+2)^2+9}}\,dx \\ &= \int \frac{3\tan u}{3\sec u}\cdot 3\sec^2 u\,du = 3\int \sec u \tan u\,du \\ &= 3\sec u + C = \sqrt{x^2+4x+13} + C \end{aligned}$$

33. $\left\{ \begin{aligned} x+1 &= 2\tan u \\ dx &= 2\sec^2 u\, du \end{aligned} \right\};$

$$\begin{aligned} \int \frac{x}{(x^2+2x+5)^2}\,dx &= \int \frac{x}{[(x+1)^2+4]^2}\,dx \\ &= \int \frac{2\tan u - 1}{(4\sec^2 u)^2} 2\sec^2 u\,du \\ &= \frac{1}{8}\int \frac{2\tan u - 1}{\sec^2 u}\,du \\ &= \frac{1}{8}\int (2\sin u\cos u - \cos^2 u)\,du \\ &= \frac{1}{8}\int \left(2\sin u\cos u - \frac{1+\cos 2u}{2}\right)\,du \\ &= \frac{1}{8}\left(\sin^2 u - \frac{u}{2} - \frac{\sin 2u}{4}\right) + C \end{aligned}$$