SECTION 7.3

29. $\left\{u = \ln x,\ du = \dfrac{dx}{x}\right\};\quad \displaystyle\int \frac{dx}{x\,(\ln x)^2} = \int \frac{du}{u^2} = -\frac{1}{u} + C = -\frac{1}{\ln x} + C$

30. $\left\{\begin{array}{l} u = 1 + \sec 2x \\ du = 2\sec\ 2x\ \tan 2x\, dx \end{array}\right\};$

$$\int \frac{\sec 2x \tan 2x}{1 + \sec 2x}\, dx = \frac{1}{2}\int \frac{1}{1+u}\, du = \frac{1}{2}\ln|1+u| + C = \frac{1}{2}\ln|1 + \sec 2x| + C$$

31. $\left\{\begin{array}{l} u = \sin x + \cos x \\ du = (\cos x - \sin x)\, dx \end{array}\right\};$

$$\int \frac{\sin x - \cos x}{\sin x + \cos x}\, dx = -\int \frac{1}{u}\, du = -\ln\ |u| + C = -\ln|\sin x + \cos x| + C$$

32. $\left\{\begin{array}{l} u = \sqrt{x} \\ du = \dfrac{1}{2\sqrt{x}}\, dx \end{array}\right\};\qquad \displaystyle\int \frac{1}{\sqrt{x}(1+\sqrt{x})}\, dx = 2\int \frac{du}{1+u} = 2\ln|1+u| + C = 2\ln|1+\sqrt{x}| + C$

33. $\left\{u = 1 + x\sqrt{x}\,,\ du = \dfrac{3}{2}x^{1/2}\, dx\right\};\quad \displaystyle\int \frac{\sqrt{x}}{1+x\sqrt{x}}\, dx = \frac{2}{3}\int \frac{du}{u} = \frac{2}{3}\ln|u| + C$

$$= \frac{2}{3}\ln\left|1 + x\sqrt{x}\ \right| + C$$

34. $\left\{u = \ln x, du = \dfrac{1}{x}dx\right\};\quad \displaystyle\int \frac{\tan(\ln x)}{x}\, dx = \int \tan u\, du = \ln|\sec u| + C = \ln|\sec(\ln x)| + C$

50. $\ln|g(x)| = \ln|x| + \ln|x+a| + \ln|x+b| + \ln|x+c|$

$$\frac{g'(x)}{g(x)} = \frac{1}{x} + \frac{1}{x+a} + \frac{1}{x+b} + \frac{1}{x+c}$$

$$g'(x) = x(x+a)(x+b)(x+c)\left(\frac{1}{x} + \frac{1}{x+a} + \frac{1}{x+b} + \frac{1}{x+c}\right); \quad g'(-b) = -b(a-b)(c-b)$$

51. $\ln|g(x)| = 4\ln|x| + \ln|x-1| - \ln|x+2| - \ln(x^2+1)$

$$\frac{g'(x)}{g(x)} = \frac{4}{x} + \frac{1}{x-1} - \frac{1}{x+2} - \frac{2x}{x^2+1}$$

$$g'(x) = \frac{x^4(x-1)}{(x+2)(x^2+1)}\left(\frac{4}{x} + \frac{1}{x-1} - \frac{1}{x+2} - \frac{2x}{x^2+1}\right); \quad g'(0) = 0$$

SECTION 7.4

37. $\left\{\begin{array}{l} u = e^x + 1 \\ du = e^x\,dx \end{array}\right\}$; $\displaystyle\int \frac{e^x}{\sqrt{e^x+1}}\,dx = \int \frac{du}{\sqrt{u}} = \int u^{-1/2}\,du = 2u^{1/2} + C = 2\sqrt{e^x+1} + C$

38. $\left\{\begin{array}{l} u = e^{ax^2} + 1 \\ du = 2axe^{ax^2}\,dx \end{array}\right\}$; $\displaystyle\int \frac{xe^{ax^2}}{e^{ax^2}+1}\,dx = \frac{1}{2a}\int \frac{du}{u} = \frac{1}{2a}\ln|u| + C = \frac{1}{2a}\ln(e^{ax^2}+1) + C$

39. $\left\{\begin{array}{l} u = 2e^{2x} + 3 \\ du = 4e^{2x}\,dx \end{array}\right\}$; $\displaystyle\int \frac{e^{2x}}{2e^{2x}+3}\,dx = \frac{1}{4}\int \frac{du}{u} = \frac{1}{4}\ln u + C = \frac{1}{4}\ln(2e^{2x}+3) + C$

40. $\left\{\begin{array}{l} u = e^{-2x} \\ du = -2e^{-2x}\,dx \end{array}\right\}$; $\displaystyle\int \frac{\sin(e^{-2x})}{e^{2x}}\,dx = -\frac{1}{2}\int \sin u\,du = \frac{1}{2}\cos u + C = \frac{1}{2}\cos(e^{-2x}) + C$

41. $\{\,u = \sin x, \quad du = \cos x\,dx\,\}$; $\displaystyle\int \cos x\,e^{\sin x}\,dx = \int e^u\,du = e^u + C = e^{\sin x} + C$

51. $\displaystyle\int_0^1 x(e^{x^2}+2)\,dx = \int_0^1 (xe^{x^2}+2x)\,dx = \left[\frac{1}{2}e^{x^2} + x^2\right]_0^1 = \left(\tfrac{1}{2}e + 1\right) - \left(\tfrac{1}{2} + 0\right) = \tfrac{1}{2}(e+1$

52. $\displaystyle\int_0^{\ln\frac{\pi}{4}} e^x \sec e^x\,dx = \left[\ln|\sec e^x + \tan e^x|\right]_0^{\ln\frac{\pi}{4}} = \ln\left(\frac{\sqrt{2}+1}{2}\right).$

SECTION 7.5

49. $y = (\sin x)^{\cos x}$

$$\ln y = (\cos x)(\ln[\sin x])$$

$$\frac{1}{y}\frac{dy}{dx} = (-\sin x)(\ln[\sin x]) + (\cos x)\left(\frac{1}{\sin x}\right)(\cos x)$$

$$\frac{dy}{dx} = (\sin x)^{\cos x}\left[\frac{\cos^2 x}{\sin x} - (\sin x)(\ln[\sin x])\right]$$

50. $y = x^{x^2}$

$$\ln y = x^2 \ln x$$

$$\frac{1}{y}\frac{dy}{dx} = 2x \ln x + x^2 \cdot \frac{1}{x}$$

$$\frac{dy}{dx} = x^{x^2+1}(2 \ln x + 1)$$

51. $y = x^{2^x}$

$$\ln y = 2^x \ln x$$

$$\frac{1}{y}\frac{dy}{dx} = 2^x \ln 2 \ln x + 2^x\left(\frac{1}{x}\right)$$

$$\frac{dy}{dx} = x^{2^x}\left[2^x \ln 2 \ln x + \frac{2^x}{x}\right]$$

52. $y = (\tan x)^{\sec x}$

$$\ln y = \sec x \ln(\tan x)$$

$$\frac{1}{y}\frac{dy}{dx} = \sec x \tan x \ln(\tan x) + \sec x \cdot \frac{\sec^2 x}{\tan x}$$

$$\frac{dy}{dx} = (\tan x)^{\sec x}\left[\sec x \tan x \ln(\tan x) + \sec^3 x \cot x\right]$$

61. $\displaystyle\int_1^4 \frac{dx}{x \ln 2} = [\log_2 x]_1^4 = \log_2 4 - 0 = 2$

62. $\displaystyle\int_0^2 p^{x/2}\,dx = \left[\frac{2p^{x/2}}{\ln p}\right]_0^2 = \frac{2(p-1)}{\ln p}$

63. $\displaystyle\int_0^1 x10^{1+x^2}\,dx = \left[\frac{1}{2\ln 10}10^{1+x^2}\right]_0^1 = \frac{1}{2\ln 10}(100 - 10) = \frac{45}{\ln 10}$

64. $\displaystyle\int_0^1 \frac{5p^{\sqrt{x+1}}}{\sqrt{x+1}}\,dx = \left[\frac{10p^{\sqrt{x+1}}}{\ln p}\right]_0^1 = \frac{10}{\ln p}(p^{\sqrt{2}} - p)$

65. $\displaystyle\int_0^1 (2^x + x^2)\,dx = \left[\frac{2^x}{\ln 2} + \frac{x^3}{3}\right]_0^1 = \frac{1}{3} + \frac{1}{\ln 2}$