SECTION 5.2

5. $L_f(P) = 1(\frac{1}{2}) + \frac{9}{8}(\frac{1}{2}) = \frac{17}{16}$, $\quad U_f(P) = \frac{9}{8}(\frac{1}{2}) + 2(\frac{1}{2}) = \frac{25}{16}$

SECTION 5.3

29. (a) $F(0) = 0$

(b) $F'(0) = 2 + \dfrac{\sin 2(0)}{1+0^2} = 2$

(c) $F''(0) = \dfrac{(1+0)^2 2\cos 2(0) - \sin 2(0)(2)(0)}{(1+0)^2} = 2$

36. (a) $F'(x) = x\int_1^x f(u)\,du$ $\quad$ (b) $F'(1) = 0$

(c) $F''(x) = xf(x) + \int_1^x f(u)\,du$ $\quad$ (d) $F''(1) = f(1)$

SECTION 5.4

53. (a) $x(t) = \displaystyle\int_0^t (10u - u^2)\,du = \left[5u^2 - \frac{u^3}{3}\right]_0^t = 5t^2 - \frac{t^3}{3}, \quad 0 \le t \le 10$

(b) $v'(t) = 10 - 2t$; v has an absolute maximum at $t = 5$. The object's position at $t = 5$ is $x(5) = \dfrac{250}{3}$.

56. $\displaystyle\int_{-2}^4 f(x)\,dx = \int_{-2}^0 (2 + x^2)\,dx + \int_0^4 (\frac{1}{2}x + 2)\,dx = \left[2x + \frac{x^3}{3}\right]_{-2}^0 + \left[\frac{x^2}{4} + 2x\right]_0^4 = \frac{56}{3}$

62. Let $F(x) = f^2(x)$. Then $F'(x) = 2f(x)f'(x)$.

Thus $\displaystyle\int_a^b f(t)f'(t)\,dt = \frac{1}{2}\int_a^b F'(t)\,dt = \frac{1}{2}[F(b) - F(a)] = \frac{1}{2}[f^2(b) - f^2(a)].$

63. $\displaystyle\frac{d}{dx}\left[\int_a^x f(t)\,dt\right] = f(x); \quad \int_a^x \frac{d}{dt}[f(t)]\,dt = f(x) - f(a)$

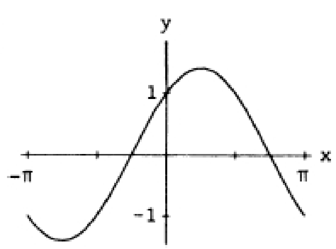
64. $\displaystyle F(x) = \int_0^x x f(t)\,dt = x\int_0^x f(t)\,dt;$ F is a product.

$\displaystyle F'(x) = x\,f(x) + \int_0^x f(t)\,dt$

SECTION 5.5

30. (a) $\displaystyle\int_{-\pi}^{\pi}(\cos x + \sin x)\,dx = [\sin x - \cos x]_{-\pi}^{\pi} = 0$

(b)



$$A = -\int_{-\pi}^{-\pi/4} f(x)\,dx + \int_{-\pi/4}^{3\pi/4} f(x)\,dx - \int_{3\pi/4}^{\pi} f(x)\,dx$$
$$= 4\sqrt{2}$$

SECTION 5.6

32. $\displaystyle f'(x) = \int(5 - 4x)\,dx = 5x - 2x^2 + C,$

$\displaystyle f(x) = \int(5x - 2x^2 + C)\,dx = \frac{5}{2}x^2 - \frac{2}{3}x^3 + Cx + K$

$\displaystyle f(1) = \frac{5}{2} - \frac{2}{3} + C + K = 1, \quad f(0) = K = -2 \implies f(x) = -\frac{2}{3}x^3 + \frac{5}{2}x^2 + \frac{7}{6}x - 2$

SECTION 5.7

10. $\left\{\begin{array}{c} u = a + bx^n \\ du = nbx^{n-1}\,dx \end{array}\right\}$; $\quad \displaystyle\int x^{n-1}\sqrt{a+bx^n}\,dx = \frac{1}{bn}\int \sqrt{u}\,du = \frac{2}{3bn}u^{3/2} + C = \frac{2}{3bn}\left(a+bx^n\right)^{3/2} + C$

16. $\left\{\begin{array}{c} u = 1 - x^4 \\ du = -4x^3\,dx \end{array}\right\}$; $\quad \displaystyle\int 2x^3(1-x^4)^{-1/4}\,dx = -\frac{1}{2}\int u^{-1/4}\,du = -\frac{2}{3}u^{3/4} + C = -\frac{2}{3}(1-x^4)^{3/4} + C$

35. $\displaystyle\int \frac{1}{\sqrt{x\sqrt{x}+x}}\,dx = \int \frac{1}{\sqrt{x}\sqrt{\sqrt{x}+1}}\,dx$

$$\left\{\begin{array}{c} u = \sqrt{x} + 1 \\ du = dx/2\sqrt{x} \end{array}\right\}; \quad \int \frac{1}{\sqrt{x}\sqrt{\sqrt{x}+1}}\,dx = 2\int u^{-1/2}\,du = 4\sqrt{u} + C = 4\sqrt{\sqrt{x}+1} + C$$

38. Set $u = x - 1$. Then $du = dx,\ x = u + 1,\ x^2 = u^2 + 2u + 1;\ u(2) = 1,\ u(5) = 4.$

$$\int_2^5 \frac{x^2}{\sqrt{x-1}}\,dx = \int_1^4 (u^{3/2} + 2u^{1/2} + u^{-1/2})\,du = \left[\frac{2}{5}\,u^{5/2} + \frac{4}{3}\,u^{3/2} + 2u^{1/2}\right]_1^4 = \frac{356}{15}$$

51. $\left\{\begin{array}{c} u = 1 + \sin x \\ du = \cos x\,dx \end{array}\right\}$; $\quad \displaystyle\int \sqrt{1+\sin x}\ \cos x\,dx = \int u^{1/2}\,du = \frac{2}{3}u^{3/2} + C = \tfrac{2}{3}(1+\sin x)^{3/2} + C$

59. $\left\{\begin{array}{c} u = 1 + \tan x \\ du = \sec^2 x\,dx \end{array}\right\}$; $\quad \displaystyle\int \frac{\sec^2 x}{\sqrt{1+\tan x}}\,dx = \int u^{-1/2}\,du = 2u^{1/2} + C = 2(1+\tan x)^{1/2} + C$

63. $\left\{\begin{array}{c} u = \tan\left(x^3 + \pi\right) \\ du = 3x^2\,\sec^2\left(x^3+\pi\right)dx \end{array}\right\}$;

$$\int x^2\,\tan\left(x^3+\pi\right)\,\sec^2(x^3+\pi)\,dx = \frac{1}{3}\int u\,du = \frac{1}{6}\,u^2 + C = \frac{1}{6}\tan^2(x^3+\pi) + C$$

65. $\left\{\begin{array}{l} u=\sin x \\ du=\cos x\,dx \end{array}\right|\left.\begin{array}{l} x=-\pi \;\Rightarrow\; u=0 \\ x=\pi \;\Rightarrow\; u=0 \end{array}\right\}$; $\displaystyle\int_{-\pi}^{\pi}\sin^4 x\,\cos x\,dx=\int_0^0 u^4\,du=0.$

68. $\displaystyle\int_0^1 \cos^2\left(\frac{\pi}{2}x\right)\sin\left(\frac{\pi}{2}x\right)\,dx=\frac{-2}{3\pi}\left[\cos^3\frac{\pi}{2}x\right]_0^1=\frac{2}{3\pi}$

SECTION 5.8

26. $\displaystyle\frac{d}{dx}\left[\int_{\sqrt{x}}^{x^2+x}\frac{dt}{2+\sqrt{t}}\right]=\frac{1}{2+\sqrt{x^2+x}}(2x+1)-\frac{1}{2+\sqrt[4]{x}}\cdot\frac{1}{2\sqrt{x}}$

35. $\displaystyle\int_{-\pi/4}^{\pi/4}(x+\sin 2x)\,dx=0$ since $f(x)=x+\sin 2x$ is an odd function.

37. $\displaystyle\int_{-\pi/3}^{\pi/3}(1+x^2-\cos x)\,dx=2\int_0^{\pi/3}(1+x^2-\cos x)\,dx$ since $f(x)=1+x^2-\cos x$ is an even function.

$$2\int_0^{\pi/3}(1+x^2-\cos x)\,dx=2\left[x+\frac{1}{3}x^3-\sin x\right]_0^{\pi/3}=\frac{2}{3}\pi+\frac{2}{81}\pi^3-\sqrt{3}$$

31. $\displaystyle H(x)=\int_{2x}^{x^3-4}\frac{x\,dt}{1+\sqrt{t}}=x\int_{2x}^{x^3-4}\frac{dt}{1+\sqrt{t}},$

$$H'(x)=x\cdot\left[\frac{3x^2}{1+\sqrt{x^3-4}}-\frac{2}{1+\sqrt{2x}}\right]+1\cdot\int_{2x}^{x^3-4}\frac{dt}{1+\sqrt{t}},$$

$$H'(2)=2\left[\frac{12}{3}-\frac{2}{3}\right]+\underbrace{\int_4^4\frac{dt}{1+\sqrt{t}}}_{=0}=\frac{20}{3}$$