MASTER THEOREM

THE PROOF OF EXACT POWERS

- $T(n) = aT(n/b) + f(n)$

LEMMA I

- Let a ≥ 1 and b>1 be constants, and let f(n) be nonnegative function defined on power of b. Define T(n) on exact power of b by the recurrence

$$T(n) = \begin{cases} O(1) & \text{if } n = 1 \\ aT(n/b) + f(n) & \text{if } n = b^j \end{cases}$$

Where j is a positive integer .

Then T (n) = $O(n^{\log_b a}) + \sum_{i=0}^{\log_b n-1} a^i f(n/b^i)$

LEMMA II

- Let a ≥1 and b>1 be constant, and let f(n) ne a nonnegative function defined on exact power of b. A function g(n) defined over exact power of b by

$$g(n) = \sum_{i=0}^{\log_b n-1} a^i f(n/b^i)$$

can then be bounded asymptotically as follows.

1. If f(n) = $O(n^{\log_b a-\varepsilon})$ for some constant ε > 0, then g(n) = $O(n^{\log_b a})$
2. If f(n) = $O(n^{\log_b a})$, then g(n) = $O(n^{\log_b a} \lg n)$
3. If af(n/b) ≤ cf(n) for some constant c < 1 and for all n ≥ b,

   then g(n) = O(f(n))

LEMMA III

- Let a ≥ 1 and b>1 be constants, and let f(n) be nonnegative function defined on power of b. Define T(n) on exact power of b by the recurrence

$$T(n) = \begin{cases} O(1) & \text{if } n = 1 \\ aT(n/b) + f(n) & \text{if } n = b^j \end{cases}$$

Then g(n) can be bounded asymptotically as follows.

1. If f(n) = $O(n^{\log_b a-\varepsilon})$ for some constant $\varepsilon > 0$, then g(n) = $O(n^{\log_b a})$
2. If f(n) = $O(n^{\log_b a})$, then g(n) = $O(n^{\log_b a}\lg n)$
3. If f(n) = $\Omega(n^{\log_b a+\varepsilon})$ for some constant $\varepsilon > 0$, and if af(n/b) $\leq$ cf(n) for some constant c<1 and all sufficiently large n,then T(n)=O(f(n))

QUESTION

We prove exact power before. How about arbitrary integer ?